Measuring the output gap, potential output growth, and natural interest rate from a semi-structural dynamic model for Peru

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Abstract

In this paper we use a calibrated version of the Quarterly Projection Model (MPT, for its acronym in Spanish) to jointly estimate the output gap, potential output growth, and natural interest rate of the Peruvian economy during most of the inflation targeting regime (between 2002 and 2017). The MPT is a semi-structural dynamic model used by the Central Reserve Bank of Peru for fore-casting and policy scenario analysis. The model functions as a multivariate filter with a sophisticated economic structure that allows us to infer the dynamics of non-observable variables from the information provided by other variables defined ex ante as observable. As the results from the Kalman filter are sensible to these variables declared as observable, we use five groups of variables to be defined as such to build probable ranges for our estimates.
The results indicate that the estimated output gap is large in amplitude and highly persistent, while potential output growth is very smooth. Therefore, most of the variation in economic activity during the inflation targeting regime can be attributed to the former. As expected from a small open economy, a historical decomposition exercise shows that output gap dynamics are mainly influenced by external factors (real and financial). The estimation of the output gap also proves that monetary policy has been extensively responsive to this leading indicator of inflation. Meanwhile, the real natural interest rate

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is estimated to be considerable stable, averaging 1.6% in the sample with only a sharp decline to 1.3% during the financial crisis.
The main finding of the paper, however, is that there has been a steady deceleration of potential output growth since 2012. A growth-accounting exercise proves that this trend is mostly explained by a reduction in total factor productivity (TFP) growth during the same time frame. Nonetheless, the drop of capital and labor contributions jointly explain almost a third part of average potential output growth slowdown between 2010-2013 and 2014-2017.

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1. INTRODUCTION

Potential (or *natural*) *output* is defined as the level of output that can be sustained indefinitely without adding pressures on inflation (Okun, 1962). Thus, periods in which inflation is stable (inflation rate on its long-term value) are associated with output on its potential level. Meanwhile, the short-term interest rate that is consistent with both inflation rate on its long-term value and output on its potential level (i.e. no transitory disturbances) is called the *natural interest rate.*

Both potential output and the natural interest rate are non-observable variables, in the sense that their dynamics can only be inferred from the behavior of other variables that can be measured (e.g. prices, gross domestic output, interest rates, exchange rate). Nonetheless, they are key components of monetary policy making. On the one hand, the difference between GDP and potential output, called the *output gap*, is considered a leading indicator of inflationary pressures. On the other hand, the natural interest rate enables policymakers to identify whether current monetary conditions are being expansionary (real interest rate below its natural level) or contractionary (real interest rate above its natural level). It is worth noting that the desirability of a specific monetary policy stance depends on inflation expectations and the stage of the business cycle, which is partly determined through the estimation of the output gap.

In this paper, we jointly estimate the output gap, potential output growth, and the natural interest rate of the Peruvian economy using quarterly data from the Inflation Targeting period (2002Q1

-2017Q4). To do so, we apply the Kalman filter on the state-space representation of a calibrated version of the Quarterly Projection Model (*MPT*, for its acronym in Spanish), a semi-structural macroeconomic model used by the Central Reserve Bank of Peru for forecasting and policy scenario analysis. The MPT follows the neo-keynesian tradition for small open economies (Phillips curve, IS curve, Taylor rule and UIP equation), but it also includes specific features of the Peruvian economy (such as partial financial dollarization and sluggish exchange rate adjustment). The Kalman filter algorithm allows us to obtain the optimal linear prediction of non-observable states using the information from other variables declared as *observable*; thus turning the MPT into a multivariate filter with an economic structure that replicates the medium-term behavior of the economy.

By construction, the results of the Kalman filter are sensible to which variables are declared as observable in the estimation process of the state variables, especially when using a calibrated model as the state equation. To account for the uncertainty risen from this feature, we use five different groups of observable variables. These groups are a combination of: real GDP growth, inflation without food and energy (core inflation), inflation expectations, impulse of business confidence (a proxy of the expected output gap), terms of trade growth, short-term interest rate (monetary policy rate), 3-Month LIBOR rate (proxy of the external interest rate), and the real effective exchange rate gap. These variables capture the main determinants of the output gap and the natural interest rate. With the different estimates we construct probable ranges for the output gap, potential output growth and natural interest rate.

From a statistical perspective, the state-space representation of the solution of a linear rational expectation macroeconomic model (such as the MPT) can be treated as a multivariate unobserved component (UC) model. In fact, the solution of the model is a statistical state vector that can be written as a restricted VAR where only some of the states are observable by the econometrician. Standard DSGE models such as Smets and Wouters (2007) introduce a measurement equation that decomposes the GDP quarterly growth rate into the first difference of the cyclical component and a stationary growth rate for the trend component. In this paper, we follow the same approach: the rational expectation solution of the MPT model is augmented with a measurement equation for GDP expressed in quarterly growth rates, with the assumption that the growth rate of the trend component follows a stationary but persistent autoregressive process.

With the results from the Kalman filter, we make two additional exercises to understand the recent behavior of the output gap and potential output growth. First, we perform a historical shock decomposition on the output gap and relate it to a narrative that we construct for its evolution considering well-know domestic and international events. Then, we adopt a growth-accounting method to decompose potential output growth into three components: (i) contribution of capital, (ii) contribution of labor, and (iii) total factor productivity (TFP) growth. This methodology assumes that aggregate output can be represented by a Cobb-Douglas function, and is useful to identify which forces are driving the trend of potential output growth.

The results show that there has been a steady decline in potential output growth since 2012. The growth-accounting exercise proves that this trend follows mostly a reduction in TFP growth. Nevertheless, the drop of capital and labor contributions also played a role, since they jointly explain almost a third part of the average potential output growth deceleration between 2010-2013 and 2014-2017. The TFP reduction may be explained by the persistent decline of terms of trade growth (a sharp fall began in 2013 with the taper tantrum), or by the lack of structural reforms throughout the last decade, but deepening in its causes is beyond the scope of this paper.

Meanwhile, the natural interest rate has remained grossly stable during the inflation targeting regime, showing only a slight reduction in recent years that probably reflects the dynamic of potential output growth. Finally, the estimation of the output gap demonstrates that the BCRP has been extensively responsive to this leading indicator of inflation, rapidly tightening or loosening its monetary policy stance depending on the position of the business cycle. Moreover, the historical shock decomposition of the output gap supports the narrative of a Peruvian economy significantly affected by foreign shocks, and one in which domestic monetary conditions (influenced by the Central Bank) have mostly moved counter-cyclically.

The remainder of this paper is arranged as follows. Section 2 discusses a brief literature review on UC models. Section 3 presents the estimation method, describing the MPT's features, the Kalman filter and smoother, and the data. Then, the main results of the Kalman filter, together with a brief analysis of output gap, potential output growth and natural interest rate dynamics are given in Section 4.

Section 5 presents the results from the historical shock decomposition performed on the output gap. Section 6 specifies the assumptions made for the growth-accounting exercise, and discusses its results. Section 7 compares our estimates of the output gap and potential output growth with other popular methods found in the empirical literature. Finally, Section 8 gives our final remarks.

2. BRIEF LITERATURE REVIEW

Univariate UC models were first used by Watson (1986) and Clark (1987) to decompose the log-level of GDP into a cycle and trend component. The structure of traditional models of these types includes a trend component modeled as a random walk with drift while the cycle component is defined as a stationary autoregressive process. A central assumption is the orthogonality restriction between trend and cycle innovations, which according to Morley et al. (2003) is fundamental to obtain a smooth trend and stationary cyclical components that explain much of the quarterly variability of GDP in the U.S economy. By contrast Beveridge and Nelson (1981) (BN), using an unrestricted ARIMA model to decompose U.S GDP, find that much of the variation in GDP is explained by fluctuations in the trend component and estimate a negative correlation between the unobserved trend and cycle innovations.

In subsequent research, Clark (1989), Kuttner (1994), Roberts (2001), and more recently Basistha and Nelson (2007), introduce multivariate UC models that employ not only information on GDP but also information on additional observable variables such as unemployment rate, inflation and inflation expectations. These models were born out of an effort towards introducing an economics-based approach into statistical methods. Thus, multivariate UC models require additional economic structure to link the different proposed observable variables with GDP dynamics.

For instance, Clark (1989) includes GDP and the unemployment rate in a bivariate UC model in order to decompose U.S. GDP into its trend and cycle components, allowing a nonzero correlation between trend and cycle innovations. The author incorporates economic structure into the estimation procedure by modelling the relationship between the cyclical component of GDP and the unemployment rate with an equation representing Okun's law. Meanwhile, Kuttner

(1994) introduces an alternative bivariate UC structure by adding inflation as an additional observable variable and assuming that inflation and the cyclical component of GDP are linked through a standard Phillips curve relationship. Along this line, Roberts (2001) uses labor hours, inflation and GDP as observable variables in a multivariate UC model with no restriction on the correlation between trend and cycle innovations. Both, Clark (1989) and Roberts (2001) find that the correlation between trend and cycle innovations is not statistically significant for U.S. data. More recently, Basistha and Nelson (2007) augment the standard UC structure for decomposing GDP with a forward looking Phillips curve using as observable variables U.S. GDP, the inflation rate and inflation expectations. The authors find a negative significant correlation between GDP trend and cycle innovations together with a cycle that is large in amplitude and highly persistent.

Recently, multivariate UC models (or multivariate filters) take into account the complete structure of a macroeconomic model and not simply additional equations that partially describe the macroeconomic dynamics at play. This is done in order to use a much richer data set when decomposing output into its cycle and trend component. The macroeconomic structure used in this new generation of multivariate UC models can be classified into semi-structural dynamic macroeconomic models and DSGE models.

The joint estimation of a set of non-observable variables that is consistent with the structure of a dynamic macroeconomic model together with a group of observable variables follows the current applied macroeconomic literature and is commonly used by Central Banks. Laubach and Williams (2003) first estimated the US output gap, trend output and natural interest rate using a backward- looking macroeconomic model consisting of two main equations: a demand or IS equation and a Phillips curve. Since then, the method has been extended by sophisticating the structure of the models and the numerical techniques. Recent exercises include Pichette et al. (2015) from the Bank of Canada, Blagrave et al. (2015) from the IMF, and Holston et al. (2017).

The preference for this multivariate filter resides in the fact that common alternatives, i.e. univariate filters such as Hodrick-Prescott or Baxter-King, only incorporate information from the GDP and do not employ the economic structure. Besides, the scarce computational requirements and the Kalman filter's recursive properties make it appealing over other filters. Furthermore, in comparison to DSGE models, semi-structural models impose fewer restrictions on the data than these structural models, thus improving the robustness of the results in case of specification errors.

3. THE MPT MODEL AS A MULTIVARIATE FILTER

3.1 Main structure of model

The MPT is a semi-structural dynamic model with rational expectations based on the neo-keynesian tradition for small open economies, and which also incorporates specific features to resemble the Peruvian economy. In this regard, the MPT structure is divided in six blocks constructed from: (i) a Phillips curve (relation between core inflation, imported inflation, and output gap); (ii) an aggregate demand curve (relation between the output gap and its determinants); (iii) a UIP equation (determination of the nominal depreciation rate from a modified version of the uncovered interest rate parity condition); (iv) a Taylor rule equation (explicit role for monetary policy); (v) an interest rate structure for US dollar interest rates denominated in soles (partial financial dollarization in the banking system is modeled by making explicit the role of long-term US dollar interest rates denominated in soles on domestic monetary conditions); and (vi) a block of equations for the external economy.

For exposition clarity, we show the main equations of the MPT model as well as its basic calibration (see Winkelried (2013) for more details). The Central Bank of Peru set the inflation target in terms of CPI inflation (i.e. headline inflation π_t) which is composed by core and non-core inflation. The Phillips curve equation is related to the core component of CPI inflation (measured with inflation without food and energy) and is given by:[1]

$$\text{1} \quad \pi_t^{\text{wfe}} = b_m \Pi_t^m + (1-b_m)\left[b_{\text{wfe}}\pi_{t-1}^{\text{wfe}} + (1-b_{\text{wfe}})\Pi_t^e\right] + b_y\left[c_y\hat{y}_t + (1-c_y)\hat{y}_{t-1}\right] + \varepsilon_t$$

[1] In all the following equations, for variables that represent a percentage variation (e.g. inflation) a capital letter such as $\prod$ designate y-o-y rates, while small letters such as π are used for quarterly annualized rates. They are related in the following way: $4\prod_t = \pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}$.

where current core inflation π_t^{wfe} is a function of imported inflation denominated in soles Π_t^m, an inertial component of inflation π_{t-1}^{wfe}, a measure of annual headline inflation expectations Π_t^e and the output gap $\hat{y}_t$. The MPT structure assumes that current core inflation depends on expectations about future headline inflation as a way to incorporate contamination of inflation expectations from the non-core component of inflation (i.e. supply shocks). Meanwhile, inflation expectations are formed as a weighted average between rational expectations of core inflation and adaptive expectations of headline inflation, as shown in the following equation:

$$\Pi_t^e = \rho_{\pi^e}\Pi_{t-1}^e + (1-\rho_{\pi^e})[(1-c_p)\mathbb{E}_t(\Pi_{t+4}^{\text{wfe}}) + c_p\Pi_{t-1}] + \varepsilon_t \qquad (2)$$

In a basic setup, imported inflation denominated in soles would be a function of both an inertial component π_{t-1}^m and a year-forward rational expectation term $\mathbb{E}_t(\Pi_{t+4}^m)$. However, due to the presence of incomplete pass-through of international prices to domestic prices, imported inflation should mainly respond to deviations from the law of one price. This is seen in the following equation, where the latter term in parenthesis measures the lagged difference between external inflation and imported inflation (both denominated in soles):

$$\pi_t^m = \left[c_{mm}\pi_{t-1}^m + (1-c_{mm})\mathbb{E}_t(\Pi_{t+4}^m)\right] + c_{mq}(\pi_{t-1}^{m*} + \lambda_{t-1} - \pi_{t-1}^m) + \varepsilon_t \qquad (3)$$

In the equation above, π^{m*} is external inflation denominated in dollars and λ is the nominal depreciation rate (soles to US dollars exchange rate). A weaker exchange rate increases the marginal costs of importers by creating a differential between the price these importers face in international markets and the price they charge domestically. That way, an increment in the exchange rate rises core inflation through its inflationary effects on domestic imported inflation.

The dynamics of the output gap and its determinants are summarized in the following forward looking IS-type equation:

$$y_t = a_y y_{t-1} + a_{y^e}(y_{t-1} + x_t^e) + a_\psi \psi_{t-1} + a_g g_t - a_t t_t + a_q q_t + a_\tau \tau_t + a_{y^*} y_{t-1}^* + \varepsilon_t \qquad (4)$$

Current output gap is a function of lagged output gap $\hat{y}_{t-1}$, which captures persistent dynamics of consumption and investment, and of the expected future output gap $y_t^e = y_{t-1} + x_t^e$, which is the sum of an adaptive term and a component that captures agents' optimism or pessimism about future economic conditions (business confidence). As in the case of inflation expectations, expectations about future output gap are a convex combination of rational and adaptive expectations:

5 $$x_t^e = \rho_{x^e} x_{t-1}^e + (1-\rho_{x^e})(\mathbb{E}_t(y_{t+1}) - y_{t-1}) + \varepsilon_t$$

Notice that x^e is modeled in a way such that if $\rho_{x^e} = 0$, i.e. all agents in the economy are fully rational, then $y_t^e = \mathbb{E}_t(y_{t+1})$.

Regarding the conventional monetary policy transmission channel, current output gap depends negatively on an lagged indicator of the long term real interest rate gap ψ_{t-1}. This indicator summarizes the domestic structure of real interest rates that determines aggregate expenditure decisions. Since the Peruvian economy is (partially) financially dollarized, the real interest rate gap that is relevant for the output gap is assumed to be a weighted average between a component that depends on the long-term interest rate denominated in domestic currency r_t^{mn} and another that depends on the long-term interest rate denominated in US dollars r_t^{me}, as follows:

6 $$\psi_t = c_r^{mn} r_t^{mn} + (1 - c_r^{mn}) r_t^{me}$$

Notice that all the components of ψ_t are expressed as deviations from their equilibrium levels. For example, the interest rate gap derived from the interest rate structure in domestic currency is $r_t^{mn} = R_t^{mn} - \bar{R}_t^{mn}$, where R_t^{mn} is the long-term real interest rate of the financial system, and $\bar{R}_t$ depends on an unobserved natural interest rate (i^n) that we are trying to estimate $\bar{R}_t^{mn} = (1-\rho_{R^{mn}})(i_t^n - \bar{i} + \bar{R}^{mn}) + \rho_{R^{mn}} \bar{R}_{t-1}^{mn} + \varepsilon_t$. R_t^{mn} is derived by taking out inflation expectations from the weighted average between the money market interest rate $I_t^{c,mn}$ and the interest rate of the banking system $I_t^{b,mn}$ as follows:

$$R_t^{mn} = c_b^{mn} I_t^{b,mn} + (1 - c_b^{mn}) I_t^{c,mn} - \Pi_t^e \tag{7}$$

The interest rate of the banking system is a function of an inertial component $I_{t-1}^{b,mn}$, and an expression that approximates the cost of funds for banks. The latter depends on an autonomous term μ_{Ib}^{mn} which measures the average margin charged by banks, the money market interest rate, and the gap of the reserve requirements rate $e_t^{mn} - e^{mn}$. The term e^{mn} should be understood as the reserve requirement rate that would be in place in "normal" times, and the inclusion of this gap expression in the banking system's interest rate structure seeks to account for the increase in funding costs due to macroprudential considerations. The equation for $I_t^{b,mn}$ is shown below.

$$I_t^{b,mn} = \rho_b^{mn} I_{t-1}^{b,mn} + (1 - \rho_b^{mn}) [\mu_{Ib}^{mn} + M^{mn} I_t^{c,mn} + c_e^{mn} (e_t^{mn} - e^{mn})] + \varepsilon_t^{b,mn} \tag{8}$$

Meanwhile, the interest rate of the money market, which represents the cost of funds faced outside the banking system (e.g. by issuing bonds), is modeled as a yield curve that rests on the liquidity premium theory (an offshoot of the market expectation hypothesis) as follows:

$$I_t^{c,mn} = \frac{1}{4} [i_t^{mn} + \mathbb{E}_t (i_{t+1}^{mn}) + \mathbb{E}_t (i_{t+2}^{mn}) + \mathbb{E}_t (i_{t+3}^{mn})] + \mu_t^{mn} + \varepsilon_t \tag{9}$$

In the equation above, i_{t+1}^{mn} is the inter-bank interest rate, which measures the cost of short-term loans between banks $i_{t+1}^{mn} = i_t + \varepsilon$, while μ_t^{mn} is the liquidity premium. That way, the expression is a sort of no-arbitrage condition, since the 1-year interest rate equals the expected return from the respective 1-year forward short-term rates plus a liquidity premium (i.e. the longer-term interest rate matches its opportunity cost).

US-dollar interest rates denominated in soles are modeled exactly with the same structure as described in equations 7 through 9.

Real external conditions affect the dynamics of the output gap via: (i) the expenditure-switching effect of the real exchange rate gap q_t; (ii) the effect of global demand over domestic exports, summarized by the output gap of Peru's main trading partners $\hat{y}_t^*$; and (iii) the terms of trade impulse τ_t, which reflects the effect of international commodity prices over economic activity. Notice that $\hat{y}_t^*$ and τ_t are assumed to follow exogenous processes since the model represents a small open economy. The expenditure-switching effect generated by changes in q_t captures movements in the tradable sector output that occur when the real multilateral exchange rate differs from its long-run equilibrium level. Therefore, when q_t is positive, the multilateral real exchange rate is above its equilibrium level and the relative price of domestic goods in terms of foreign goods falls, inducing an increase in exports.

Finally, fiscal policy variables, such as government expenditures g_t and taxes t_t, enter the output gap equation in the form of impulses and are considered to be exogenous.

Regarding the nominal exchange rate, the MPT takes a standard version of the uncovered interest rate parity (UIP) equation adjusted by a risk premium for investing in the domestic asset, and incorporates sluggish adjustments of the nominal depreciation rate. This last feature simulates the effects of FX intervention over the dynamics of the exchange rate.[2] We proceed to briefly explain how this version of the UIP is derived.

The conventional UIP equation (adjusted by a risk premium ξ_t) is given by:

$$i_t^{mn} = i_t^{me} + \xi_t + 4(s_{t+1}^e - s_t)$$

Where s_t denotes the logarithm of the nominal exchange rate, s_{t+1}^e is the expected nominal exchange rate, i_t^{mn} is the short-run nominal return of the domestic assets and i_t^{me} corresponds to the dollar denominated asset return. As it is well known, the UIP equation is an arbitrage equation that equalizes the nominal rate of return of domestic and foreign currency denominated assets. We are implicitly assuming that domestic and foreign assets are imperfect substitutes

2 The BCRP intervenes in the FX market to reduce FX volatility in a context of persistent partial financial dollarization. Although we do not model FX intervention explicitly, the sluggish adjustment of the the nominal depreciation rate allows us to incorporate its effects.

in the sense that the return on the dollar is adjusted by an exchange rate and country risk premium.

To incorporate sluggish adjustments of the depreciation rate, the MPT assumes that expected exchange rate is a weighted average of rational and 'naive' expectations (agents of this type expect future exchange rate to be equal to the observed exchange rate plus a random walk term ϵ). Adaptive expectations should be more relevant if the effects of FX intervention are stronger. The expected exchange rate is thus modeled as:

$$s_{t+1}^{e} = \left(\frac{\rho_\lambda}{1+\rho_\lambda}\right)\mathbb{E}_t(s_{t+1}^{e}) + \left(\frac{1}{1+\rho_\lambda}\right)(s_{t+1} + \epsilon_t)$$

The above equations can be combined and written in quarterly annualized variations. By defining λ_t as the exchange rate variation, we get that $\lambda_t = s_{t+1} - s_t$, and so the final expression for the nominal depreciation rate in the MPT is given by:

10
$$\lambda_t = \rho_\lambda \mathbb{E}_t(\lambda_{t+1}) + (1+\rho_\lambda)(i_t^{me} + \xi_t - i_t^{mn} + \epsilon_t)$$

Finally, the Taylor rule shows that the Central Bank responds to future deviations of the core inflation (four quarters ahead) from the target rate $\left(\tilde{\Pi}_t = \mathbb{E}\left[\Pi_{t+4}^{wfe}\right] - \bar{\Pi} = \Pi_{t+4}^{wfe} - 2\right)$, and to the current and lagged output gap $c_{f_y} = 0{,}5$. It also has an inertial component as shown below:

11
$$i_t = f_i i_{t-1} + (1-f_i)[i_t^n + f_\pi \tilde{\Pi}_t + f_y(c_{fy} y_t + (1-c_{fy})y_{t-1}] + \varepsilon_t$$

The natural interest rate that we intend to estimate i_t^n appears in the Taylor rule as a drifting intercept, and can be rationalized as the trend interest rate that serves as a guideline for monetary policy. This trend interest rate exists when the output gap and the core inflation rate are placed on their equilibrium values. Thus, i_t^n is consistent in the model with an economy with no transitory disturbances (similar to the definition of the natural interest rate).

The natural interest rate follows an autoregressive process, where the unconditional mean $\bar{i}$ is calibrated on 4.5%.[3]

$$i_t^n = (1-\rho_{i^n})\bar{i} + \rho_{i^n} i_{t-1}^n + \varepsilon_t \tag{12}$$

The above description means that the MPT has no explicit role for potential output. In fact, it only determines the dynamic of the output gap (which by definition is the difference between real GDP and the potential output) among other modeled variables such as inflation, exchange rate, and interest rate. Therefore, we need to add a *measurement* equation that links the output gap with potential output growth. By definition, given the potential output Y_t^* and real GDP Y_t, the output gap $\hat{y}_t$ is defined as:

$$\hat{y}_t = \frac{Y_t - Y_t^*}{Y_t^*} = \frac{Y_t}{Y_t^*} - 1 \approx \ln Y_t - \ln Y_t^*$$

Subtracting the output gap from the previous period, we get:

$$\hat{y}_t - \hat{y}_{t-1} = (\ln Y_t - \ln Y_{t-1}) - (\ln Y_t^* - \ln Y_{t-1}^*) \approx \Delta\% Y_t - \Delta\% Y_t^*$$

The measurement equation that we add on the MPT is then given by the equation below. This equation states that real GDP growth (observed variable) equals potential output growth plus the variation in the output gap.

$$\Delta\% Y_t = \Delta\% Y_t^* + (\hat{y}_t - \hat{y}_{t-1}) \tag{13}$$

However, as we are introducing two new variables to the model, we need to incorporate an additional equation, defined below. This

[3] This way of modelling the equilibrium interest rate is also followed by the IMF in their Global Projection Model (see for example Carabenciov et al. (2013)).

equation states that potential output growth follows an autoregressive process.

$$\Delta\%Y_t^* = \rho_{Y^*}\Delta\%Y_{t-1}^* + \xi_t^{Y^*} \quad (14)$$

By the same token, notice that equations 11 and 12 can also be interpreted as a measurement equation for the nominal interest rate. Under this assumption, the Taylor rule (equation 11) may be viewed as decomposing the observed interest rate i_t into a systematic component (cycle component for the nominal interest rate) and a nonobserved component related to the trend interest rate.

Table 1 summarizes the calibration used for our estimation purposes. The values assigned for the calibrated parameters are in line with the estimation results shown in Winkelried (2013), and are in fact the result of extensive judgment by the technical staff to improve the forecasting and explanatory power of the model. Using a set of calibrate parameters (instead of estimating them all) allows us to restrict uncertainty to the estimation of latent variables.

Table 1

CALIBRATION OF MPT'S MAIN PARAMETERS

Phillips Curve		*Agreggate Demand*		*Interest rates*	
b_m	0.09	a_y	0.62	μ_{Ib}^{mn}	10.86
b_{wfe}	0.35	a_{y^e}	0.19	M^{mn}	0.57
b_y	0.19	a_ψ	0.16	ρ_b^{mn}	0.72
c_y	0	a_t	0.01	c_e^{mn}	0.30
Taylor Rule		a_g	0.12	c_b^{mn}	0.50
f_i	0.70	a_τ	0.04	ρ_R^{mn}	0
f_p	1.50	a_{y^*}	0.15	c_r^{mn}	0.60
f_y	0.50	a_q	0.04	μ_{Ib}^{me}	4.03
c_{fy}	0.50	Expectations		M^{me}	0.99
UIP Equation		ρ_{π^e}	0.70	ρ_b^{me}	0.75
ρ_λ	0.40	c_p	0.15	c_e^{me}	0.1
Natural Interest Rate		ρ_{x^e}	0.50	c_b^{me}	0.50
ρ_{i^n}	0.50	Imported inflation		ρ_R^{me}	0
Potential Output Growth		c_{mm}	0.46	c_r^{me}	0.40
ρ_{Y^*}	0.99	c_{mq}	0.44		

3.2 State-space representation of the model and the Kalman filter

In mathematical terms, the MPT is a system of 55 linear stochastic difference equations with weighted averages of rational and adaptive expectations, where the unknowns are sums over infinite sequences of exogenous shocks across time for all the endogenous variables. This type of models require a numerical solution whose solving algorithms are usually modified versions of Blanchard and Kahn (1980). These algorithms classify variables into state and control variables. State variables define the system's stance in each period of time (as previously mentioned), and are further categorized into endogenous and exogenous states (random shocks that affect the dynamic of endogenous variables). The model's numerical solution is represented with policy functions, leaving all control and endogenous state variables as a linear function of state variables. It is worth mentioning that the rational expectations assumption means that the model's economic agents know these policy functions, and that they compute their expectations using them.

In compact form, the MPT can be written as:

$$E\left[X_{t+1} \mid \mathcal{F}_t\right] = AX_t + B\xi_t$$

where X_t is the vector of endogenous variables, ξ_t is a random vector of structural innovations or exogenous forcing variables assumed to be $\xi_t \sim iid(0,\Sigma)$, while matrices A and B store all the parameters of the model. The rational expectation operator applied to the stochastic process X_t is given by the term $E\left[X_{t+1} \mid \mathcal{F}_t\right]$ and it is defined as the conditional expectation of X_t with respect to the information set $\mathcal{F}_t$. The vector of endogenous variables can be partitioned as $X_t = [S_t C_t]'$, where S_t is the vector of endogenous state variables and C_t is the vector of control variables. The state vector is composed by the endogenous states, S_t, as well as by the exogenous states ξ_t. The rational expectation solution of the model is given by the following linear policy function:

$$X_t = \mathbf{\Psi} S_{t-1} + \mathbf{\Omega}\ \xi_t$$

where matrices Ψ and $\mathbf{\Omega}$ are composed by non-linear combinations of the parameters in the model.

As we have already mentioned, the state-space representation of the models' solution is composed by a state equation and a measurement equation. The state equation is formed by rewriting the above rational expectation solution of the model as a restricted VAR. Taking into account the definition of X_t, the solution of the model can be partitioned into a policy function for the endogenous state vector and a policy function for the control variables as follows:

$$S_t = \Psi_s S_{t-1} + \mathbf{\Omega}_s \xi_t$$

$$C_t = \Psi_c S_{t-1} + \mathbf{\Omega}_c \xi_t$$

where Ψ_s,Ψ_c,$\mathbf{\Omega}_s$ and $\mathbf{\Omega}_c$ are the corresponding partitions of matrices Ψ and $\mathbf{\Omega}$. Therefore, the state equation of the model is given by the following restricted VAR:

$$\begin{bmatrix} S_t \\ C_t \end{bmatrix} = \begin{bmatrix} \Psi_s & \mathbf{0} \\ \Psi_c & \mathbf{0} \end{bmatrix} \begin{bmatrix} S_{t-1} \\ C_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{\Omega}_s \\ \mathbf{\Omega}_c \end{bmatrix} \xi_t$$

Finally, the measurement equation uses a rectangular selection matrix $\mathbf{H}$ applied on the vector X_t to define the set of observable variables Y_t that are used to estimate the non-observable variables with the Kalman filter. The state-space representation of the rational expectation solution of the MPT model is of the following form:

$$X_t = \begin{bmatrix} \Psi_s & \mathbf{0} \\ \Psi_c & \mathbf{0} \end{bmatrix} X_{t-1} + \begin{bmatrix} \mathbf{\Omega}_s \\ \mathbf{\Omega}_c \end{bmatrix} \xi_t$$

$$Y_t = \mathbf{H} X_t$$

Thus, with the historical data of observable variables Y_t, the Kalman filter and the smoother can be applied on the state-space representation of MPT's solution to estimate the state variables.

3.3 The observable variables

From the explanation of the Kalman filter, it is straightforward to conclude that the estimation results are sensible to the group of variables that are declared as observable (the ones that are defined in the measurement equation). Therefore, the selection of variables must be done aiming to capture the main drivers of the non-observable variables to be estimated. It is also worth noting that if a time series used as input for the Kalman filter is updated (e.g. a historical revision or new data for subsequent periods), the results will also vary.

To account for the uncertainty risen by the selection of *observable* variables, we employ five groups of variables to be declared as such. This way, we get a set of estimation results which we can use to define probable ranges for the non-observable variables.[4] The five groups of variables (all of which are plotted in Figure 1) are:

i) **Group 1:** Real GDP growth, inflation without food and energy, inflation expectations (1-year forward), impulse of business confidence (proxy of the expected output gap), terms of trade growth and real effective exchange rate gap.[5]

ii) **Group 2:** Real GDP growth, inflation without food and energy, inflation expectations (1-year forward), impulse of business confidence, terms of trade growth, real effective exchange rate gap, short-term interest rate (BCRP's monetary policy rate) and 3-Month LIBOR rate (proxy of external interest rate).

iii) **Group 3:** Real GDP growth, inflation without food and energy, inflation expectations (1-year forward), impulse of business confidence, terms of trade growth, real effective exchange rate gap and short-term interest rate (BCRP's monetary policy rate).

[4] The probable ranges are the difference between the maximum and minimum value of the five estimates at each period of time.

[5] The real effective exchange rate (REER) gap is actually also a non-observable variable. However, it is one of the most important determinants of output gap dynamics. Thus, we use a satellite model based on cointegration relations (Behavioral Equilibrium Exchange Rate or *BEER* model) to estimate it and declare the REER gap as an *observable* variable. For more references on the BEER methodology, see MacDonald and Clark (1998).

Figure 1

OBSERVABLE VARIABLES

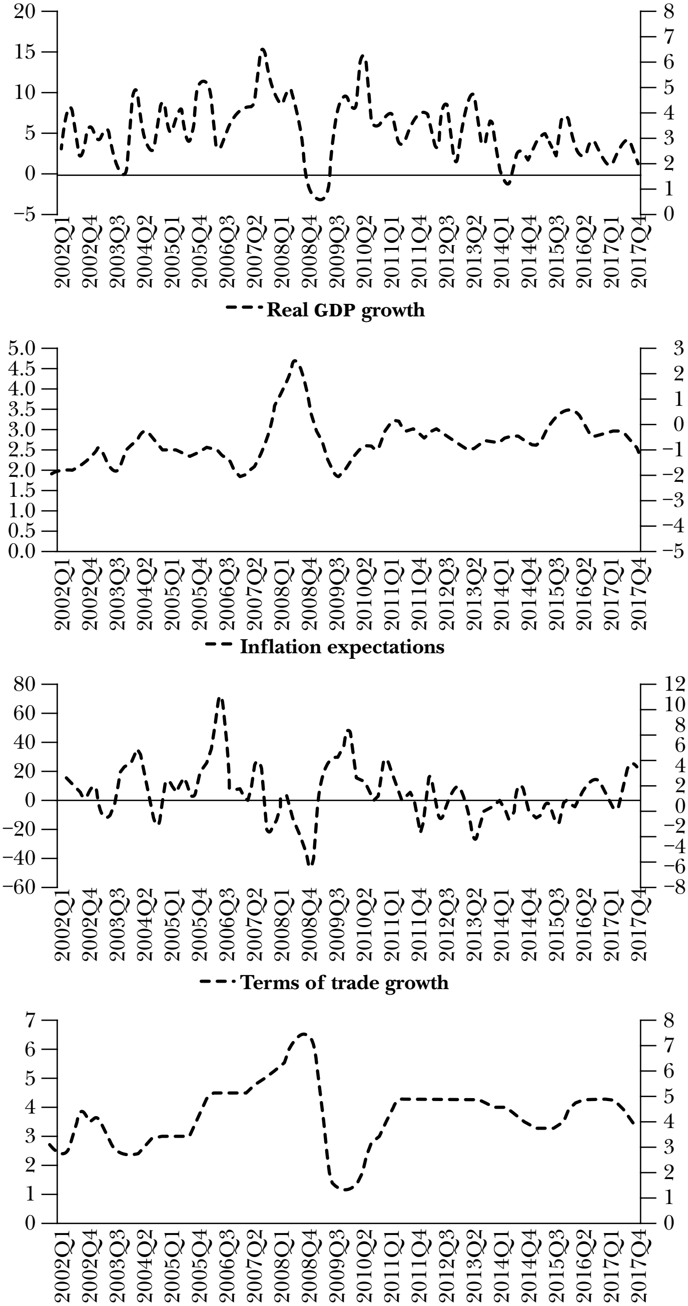
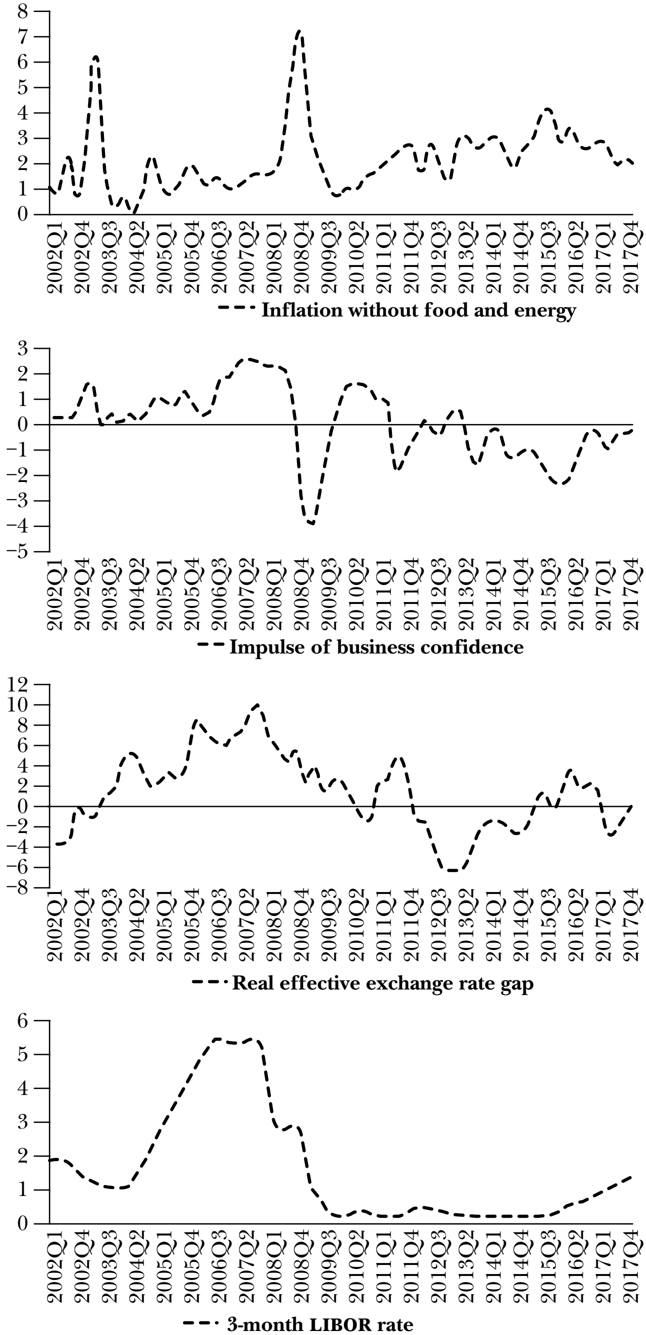


Figure 1 (cont.)

OBSERVABLE VARIABLES



iv) **Group 4:** Real GDP growth, inflation without food and energy, inflation expectations (1- year forward), impulse of business confidence, terms of trade growth and short-term interest rate (BCRP's monetary policy rate).

v) **Group 5:** Real GDP growth, inflation without food and energy, inflation expectations (1-year forward), terms of trade growth, and short-term interest rate (BCRP's monetary policy rate).

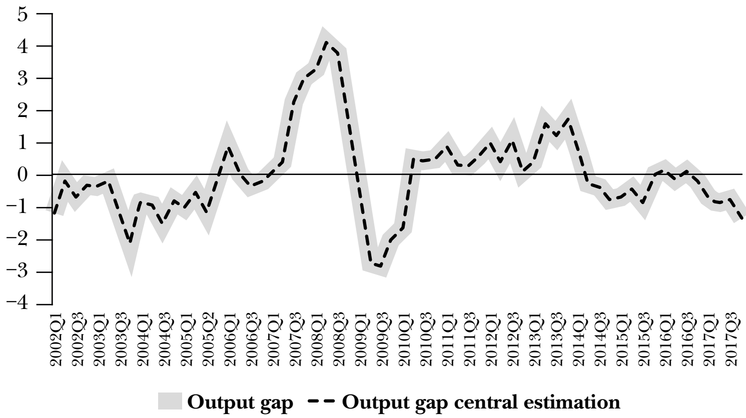
All the quarterly data is published on the BCRP's website. We exclusively analyze the inflation targeting period (2002Q1-2017Q4) to avoid estimation problems from regime changes, and we accordingly calibrate the MPT for these dates. However, all the variables are forecasted until 2019Q4 with the information available until August 2018, to improve the accuracy of the filtering process (estimation of non-observable variables) at the end of sample.[6]

4. MAIN RESULTS

Figure 2 displays the probable range as well as the central estimation for the output gap.[7] It shows that the period right before the Financial Crisis (September 2008) was characterized by its marked expansion (the output gap rose from -1.5% to 4.1% on average between the third quarter of 2004 to the second quarter of 2008). In fact, as it is documented by Quispe et al (2009), the economy grew considerably (between 8.0% and 10.0%) on the quarters right before the Crisis, and inflation was high (above 3.5%) mainly due to aggregate demand expansion. This behaviour was explained by a sustained increase in terms of trade that boosted business confidence and a massive inflow of short-term foreign capital which loosened credit conditions. On a yearly basis, Peruvian terms of trade experienced increasing average growth rates between 2003 until 2007.

Figure 2

ESTIMATED OUTPUT GAP



The expansion ended when the financial crisis brought economic recession for Peru's main trading partners, a reversion of terms of trade growth and a capital flight. As consequence, the output gap fell from an average peak of 4.1% in the second quarter of 2008 to its lowest point of -2.8% in the third quarter of 2009, remaining negative for about five consecutive quarters until the first semester of 2010. Then, the output gap bounced and remained positive between zero and one percent with no visible trend until 2013 (the average of our central estimation between 2010 and 2013 is 0.5%). This behaviour was sustained by loose monetary conditions (the BCRP eased its policy stance, while developed economies did the same with traditional monetary policy instruments and the QE). During 2013, the output gap briefly rose but then, a downward trend started as international financial conditions tightened following the taper tantrum (which started on May 2013 with the Fed's tapering announcement), and as the price of commodities dropped (this last event partially caused by the taper tantrum, but also due to the deceleration of China).

It is worth mentioning that the contraction of the output gap observed on the 2015-2017 period is also consequence of political turmoil (2015 and late-2017 were periods of political uncertainty that negatively affected business confidence), fiscal adjustments (there was a contraction of fiscal spending on the last quarter of 2016 as part of a strategy to reduce fiscal deficit), and the natural disasters caused by El Niño phenomenon between February and March 2017. All this *intuitive* narrative finds its support in the model with the historical shock decomposition of Section 5.

Figure 3 shows how responsive the monetary authority has been to the position of the business cycle measured as the output gap. The BCRP has adjusted its monetary policy stance rapidly both upward or downward depending if the economy was heating or cooling, respectively. As the output gap is a leading indicator of inflationary pressures, the responsiveness of the BCRP goes in line with the behavior expected from a Central Bank following an inflation targeting regime.

The estimation of potential output growth (annualized quarterly growth rate) is presented in Figure 4. Potential output growth accelerated in the period right before the Financial Crisis. It jumped from around 4.0% in 2002 to almost 8.0% by the end of 2007. After that, potential output growth experienced a gradual and persistent decline. Between 2008 and 2010, average potential output growth rate was 6.6%, while between 2011 and 2013 it was 5.6%. In the latest period (2014-2017), the economy experienced an average potential output growth rate of 3.4%. This may be the result of less investment, labor participation or lower productivity. To better understand the phenomenon, Section 5 presents a growth-accounting exercise that decomposes potential output growth into these determinants.

Finally, Figures 5 and 6 show the probable ranges for the nominal and real natural interest rate, respectively. Each range is presented with its corresponding observed policy rate (in nominal and real terms, respectively). The real natural rate is constructed by subtracting the steady-state value of inflation expectations from the estimated nominal natural interest rate. The MPT calibration for the steady-state or long-run equilibrium inflation rate and inflation expectations is 2.0%, which is consistent with the center of the BCRP's inflation target range. The most salient feature is that both nominal and real natural interest rates have been considerable stable along the inflation targeting regime. The average nominal natural rate in the sample is 3.6%, and, consequently, the average real natural rate is 1.6%.

Figure 3

OUTPUT GAP AND MONETARY POLICY RATE

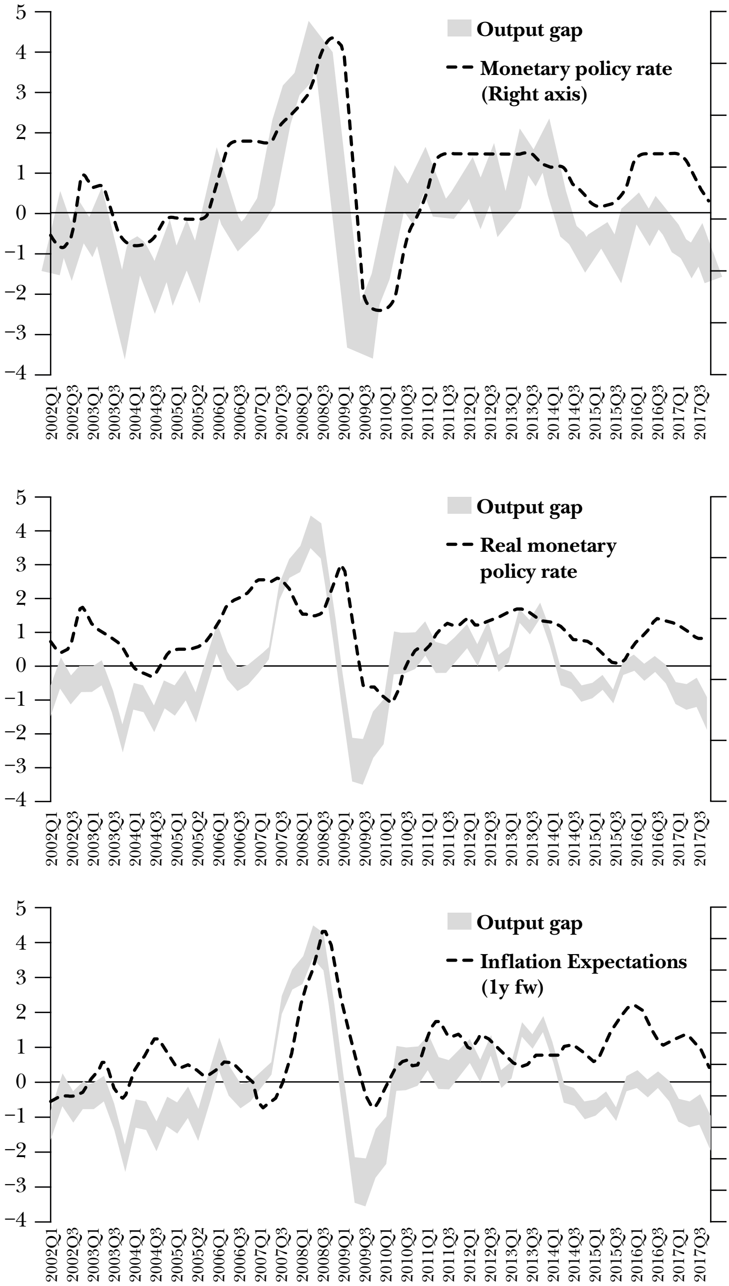


Figure 4

ESTIMATED POTENTIAL OUTPUT GROWTH (ANNUALIZED QUARTELY RATE) %

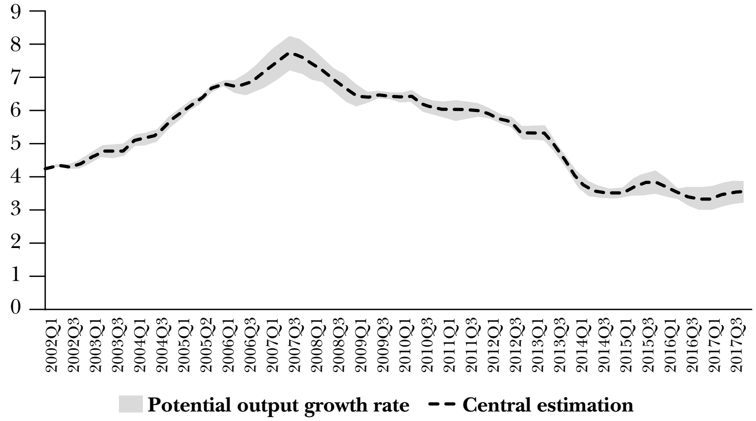
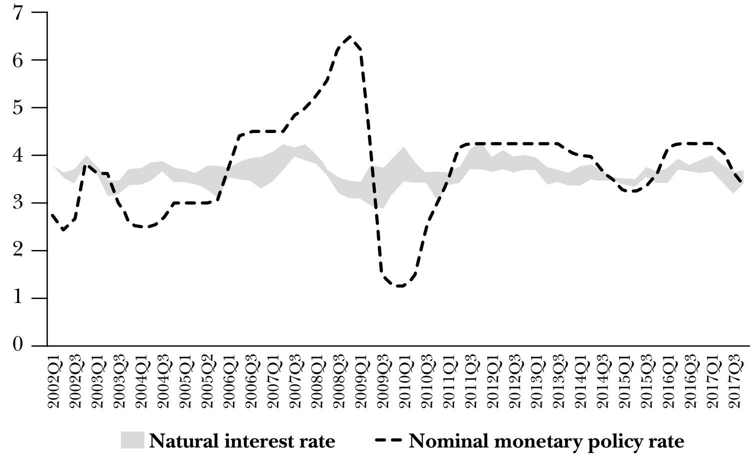


Figure 5

NOMINAL NATURAL AND MONETARY POLICY RATES (%)



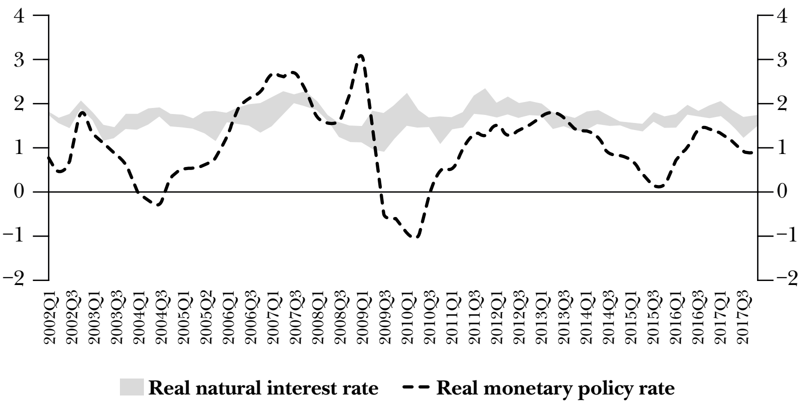
The stability of the natural rate is consistent with the fact that neither the observed nominal nor real monetary policy rates have presented any clear trend since inflation targeting was adopted. Before the financial crisis, the real natural rate rose from an average of 1.5% in 2005 to 2.1% at the end of 2008, consistent with the estimated higher potential output growth during the same years. During 2008, the natural rate fell swiftly to its lowest historical level of 1.3%, and then reverted to an average of 1.6%, remaining grossly stable since.

The difference between the real natural and observed monetary policy rate serves as an indicator of the monetary policy stance. Our results suggest that during the inflation targeting regime, the BCRP has mostly sustained expansive monetary conditions. Only on the years preceding the Financial Crisis (2006-2008) did the BCRP hold a contractionary stance with a real monetary policy rate above the estimated real natural rate range, clearly responding to the high inflationary pressures rising from the heated economy. The monetary authority then adjusted its policy stance downward to respond to the effects of the Financial Crisis. During the 2010-2013 period, the Central Bank tried to reverse its stance, gradually tightening monetary conditions. Nevertheless, before monetary policy could be normalized, 2013 brought the beginning of the taper tantrum and the sharp decline in commodity prices, thereby inciting the BCRP to loosen its position again. The economic slowdown seen during 2016 and 2017 has accordingly been responded with a period of monetary policy easing after an attempt to normalize the stance.

However, one may argue that the equilibrium expected inflation rate does not necessarily coincide with our normative assumption of the equilibrium inflation rate. This argument becomes particularly relevant when evaluating historic monetary policy stance: if inflation expectations were not perfectly anchored to the center of the inflation target range, subtracting 2.0% would possibly not yield the real conditions at the time. In this regard, we construct another real natural interest rate series by subtracting the average value of inflation expectations between 2002 and 2017 (2.7%), mainly for robustness purposes. The result is shown in Figure 7. It is straightforward to notice that the narrative surrounding the monetary policy stance changes in this graph. Most significantly, monetary policy would not have been mostly expansive during the inflation targeting regime. For instance, the 2012Q1-2014Q2 period would turn out to

Figure 6

REAL NATURAL AND MONETARY POLICY RATE (%)

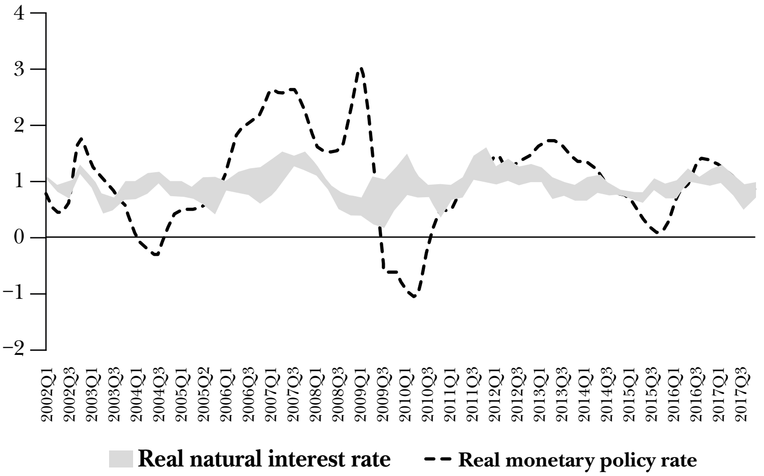


be a contractionary episode, showing the effectiveness of the Central Bank in responding to the spike in the output gap. Moreover, the 2016Q2-2017Q1 would have also been a period of tightening, thereby given stronger motives for the subsequent cuts in the monetary policy rate as demand expansion was still timid.

Finally, it is worth mentioning that output gap dynamics in the MPT model depend on a richer interest rate structure, not solely given by the short-term domestic policy interest rate. On the one hand, this short-term real interest rate affects the output gap indirectly as it first influences longer-term real rates. On the other hand, as the economy suffers from persistent partial financial dollarization (bank loans and deposits), variations in the external interest rate and in depreciation expectations also affect the longer-term real rates denominated in dollars. Thus, even if the short-term real interest rate has remained below its natural position, aggregate monetary conditions could have been contractionary in specific intervals due to external factors or domestic forces that stir long-term rates. Section 5 sheds evidence on how domestic monetary conditions affected the dynamic of the output gap.

Figure 7

REAL NATURAL AND MONETARY POLICY RATES (ALTERNATIVE METHOD)
Percentages



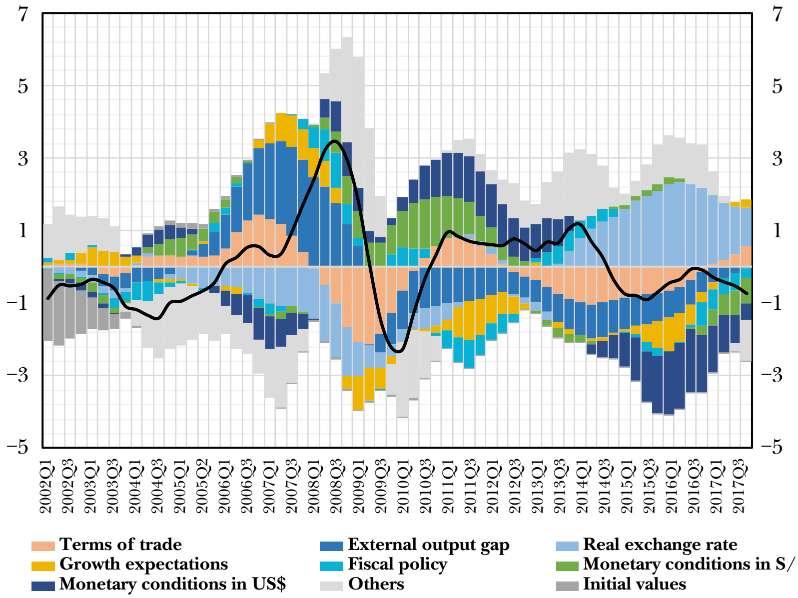
5. HISTORICAL SHOCK DECOMPOSITION: EXPLAINING OUTPUT GAP DYNAMICS

The main advantage of estimating non-observable variables using a macroeconomic model as a multivariate filter is that we can perform a historical shock decomposition on them. This means that we can decompose their historical deviations from their respective steady state values into the contributions coming from all the shocks defined in the model. This becomes particularly relevant for the output gap, as it is modeled with various determinants in an IS-type equation (see Section 3.1). Figure 8 presents the output gap's historical shock decomposition, having grouped all the MPT's shocks into eight groups: terms of trade, external output gap, real exchange rate, growth expectations, fiscal policy, monetary conditions in domestic currency (S/), monetary conditions in foreign currency (US$), and other shocks.

As it is shown, much of the output gap's narrative described in Section 4 is supported by the shock decomposition. In the model, the expansion of the output gap right before the Financial Crisis (last quarter of 2008) is explained by propitious external conditions (positive contributions of terms of trade and external output gap since 2005Q2, while monetary conditions in US$ contributed significantly to its expansion since 2008Q3). Immediately after the crisis, the output gap was favoured by loose monetary conditions (domestic and foreign) which offset the negative effects of the external output gap and growth expectations. However, eventually the downward trend began in 2014 as the contribution of terms of trade and monetary conditions in foreign currency turned adverse. Finally, the shock decomposition reveals that monetary conditions in domestic currency where actually contributing negatively to output gap since mid-2016 (a process of tightening had began that year), and thus supports BCRP's decision of loosening the monetary policy stance in 2017. For further clarity in the analysis, we present detailed plots of selected group of shocks' contributions in Appendix 9.2.

Figure 8

HISTORICAL SHOCK DECOMPOSITION OF THE OUTPUT GAP
Central estimation (%)



6. GROWTH-ACCOUNTING: EXPLAINING THE RECENT SLOWDOWN IN POTENTIAL OUTPUT GROWTH

To better understand the recent trend of annual potential output growth, we decompose it through the growth-accounting method. This method rests on the assumption that potential output can be modeled with a Cobb-Douglas function such as:

15
$$Y_t^* = A_t K_t^{\alpha} L_t^{1-\alpha}$$

In the above equation, Y_t^* is the potential ouput, K_t is the physical aggregate capital, L_t is the aggregate labor, and A_t is total factor productivity (TFP). TFP measures how much potential output will rise in addition to the effect of a one-unit increment in labor or capital, and is thus a proxy of economic efficiency. Meanwhile, the parameter α is both a measure of the elasticity of potential output to capital and of the share of total income that goes to this input (1–α denotes exactly the same for labor).

Equation 15 can be re-written on logarithm terms with annual variations as follows:

16
$$y_t^* - y_{t-1}^* = (a_t - a_{t-1}) + \alpha(k_t - k_{t-1}) + (1-\alpha)(l_t - l_{t-1})$$

This way, potential output growth $y_t^* - y_{t-1}^*$ is decomposed into TFP growth $a_t - a_{t-1}$ and the weighted average of factors of production growth. We set the parameter α on 0.485, which corresponds approximately to the middle value of the range of estimations for the Peruvian economy (see Céspedes and Rondán (2016) for a summary of these estimates).

In terms of data, we use the annual series of Economically Active Population published by the National Institute of Statistics and Information (INEI, for its acronym in Spanish) as a proxy of labor. Since we are modelling potential output, we compute its trend component using the Hodrick-Prescott filter and employ it for our estimates.

Meanwhile, as it is standard in the growth accounting literature, the capital stock is built using the perpetual inventory method. The law of motion for the capital stock is given by:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{17}$$

where, I_t is the gross fixed capital formation (GFCF) and the parameter δ denotes the depreciation rate of capital. Annual GFCF series is published by the BCRP. Meanwhile, the depreciation rate is set on 5.0%, which is a standard in macroeconomic literature. This method also requires an assumption for the initial capital stock (K_0). As it is common in other exercises for the Peruvian economy, we take the initial capital stock to be 42.2 billion soles of 1994 in 1950 (see Céspedes and Rondán (2016)).

Table 2

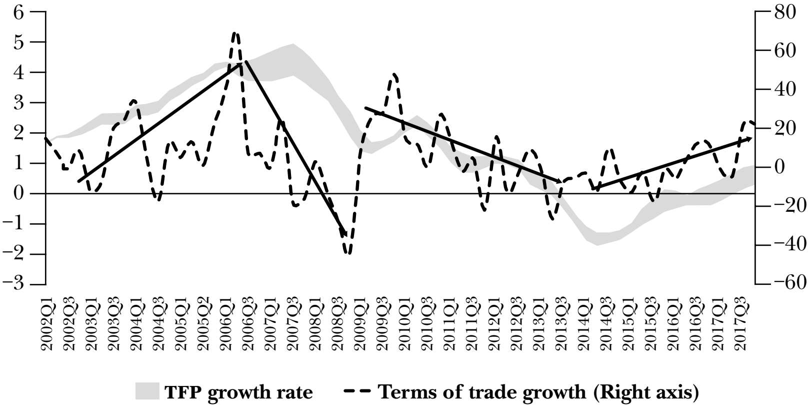
GROWTH-ACCOUNTING (CENTRAL ESTIMATION) (%)

Year	*Potential output*	*Labor participation*	*Capital participation*	*TFP*
2002	4.3	1.5	0.8	1.9
2003	4.7	1.5	0.8	2.4
2004	5.3	1.5	0.9	3.0
2005	6.2	1.4	1.0	3.8
2006	6.8	1.3	1.3	4.1
2007	7.5	1.3	2.0	4.3
2008	7.1	1.2	2.7	3.2
2009	6.5	1.1	3.6	1.7
2010	6.2	1.0	3.1	2.1
2011	6.0	0.9	4.0	1.1
2012	5.7	0.8	3.9	1.0
2013	5.0	0.8	4.3	-0.1
2014	3.5	0.7	4.3	-1.4
2015	3.7	0.7	3.6	-0.5
2016	3.4	0.7	2.9	-0.2
2017	3.3	0.7	2.4	0.3

The results of this exercise are shown in Table 2 above. It is then clear that most of the recent decline in potential output growth is explained by a contraction of TFP growth. In fact, between 2010-2013 and 2014-2017, the reduction in capital and labor contributions only accounted for one third of the decrease in average potential output growth rate. Figure 9 shows that the TFP slowdown began in 2010, two years prior to the start of the declining of potential output growth. However, TFP reduction did turned sharper in 2012.

Figure 9

TFP GROWTH AND TERMS OF TRADE GROWTH (%)



The contraction of aggregate productivity is related to structural factors. For example, the lack of structural reforms regarding institutions, human capital, infrastructure, business regulation, financial depth, and technological innovation may have contributed to the decline in productivity (see Loayza et al. (2005) and Levine (2005)).

However, TFP is also affected by external conditions. For instance, Castillo and Rojas (2014) find that terms of trade shocks bring important productivity gains in the short and long-run for Mexico, Peru and Chile. This could be related to the fact that an expansion in terms of trade increases the intensity and incites improvements in the use of factors of productions. Figure 9 above shows that there is in fact a close relation between TFP and terms of trade growth during the inflation targeting regime. The transitory sharp decline in terms of trade during the Financial Crisis did not affect TFP growth in the expected magnitude, probably due to the ephemeral nature of the shock. However, since 2010, as terms of trade decelerated and then contracted, TFP dropped as well.

7. COMPARISON WITH UNIVARIATE FILTERS

How much would our results differ if we had instead used univariate filters? We have already discussed that multivariate filters have the advantage of using multiple sources of information at the same time and defining an economic structure for the variables at play. However, it is worth comparing our estimates with the ones obtained with univariate filters due to their widespread use. In this section, we decompose GDP and the ex ante real monetary policy rate into a cycle and trend component using the standard Hodrick-Prescott filter (HP) with a lambda parameter of 1600, the Baxter-King band-pass filter (BK) considering business cycle frequencies between 6 and 32 quarters, and different specifications of unobserved component (UC) models estimated with Bayesian techniques following Grant and Chan (2017). Prior to presenting and comparing the results, we briefly discuss the structure of the UC models employed in this section.

7.1 Decomposing Real GDP

We use two different specifications of UC models. Both specifications are unconstrained in the sense that there is no restriction imposed on the correlation between innovations of the cyclical and trend components. Following the literature, we label these models as UCUR (Unobserved Components Unrestricted Model). The first UCUR specification assumes an stochastic growth rate for the trend component and is based on Grant and Chan (2017). More precisely, the growth rate for the trend component follows a random walk, in contrast with more standard UC models in which the trend level is modeled this way. Since the marginal likelihood of the UCUR model is sensitive to prior specifications, we use three different sets of priors.[8] Each set yields a particular GDP decomposition that we label as UCUR 1, UCUR 2 and UCUR 3, respectively.

The structure of these UCUR models has the following log-specification:

18 $$y_t = \tau_t + c_t$$

where y_t denotes quarterly GDP, τ_t is the trend component and c_t is the stationary cyclical component. The trend growth rate $\Delta\tau_t \equiv \tau_t - \tau_{t-1}$ is modeled as a random walk whereas the cyclical component is modeled as a stationary AR(2) process with zero mean, as shown below:

19 $$\Delta\tau_t = \Delta\tau_{t-1} + u_t^\tau$$

20 $$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + u_t^c$$

According to Grant and Chan, the random walk specification for $\Delta\tau_t$ is more flexible since it can accommodate breaks in trend output growth, in contrast with the standard specification of a random walk for the trend process τ_t. Finally, the initial trend points, τ_0 and τ_{-1}, are treated as unknown parameters and the innovations u^c and u^τ are assumed to be jointly normal as follows:

$$\begin{pmatrix} u_t^c \\ u_t^\tau \end{pmatrix} \sim \mathcal{N}\left(0, \begin{pmatrix} \sigma_c^2 & \rho\sigma_c\sigma_\tau \\ \rho\sigma_c\sigma_\tau & \sigma_\tau^2 \end{pmatrix}\right)$$

The ρ parameter reflects the correlation between the innovations of each GDP component (in standard UC models, ρ is assumed to be zero). For the estimation procedure, we used quarterly real GDP from 1980-2017, and forecasted it until 2019 with ARIMA models.

Meanwhile, the second UCUR specification, based on Perron and Wada (2009), assumes that the trend level follows a random walk and adds two exogenous breaks for the trend component. The breaks are modeled as a change in the deterministic component of the trend. We label this specification UCUR2BP (UCUR with two breaks points). The trend component for this model is specified as follows:

21 $$\tau_t = \mu_1 1(t < t_1) + \mu_2 1(t_1 \leq t < t_2) + \mu_3 1(t_2 \leq t) + \tau_{t-1} + u_t^{\tau}$$

where t_1 and t_2 denote the two break points considered for this specification, while $1(A)$ is an indicator function that takes the value of 1 if condition A is true and 0 otherwise. Following Guillén and Rodríguez (2014), we set t_1 to be the third quarter of 1990 and t_2 to be the first quarter of 2002. Appendix 9.3 shows the prior distributions for the estimated parameters, as well as the respective posterior means.

Figures 10 and 11 compare the estimated range of the output gap and potential output growth under the MPT multivariate filter with the respective results from each univariate filter.

As it is shown in the figures, the HP and the first two UCUR models are the closest to our estimates. This is validated on Table 3 below, where we present the correlations between our central estimations of output gap and potential growth rate with the ones from univariate filters.

Table 3

CORRELATION OF OUTPUT GAP CENTRAL ESTIMATION WITH UNIVARIATE FILTERS

Univariate Filter	*Output Gap*	*Potential Output Growth*
HP	0.96	0.97
BK	0.87	0.87
UCUR 1	0.94	0.97
UCUR 2	0.90	0.97
UCUR 3	0.83	0.96
UCUR 2BP	0.84	0.83

Figure 10

OUTPUT GAP VS UNIVARIATE FILTERS (%)

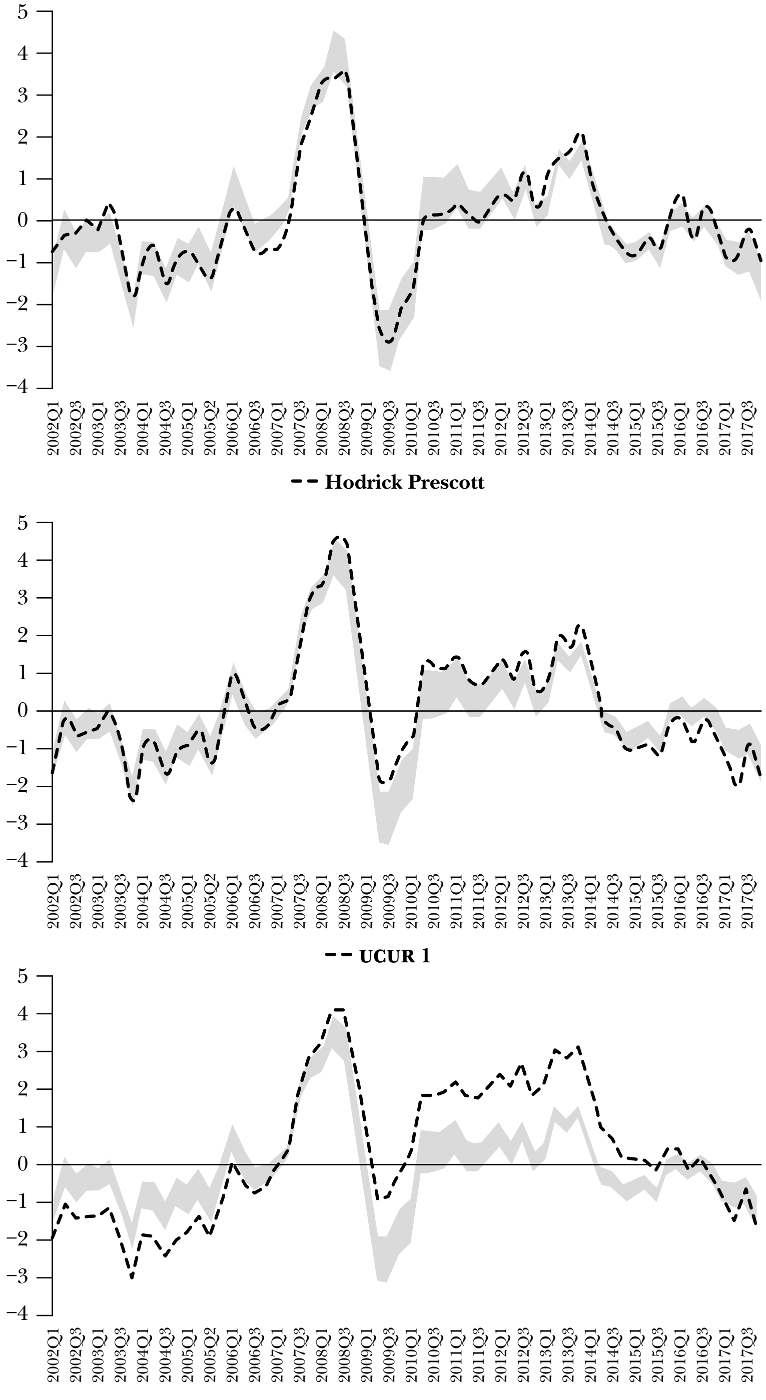
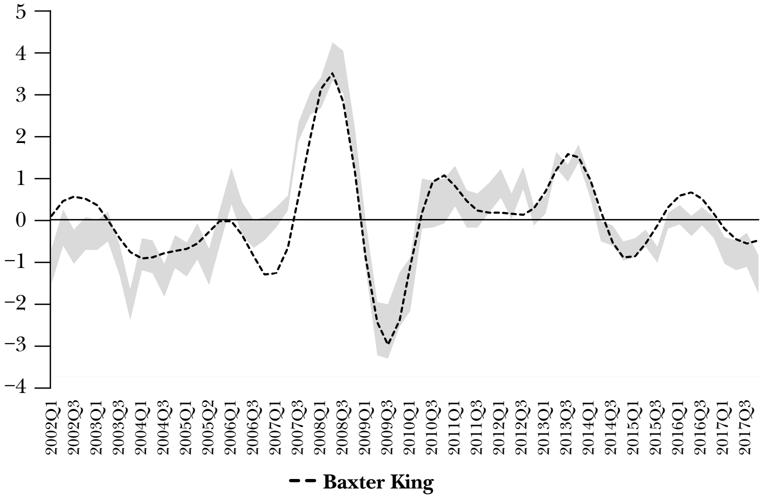
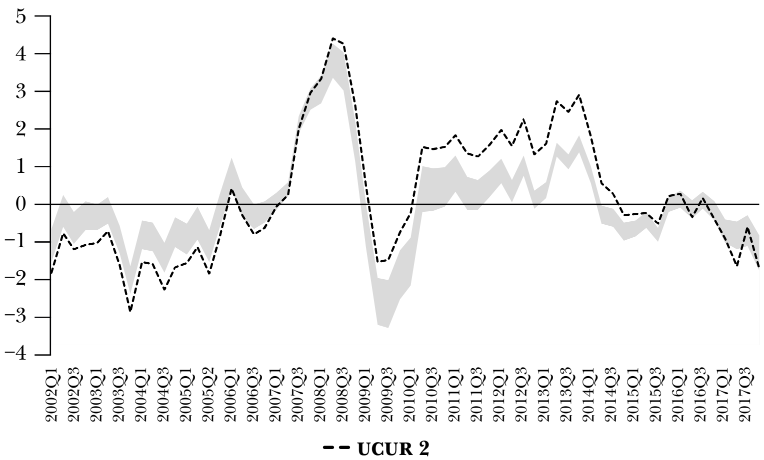


Figure 10 (cont.)

OUTPUT GAP VS UNIVARIATE FILTERS (%)





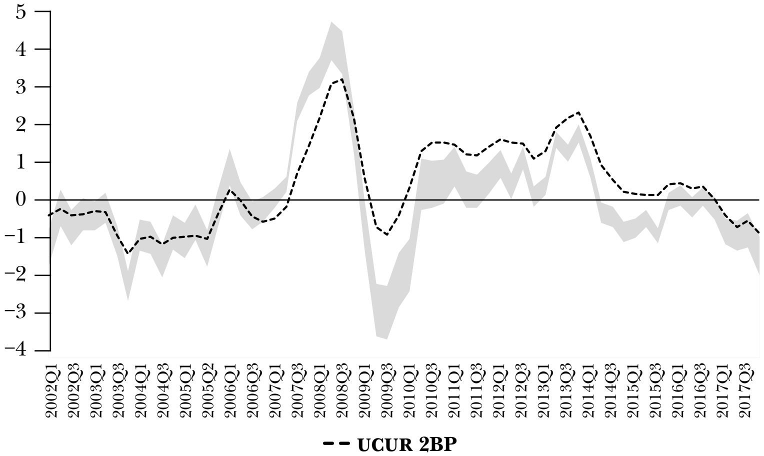


Figure 11

POTENTIAL OUTPUT GROWTH VS UNIVARIATE FILTERS (%)

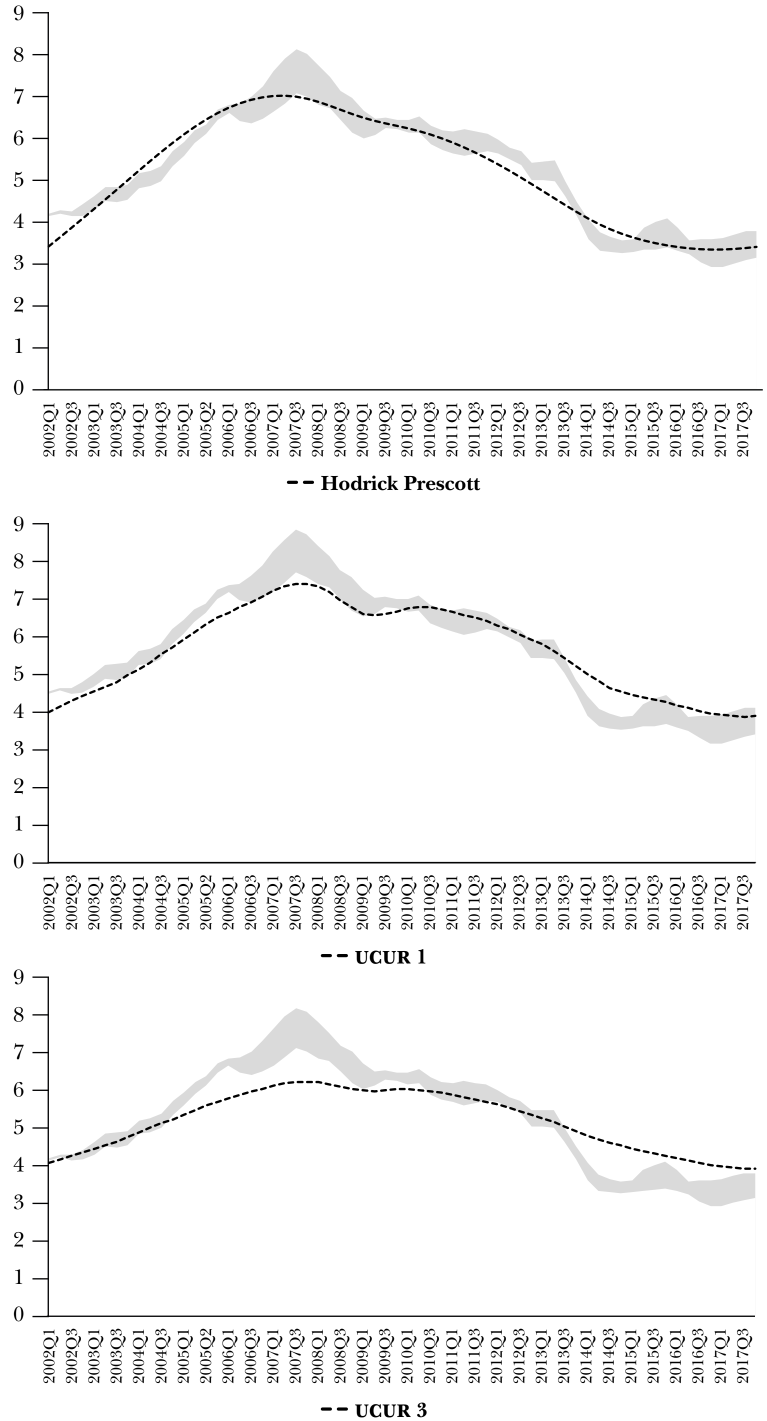
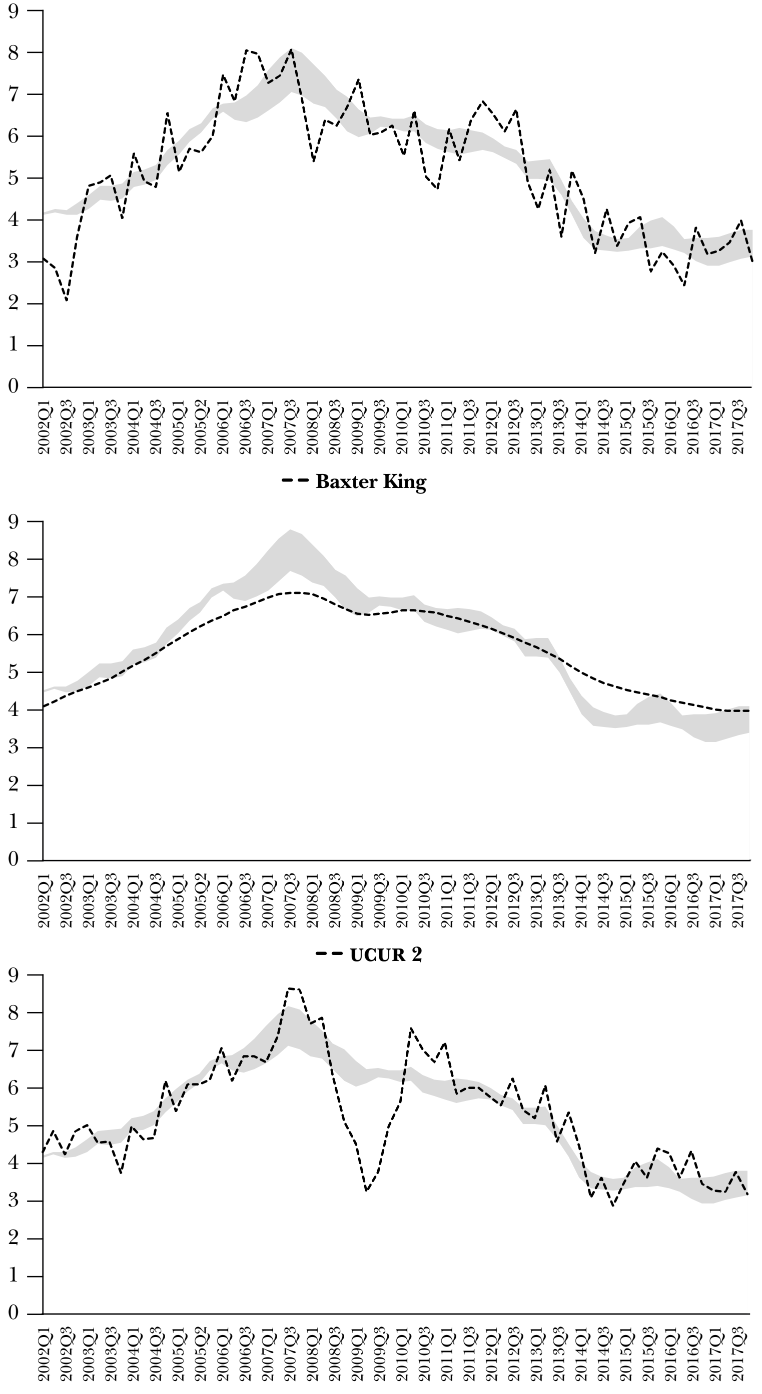


Figure 11 (cont.)

POTENTIAL OUTPUT GROWTH VS UNIVARIATE FILTERS (%)



7.2 Decomposing the ex ante real monetary rate

Our UC model for the estimation of the ex ante real monetary policy rate is based on Fiorentini et al. (2018), who make use of a local level model like Watson (1986) for the natural interest rate. In this model, the real monetary policy rate r_t is assumed to be the sum of a permanent (or trend component) r_t^* and a transitory component r_t^c. Again, due to the fact that the marginal likelihood is sensitive to prior specifications, we use two alternative priors, and label the results as UC1 and UC2 (see Appendix 9.4).[9] The structure of the models is given by the following equations:

22 $$r_t = r_t^* + r_t^c$$

23 $$r_t^* = r_{t-1}^* + u_t^{r^*}$$

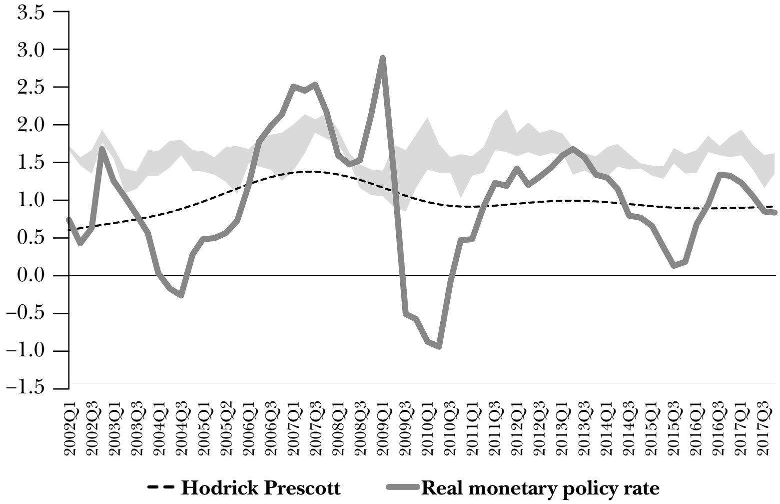
24 $$r_t^c = \alpha r_{t-1}^c + u_t^{r^c},$$

The innovations $u_t^{r^*}$ and $u_t^{r^c}$ are assumed to be uncorrelated and jointly normal $\rho = 0$.

We compare the results from these UC model estimations with our multivariate estimation of the real natural interest rate in Figure 12 (where the results from the HP and BK filters are also shown for comparison purposes). As it was explained in Section 4, we compute the real natural interest rate by subtracting 2.0% from the nominal natural interest rate that we get from the MPT multivariate filter. However, following the previous discussion that the equilibrium expected inflation rate does not necessarily coincide with this normative assumption, we also proceed to compare the results with a real natural interest rate constructed by subtracting 2.7% (average inflation expectations between 2002 and 2017) in Figure 13. Finally, Table 4 presents the correlations between our central estimation of the real natural interest rate and the univariate results (the correlation coefficient is the same with any of the two assumptions on the equilibrium expected inflation).

Figure 12

REAL NATURAL INTEREST RATE VS UNIVARIATE FILTERS (%)



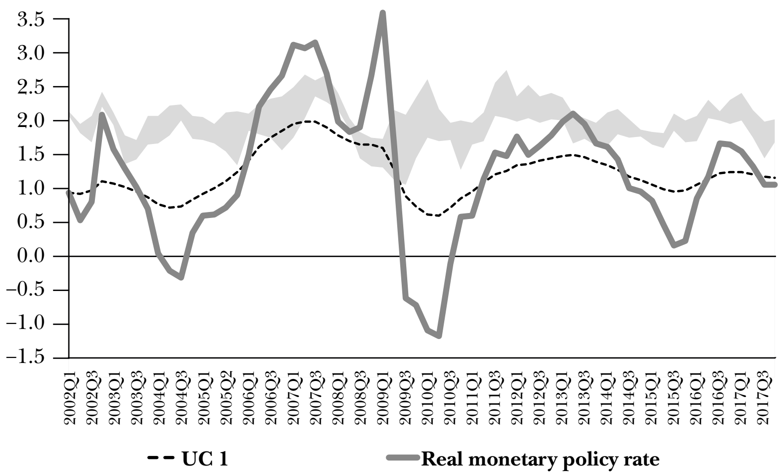
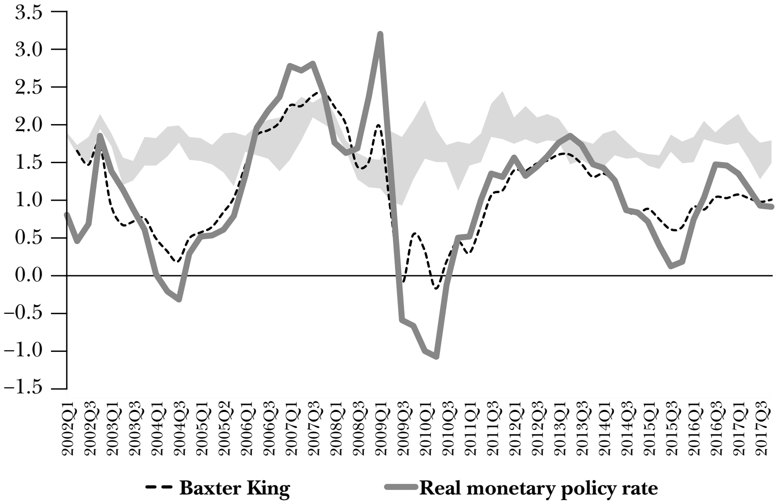


Figure 12 (cont.)

REAL NATURAL INTEREST RATE VS UNIVARIATE FILTERS (%)



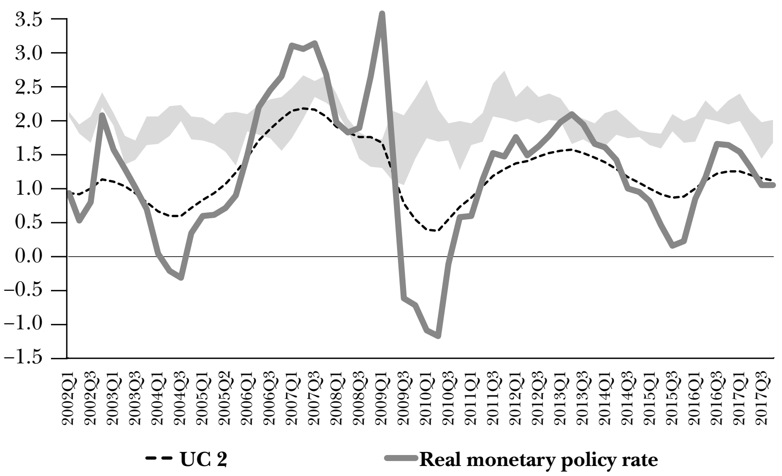
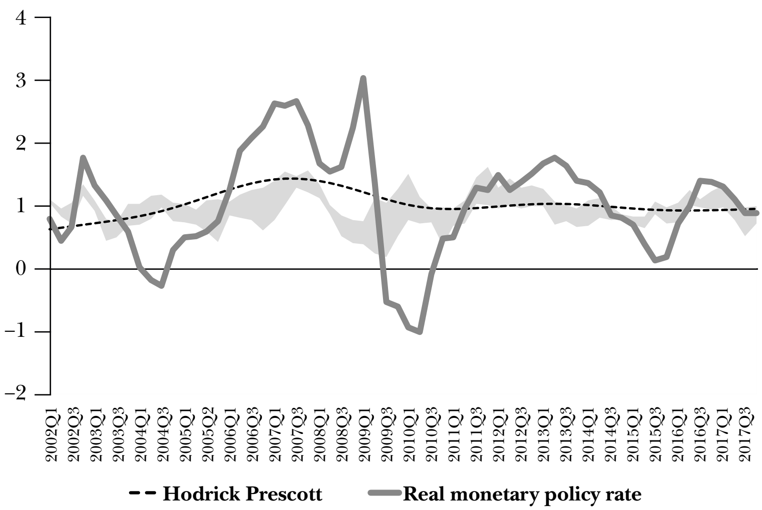


Figure 13

REAL NATURAL INTEREST RATE USING MARKET INFLATION EXPECTATIONS VS UNIVARIATE FILTERS (%)



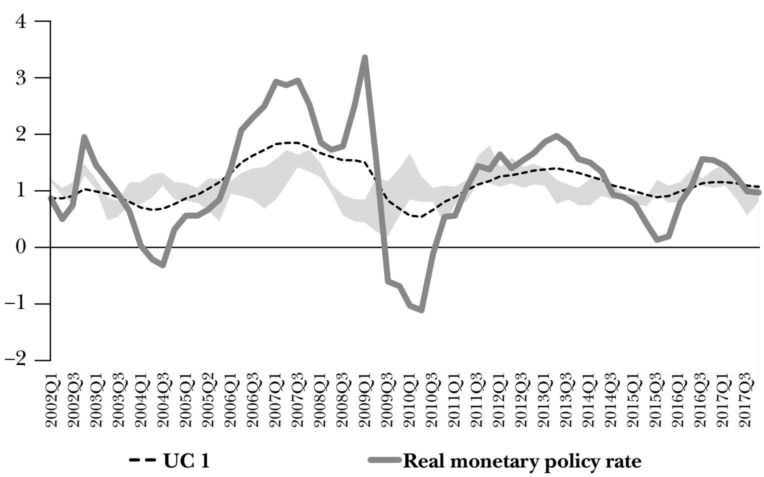
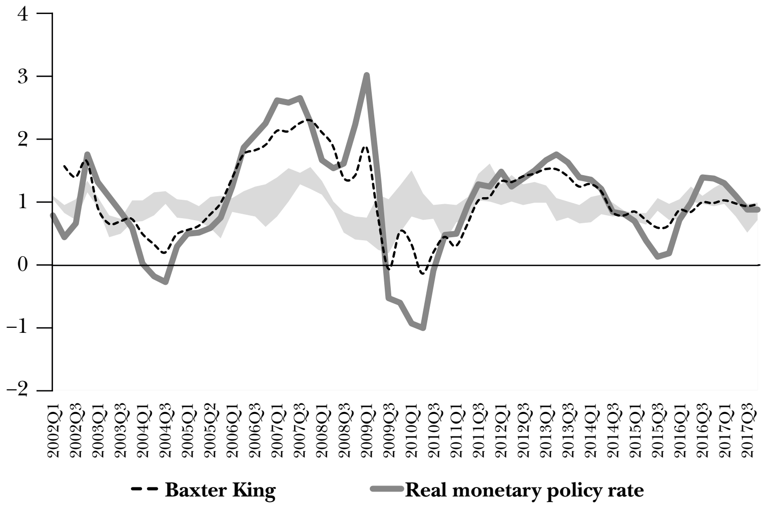


Figure 13 (cont.)

REAL NATURAL INTEREST RATE USING MARKET INFLATION EXPECTATIONS VS UNIVARIATE FILTERS (%)



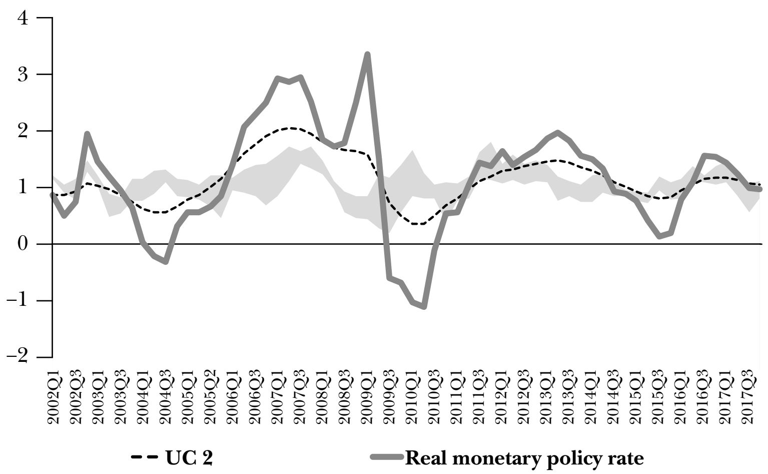


Table 4

CORRELATION OF NATURAL INTEREST RATE CENTRAL ESTIMATION WITH UNIVARIATE

Univariate Filter	*Natural Interest Rate* ($\Phi^e = 2\%$)
HP	0.22
BK	0.50
UC 1	0.40
UC 2	0.41

8. FINAL REMARKS

In this paper we employed a semi-structural dynamic model of the Peruvian economy (the MPT) to estimate the output gap, potential output growth and natural interest rate during the Inflation Targeting regime (2002Q1 - 2017Q4). This was accomplished by applying the Kalman filter and a smoother on the model, declaring different groups of variables as observable to account for the uncertainty risen from the selection of these variables.

From the results, we conclude that monetary policy has been very responsive to movements in the output gap, a trait that is desirable from any Central Bank with an inflation targeting mandate because the gap is a leading indicator of inflationary pressures. In fact, as the business cycle has changed position due to external and domestic events, the monetary policy rate has moved rapid and counter-cyclically to maintain monetary stability. Similarly, the results show that the natural interest rate has remained grossly stable, and that there has been loose domestic monetary conditions during most of the inflation targeting regime (real interest rate below the natural rate). Nevertheless, the BCRP has tightened or loosened monetary conditions according to the position of the business cycle.

The main finding, however, is that there has been a steady decline in potential output growth since 2012. A growth-accounting exercise, conducted to explain this phenomenon, shows that this decreasing trend follows mostly a reduction in TFP growth. Capital and labor also played a role with diminishing contributions between 2010-2013 and 2014-2017. However, this reduction only explains a third part of the average potential output growth slowdown across these periods. We do not deepen in the drivers behind TFP behaviour, leaving the analysis of this phenomenon for future studies. It is most likely that TFP reduction may reflect the persistent decline of terms of trade growth, or the lack of structural reforms throughout the last decades.

Therefore, the upward trend seen in commodity prices during 2017 and early 2018 may contribute to rise potential output growth by favouring investment in capital and by having positive effects on the TFP. These productivity gains may be more enduring if they are accompanied by: (i) reforms oriented toward improving infrastructure, connectivity and access to public services in Peru; (ii) the expansion of human capital by increasing the quality of educational and health services, and by fostering well-thought flexibility of the labor market; and (iii) the implementation of public policies oriented towards technology diffusion and knowledge transfer.

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9. APPENDIX

9.1 Data description

Table 5

STATISTICS AND DEFINITIONS

Variable	*Mean*	*St.Dev.*	*Definition*
Real GDP Growth	5.2	2.5	Annualized rate of q-o-q variation of seasonally-adjusted real GDP
Inflation without food and energy	2.1	1.2	Annualized rate of q-o-q variation of seasonally-adjusted CPI without food and energy
Inflation expectations	2.6	0.6	Quarterly average of 1-year forward headline inflation expectations
Impulse of business confidence	0	1.3	Gap of the 3-month in advance sector expectations index
Terms of trade growth	4.8	17.8	Annualized rate of q-o-q variation of terms of trade index
Short-term domestic interest rate	3.7	1.0	Quarterly average of BCRP's nominal monetary policy rate
Short-term foreign interest rate	1.7	1.6	Quarterly average of 3-Month LIBORrate
Real effective exchange rate gap	1.8	4.0	Gap of the real effective exchange rate

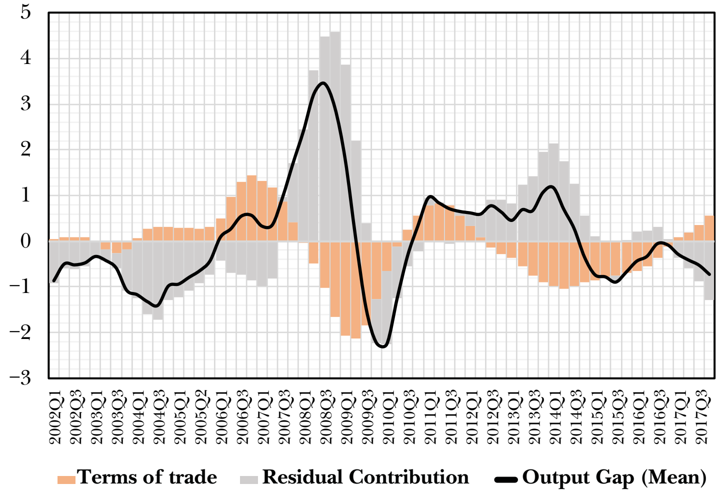
Note: Mean and standard deviation are calculated considering our forecast horizon (i.e. sample covers 2002Q1-2019Q4).

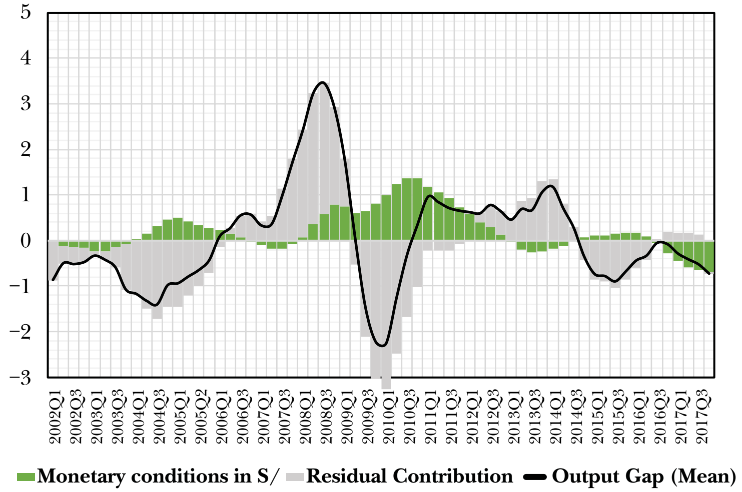
Table 6

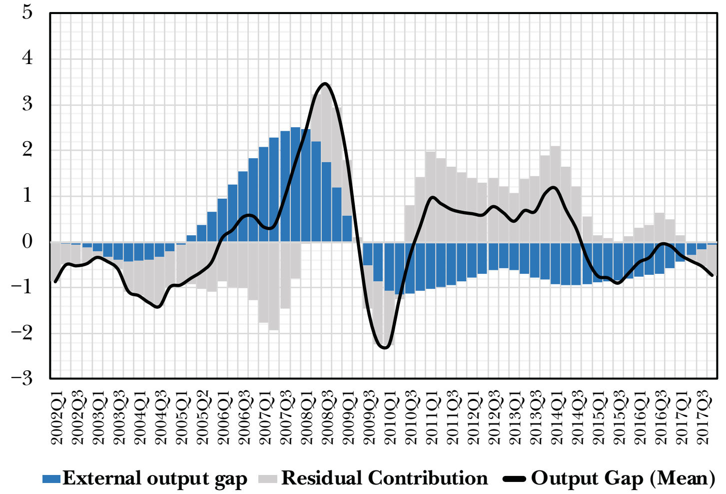
SOURCES AND NOTES

Variable	*Series code in BCRPData*	*Note*
Real GDP Growth	PN02516AQ	Seasonal adjustment is made with an X-13 ARIMA model in Eviews 9
Inflation without food and energy	PN01289PM	Seasonal adjustment is made with TRAMO-SEATS. Quarterly CPI is built by taking the quarter average of the monthly data
Inflation expectations	PD12912AM	
Impulse of business confidence	Encuesta de Expectativas Macroeconómicas Índices de confianza empresarial	Gap is built by substracting 60 (mean) and dividing by 5 (standard deviation)
Terms of trade growth	PN10029BQ	
Short-term domestic interest rate	PD04722MM	
Short-term foreign interest rate	PD31892XM	
Real effective exchange rate gap	PN01259PM	Equilibrium real effective exchange rate is estimated with the BEER method (cointegration relations built with terms of trade, trade openness, public spending and relative output per worker)

9.2 Historical shock decomposition: Detailed plots (%)






5
4
3
2
1
0
−1
−2
−3
2002Q1
2002Q3
2003Q1
2003Q3
2004Q1
2004Q3
2005Q1
2005Q2
2006Q1
2006Q3
2007Q1
2007Q3
2008Q1
2008Q3
2009Q1
2009Q3
2010Q1
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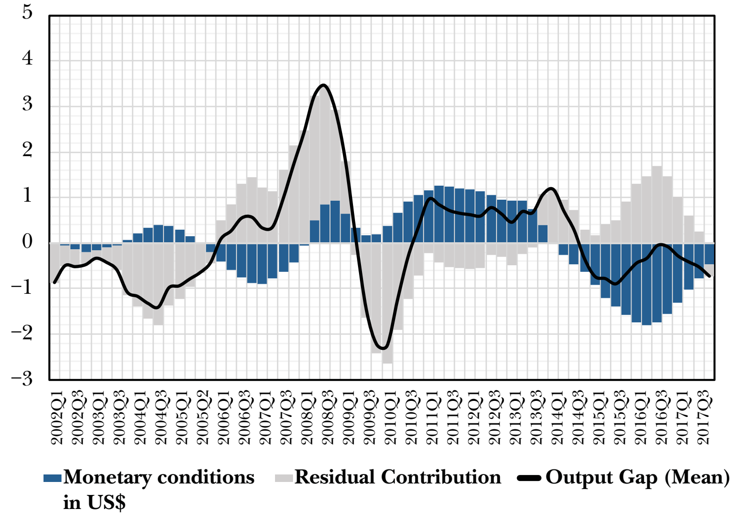
2011Q1
2011Q3
2012Q1
2012Q3
2013Q1
2013Q3
2014Q1
2014Q3
2015Q1
2015Q3
2016Q1
2016Q3
2017Q1
2017Q3
External output gap
Residual Contribution
Output Gap (Mean)


5
4
3
2
1
0
−1
−2
−3
2002Q1
2002Q3
2003Q1
2003Q3
2004Q1
2004Q3
2005Q1
2005Q2
2006Q1
2006Q3
2007Q1
2007Q3
2008Q1
2008Q3
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2009Q3
2010Q1
2010Q3
2011Q1
2011Q3
2012Q1
2012Q3
2013Q1
2013Q3
2014Q1
2014Q3
2015Q1
2015Q3
2016Q1
2016Q3
2017Q1
2017Q3
Monetary conditions in US$
Residual Contribution
Output Gap (Mean)

9.3 Prior distributions-Decomposition of GDP

Parameters	*Prior distributions*	*Hyperparameters - UCUR 1*	*Hyperparameters - UCUR 2*	*Hyperparameters - UCUR 3*
$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$	$\mathcal{N}(\Phi;\mathbb{V})$	$\Phi = \begin{bmatrix} 1.3 \\ -0.4 \end{bmatrix}$ $\mathbb{V} = \mathbb{I}$	$\Phi = \begin{bmatrix} 1.3 \\ -0.4 \end{bmatrix}$ $\mathbb{V} = \mathbb{I}$	$\Phi = \begin{bmatrix} 1.3 \\ -0.4 \end{bmatrix}$ $\mathbb{V} = \mathbb{I}$
τ_0	$\mathcal{N}(\mu; v)$	$\mu = 3.9$; $v = 100$	$\mu = 3.9$; $v = 100$	$\mu = 3.9$; $v = 100$
τ_{-1}	$\mathcal{N}(\mu; v)$	$\mu = 3.9$; $v = 100$	$\mu = 3.9$; $v = 100$	$\mu = 3.9$; $v = 100$
σ_c^2	U [a; b]	a = 0; b = 4.25	a = 0; b = 4.5	a = 0; b = 5.0
σ_τ^2	U [a; b]	a = 0; b = 0.1	a = 0; b = 0.01	a = 0; b = 0.25
ρ	U [a; b]	a = −1; b = 1	a = −1; b = 1	a = −1; b = 1

9.4 Prior distributions-Decomposition of ex ante real monetary rate

Parameters	*Prior distributions*	*Hyperparameters - UC 1*	*Hyperparameters - UC 2*
α	$\mathcal{N}(\mu; v)$	$\mu = 0.7$; $v = 1$	$\mu = 0.9$; $v = 1$
$\sigma_{r^c}^2$	U [a; b]	a = 0; b = 0.5	a = 0; b = 0.5
$\sigma_{r^*}^2$	U [a; b]	a = 0; b = 0.1	a = 0; b = 0.15