THE INTERDEPENDENCE OF FISCAL AND MONETARY POLICY

DEBT SUSTAINABILITY AND FISCAL SPACE IN A HETEROGENEOUS MONETARY UNION: NORMAL TIMES VS THE ZERO LOWER BOUND

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Editors:
Santiago García-Verdú and Jorge Ponce

2020 Joint Research Program

XXV Meeting of the Central Bank
Researchers Network

Debt Sustainability and Fiscal Space in a Heterogeneous Monetary Union: Normal Times vs the Zero Lower Bound*

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August, 2021

Abstract

In this paper we study fiscal policy effects and fiscal space for countries in a monetary union with different levels of public debt. We develop a dynamic stochastic general equilibrium (DSGE) model of a two-country monetary union, calibrated to match characteristics of Spain and Germany, in which debt sustainability is endogenously determined a la Bi (2012) to shape the responses of the risk premium on public debt. Policy shocks change the market's expectation about future primary surplus, producing a direct effect on the sovereign risk premium and macroeconomic responses of the economy. In normal times, the costs of a government spending driven fiscal consolidation in the high-debt country are greatly diminished when the consolidation improves its debt sustainability prospects. Fiscal consolidations in both members of the monetary union decrease real interest rates and amplify the reduction in risk premium in the highly-indebted country, improving union-wide output in the long run, at the cost of lower output in the low-debt country in the short run. On the contrary, when monetary policy is constrained at the zero lower bound, the risk premium channel arising from the endogenous determination of debt sustainability becomes muted.

JEL Codes: E31, E62, H30.

Keywords: fiscal multiplier, fiscal coordination, nonlinear DSGE models, zero lower bound.

*We thank Dennis Bonam, seminar participants at the Banco de España, the 21st Bank of Italy Workshop on Public Finance, the Fiscal Policy Seminar 2018 at the Federal Ministry of Finance in Berlin, the 2019 Computing in Economics and Finance Conference and an anonymous referee for their comments and suggestions. Javier Andrés acknowledges the financial support by the Spanish Ministry of Economy and Competitiveness (grant ECO2017-84632-R) and Generalitat Valenciana (Conselleria d'Educació, Investigació, Cultura i Esports, grant GVPROMETEO2016-097).

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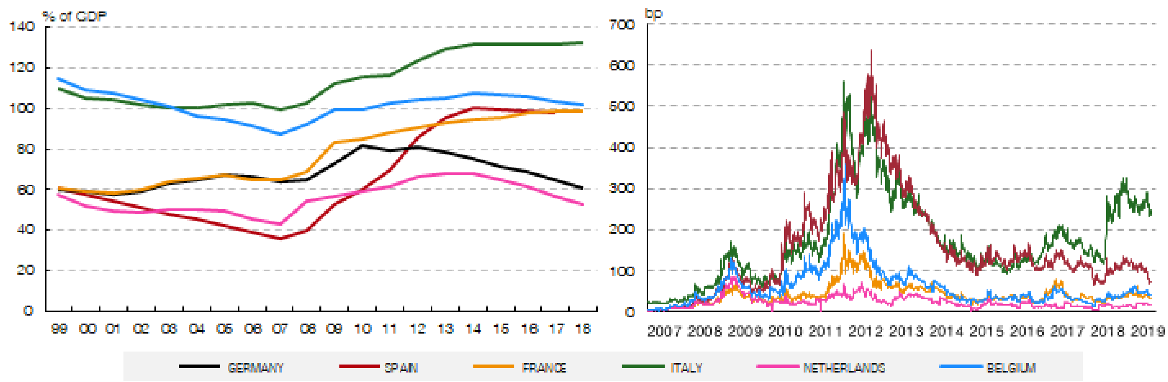
1 Introduction

The global financial and economic crisis left a legacy of historically high levels of public debt in advanced economies, at a scale unseen during modern peace time. Keeping public debt at high levels, however, is a source of vulnerability in itself, particularly given the arising fiscal and economic pressures from ageing. A high public debt burden is even more problematic in a monetary union like the euro area (EA), as monetary policy focuses on the EA aggregate while fiscal policy decisions remain at the national level. As shown in Figure (1), although the average debt-to-GDP ratio in the EA stays high at 90 percent, a great dispersion exists across countries. Countries with reasonably low debt levels by the end of 2018, standing at around or below 60 percent, including Germany (61 percent) and The Netherlands (52 percent), coexist with others characterized by high debt levels, around or above 100 percent of GDP, including France (98 percent), Italy (132 percent) and Spain (97 percent). In those highly indebted countries, borrowing costs have increased sharply, which undermine their solvency, all the more if they have to face a severe slowdown in economic growth in the near future. Moreover, risks to debt sustainability in a large Member State, like the ones recently experienced in Italy, can entail risks to the stabilization of the monetary union as a whole, while cross-country spillovers of disorderly default can threaten the very existence of the EA.

Figure 1: Evolution of government debt and spreads in the euro area



a) General Government Debt (% of GDP)
b) 10-year bond spreads

Despite the difficult financial position in some countries, the debate on fiscal strategies is far from settled in the Euro area. The mandate to reduce public deficits and debt is the official policy although there is some disagreement about the timing of that process. Also the weakening of the recovery phase has stirred up some fears that unconventional monetary stimuli might be losing steam and many voices have been raised in favor of a more expansionary fiscal stance, at least in the core countries of the EMU in which sustainability issues are less pressing. In this paper, we assess the effect of fiscal policy actions in a monetary union in which member countries differ in the risk of debt sustainability. Countries with low and high public debt-to-GDP ratios coexist.[1] Fiscal actions can be either unilateral or coordinated, and may happen in normal monetary policy times or at the zero lower bound (ZLB). We pay special attention to the cross-country spillovers of fiscal polices.

We extend a dynamic stochastic general equilibrium (DSGE) model of a two-country monetary union along the lines of Benigno & Benigno (2006), modified to allow for debt sustainability to be endogenously determined. In particular, (partial) government default may occur, so that a haircut is applied whenever debt in one member country of the monetary union becomes unsustainable. In the model, debt sustainability is captured by the "fiscal limit," defined as the expected discounted sum of maximum primary surplus that can be generated in the future (Bi 2012). Given fluctuations in fiscal policy shocks and political risk, the fiscal limit is stochastic and is represented by a distribution of the maximum debt-GDP levels that can be supported. Therefore, investors may demand risk premia on government debt before reaching the fiscal limit, generating a nonlinear relationship between sovereign risk premia and the level of government debt. As the high-debt (home) country approaches its fiscal limit, it pays a higher risk premium on its public debt. The low-debt (foreign) country, however, is far away from its fiscal limit and hence pays the risk free rate.

The simulated fiscal limits are state-dependent, accounting for the underlying economic structural and future policy uncertainties of both the home and foreign economics. Policy decisions affect the fiscal limit distribution and the sovereign risk premium.[2] The endogeneity of the fiscal limit gives rise to a sovereign risk premium channel and thus to cross-country spillovers (fiscal or otherwise) in the monetary union. These spillovers are asymmetric, inducing quite different macroeconomic responses in the country with high debt as compared with an economy operating well away from its fiscal limit.

With the simulated state-dependent fiscal limit distributions, we calibrate the model to match characteristics of Spain and Germany and analyze three fiscal issues in the European policy debate: the long-run consolidation process that a high-debt country must endure to converge back towards more sustainable debt levels, the impact of short-run discretionary fiscal policy in countries with strained public finances, and the effect of fiscal policy coordination between the high-debt and low-debt countries. For the high-debt country, adjusting to lower levels of public debt is always a costly and lengthy process. However, we find that under the Taylor rule, a discretionary consolidation effort can speed up such process in the high-debt country, as long as it helps take the debt-to-GDP ratio away from the risky zone (the proximity of the fiscal limit). A fiscal consolidation today implies higher future primary surpluses, shifting the fiscal limit distribution to the

[1] High and low debt will be defined more precisely later on.

[2] Our approach differs from the classic strategic sovereign default approach (Eaton & Gersovitz 1981, Aguiar & Gropinath 2006, Arellano 2008).

right and reducing the probability of default on impact. This effect on the risk premium is small but persistent in time, inducing sizable output effects. In our simulations, a 1 percent transitory government spending cut reduces the risk premium by 1.5 basis points (bps) on impact, reaching a maximum of 4 bps after 5 years, and it is still 2 bps lower after 10 years. The risk premium channel reduces financing costs and generates a cumulative output multiplier of -0.50 after 10 years (compared to 0.24 in a model without default risk), suggesting expansionary fiscal consolidation, consistent with the empirical findings in Giavazzi & Pagano (1990), Alesina et al. (1998), and Alesina & Ardagna (2010). Next, we analyze the spillover effects on a high debt country from the fiscal decisions of the low-debt country and fiscal policy coordination in the union. A fiscal expansion in the foreign (low-debt) country leads to inflation, and given its large size, increases the nominal interest rate in the Euro Area and real interest rates in both countries. The higher real rates lower demand in the home (high-debt) country, thus worsening its fiscal sustainability prospects and raising the risk premium. As debt accumulates faster, high tax rates further depress output in the home country and reduce union output in the long run. Alternatively, a fiscal consolidation in the foreign country lowers real interest rates. Higher demand and lower debt services increase home output, leading to long-run output gain in the union, at a cost of short-run output loss in the foreign country.

When monetary policy is constrained at the zero lower bound (ZLB), the real interest rate channel works against the risk premium channel arising from the state-dependent fiscal limit, which makes the response in risk premium muted. On the one hand, a discretionary fiscal consolidation at home improves future primary surplus and lowers the default probability. On the other hand, a fiscal consolidation generates deflation expectations that increase the real interest rate and the cost of rolling over debt, increasing the default probability. These two effects offset each other. Under our calibration, the effect of the increase in real interest rates dominates the improvement in risk premium, making fiscal consolidation counterproductive.

Our paper is related to several studies that connect sovereign risk premia and fiscal sustainability (Uribe & Yue 2006, Garcia-Cicco et al. 2010, Daniel & Shiamptanis 2012, Corsetti et al. 2013, Polito & Wickens 2015). These papers often assume fiscal limit distribution a function of debt, which we call "exogenous fiscal limit".[3] Our paper constructs model-consistent state-dependent fiscal limits and captures endogenous responses of fiscal limits to economic disturbances, which significantly strengthen the risk premium channel. In particular, we show that making the fiscal limit state-dependent reduces the government spending multiplier by 60 percent compared with what is obtained in the model with an exogenous fiscal limit.[4]

[3]For example, Corsetti et al. (2013) and Batini et al. (2018) propose a model where the euro area periphery government is faced with a fiscal limit following a beta distribution calibrated to Greece data.

[4]Similarly, Battistini et al. (2019) construct a dynamic Laffer Curve that incorporates the endogenous responses of risk premium to economic shocks.

Our analysis is also related to papers that study cross-border spillovers from fiscal stimulus, such as Corsetti et al. (2010), Arce et al. (2016), Blanchard et al. (2017) and Farhi & Werning (2016). These works find that fiscal adjustment instruments, structural reforms, and monetary policy all matter for the magnitude of fiscal spillovers in the Euro Area, but they do not incorporate default risk.

Finally, our paper is not meant to add to the theory of sovereign default, as in Eaton & Gersovitz (1981), Arellano (2008), Mendoza & Yue (2012) and Dovis (2018). Rather than making the default decision a strategic choice, we opt to treat the intrinsically political decision as a random draw from the fiscal limit distribution. The reason for this modelling choice is twofold. First, in a monetary union like the euro area a strategical default on its debt would imply leaving the union. This research question is related to the reasons to form a currency union, as in Chari et al. (2020), but beyond the scope of this paper. Second, our model retains the DSGE framework convenient for incorporating several economic and policy shocks and conducting fiscal experiments, without explicitly modeling the strategic default decision.

The paper is organized as follows. Section 2 describes the baseline model, while Section 3 details the model calibration. Section 4 explains how the model-consistent state-dependent fiscal limit is derived and how it is affected by macroeconomic fundamentals and policy decisions. In Section 5 the main fiscal policy experiments are described. Finally, Section 6 concludes.

2 Model

We use a two-country New Keynesian model to analyze a monetary union along the lines of Benigno & Benigno (2006), augmented with state-dependent fiscal limits and interest risk premia. Specifically, the monetary union consists of two countries, home and foreign, each inhabited by a continuum of households, with parameter s determining the relative size of the home country. The foreign variables are defined with an asterisk. To incorporate sovereign default risk and capture different fiscal positions of union members, our model allows for the possibility of (partial) government default in the home country: a haircut is applied when the home country's level of debt becomes unsustainable. The concept of debt sustainability is operationalized by the fiscal limit, which will be defined later. As debt approaches the fiscal limit, households in the home country may demand risk premia on government debt. We assume that the public debt-to-GDP ratio in the foreign country is sufficiently low so that, for simplicity, there is no fiscal limit in this country and it always pays the risk free rate. In the rest of the section we will describe the model for the home country and only mention the rest of the union when there is an asymmetry.

2.1 Households

The home country is populated by a large number of households indexed by $h \in [0, s)$, while those living in the foreign country are indexed by $f \in [s, 1]$. Preferences are given by:

$$\max_{c_t, B_t, n_t} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\varphi}}{1+\varphi} \right], \tag{1}$$

where β is the households' subjective discount factor, c_t is consumption and n_t the households' labor supply. The inverse of intertemporal elasticity of substitution, σ, measures relative risk aversion. The parameter φ governs the Frisch elasticity of labor supply. The household receives nominal wages W_t and monopoly profits Υ_t from the firm, both of which are taxed at the rate τ_t. The household maximizes utility subject to the budget constraint,

$$P_t c_t + \frac{B_t}{R_t} + \frac{D_t}{R_t^f} = (1 - \delta_t) B_{t-1} + D_{t-1} + (1 - \tau_t)(W_t n_t + P_{H,t} \Upsilon_t), \tag{2}$$

where P_t is the CPI and $P_{H,t}$ is the PPI.

The government debt in the home country, B_t, is subject to default risk. The default decisions depend on a realized effective fiscal limit, $\mathcal{B}_t^H$, drawn from a fiscal limit distribution $\mathcal{B}^H(\mathcal{S}_t)$, conditional on the state $\mathcal{S}_t$. Specifically,

$$\delta_t = \begin{cases} 0 & \text{if} \quad b_{t-1} < \mathcal{B}^H(\mathcal{S}_t) \\ \delta & \text{if} \quad b_{t-1} \geq \mathcal{B}^H(\mathcal{S}_t), \end{cases} \tag{3}$$

where $b_{t-1} = \frac{B_{t-1}}{P_t}$ is the real government debt. If the real value of debt at the beginning of period t, b_{t-1}, exceeds the effective fiscal limit, $\mathcal{B}_t^H$, then the government partially defaults and outstanding debt at the beginning of period t becomes $(1 - \delta) b_{t-1}$, otherwise it repays in full amount with $\delta_t = 0$. The derivation of $\mathcal{B}^H(\mathcal{S}_t)$ is described in Section (4.1). Government debt in the home country, therefore, pays a risky yield of R_t. In addition, households may also hold a risk-free bond, D_t, that pays the risk-free rate, R_t^f, with an aggregate zero net supply.

Optimization conditions for households in the home country are:

$$n_t^{\varphi} = \lambda_t (1 - \tau_t) w_t, \tag{4}$$

$$\lambda_t = \beta R_t E_t \frac{(1 - \delta_{t+1}) \lambda_{t+1}}{\pi_{t+1}}, \tag{5}$$

$$\lambda_t = \beta R_t^f E_t \frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{6}$$

where $\lambda_t = c_t^{-\sigma}$, $\pi_{t+1} = \frac{P_{t+1}}{P_t}$, and $w_t = \frac{W_t}{P_t}$ is the real wage. The latter two equations determine the interest rate spread on risky government debt. The households' optimization problem must also satisfy the following transversality condition:

$$\lim_{j\to\infty} E_t \beta^{j+1} \frac{\lambda_{t+j+1}}{\lambda_t}(1-\delta_{t+j+1})b_{t+j} = 0. \tag{7}$$

Since the foreign government will never default on its debt, foreign bonds (B_t^*) pay the risk-free rate (R_t^f). In this case, we have the standard intertemporal Euler equation:

$$\lambda_t^* = \beta R_t^f E_t \frac{\lambda_{t+1}^*}{\pi_{t+1}^*}, \tag{8}$$

where $\lambda_t^* = c_t^{*-\sigma}$, $\pi_{t+1}^* = \frac{P_{t+1}^*}{P_t^*}$. Using Euler equations in both countries ((6) and (8)), we can derive an arbitrage condition linking the real exchange rate, $RER_t = \frac{P_t^*}{P_t}$, to differences in nominal interest rates and consumption levels

$$RER_t = \Gamma_0 \frac{\lambda_t}{\lambda_t^*} \frac{R_{t-1}^f}{R_{t-1}\left(1-\delta_t\right)}, \tag{9}$$

where $\Gamma_0 = RER_0 \frac{\lambda_0}{\lambda_0^*} \frac{R_0^f}{R_0(1-\delta_1)}$, is a constant including only initial conditions for asset holdings and interest rates, which we assume equal to 1 to simplify the analysis.

2.2 Final Consumption Goods

Households consume the following basket of final goods produced at home, $c_{H,t}$, and abroad, $c_{F,t}$,

$$c_t = \left(\frac{c_{H,t}}{\eta}\right)^{\eta} \left(\frac{c_{F,t}}{1-\eta}\right)^{1-\eta}, \tag{10}$$

where η represents the preference by home consumers for goods produced at home. There exists home bias in consumption when $\eta > \frac{1}{2}$. The demand for final goods produced at home and abroad and the home consumer price index are

$$c_{H,t} = \eta \left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\eta} c_t = \eta tot_t^{1-\eta} c_t, \tag{11}$$

$$c_{F,t} = (1-\eta)\left(\frac{P_{F,t}}{P_{H,t}}\right)^{-\eta} c_t = (1-\eta) tot_t^{-\eta} c_t, \tag{12}$$

$$P_t = P_{H,t}^{\eta} P_{F,t}^{1-\eta}, \tag{13}$$

where $tot_t = P_{F,t}/P_{H,t}$ represents the relative terms of trade.

2.3 Final Intermediate Goods

Differentiated Intermediate goods produced at home $y_{H,t}(h)$ are bundled together into final home intermediate goods $y_{H,t}$, according to the following technology:

$$y_{H,t} = \left[\left(\frac{1}{s} \right)^{\frac{1}{\theta}} \int_0^s y_{H,t}(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \tag{14}$$

where θ represents the elasticity of substitution between different good-varieties, equal across regions, and $\frac{\theta}{\theta-1}$ is the price mark-up. These final intermediate goods can be used to produce final home or foreign consumption goods ($c_{H,t}(h)$ or $c^*_{H,t}(h)$) and home public spending (g_t). Cost minimization on the part of final goods producers results in the following demand curve for the intermediate home good, $y_{H,t}(h)$, and the corresponding home producer price index, $P_{H,t}$,

$$y_{H,t}(h) = \frac{1}{s} \left(\frac{p_{H,t}(h)}{P_{H,t}} \right)^{-\theta} y_{H,t}, \tag{15}$$

$$P_{H,t} = \left[\frac{1}{s} \int_0^s p_{H,t}(h)^{1-\theta} dh \right]^{\frac{1}{1-\theta}}. \tag{16}$$

2.4 Intermediate Goods Production

Intermediate goods producers adopt a linear production technology, $y_t(h) = a_t n_t(h)$, with real marginal costs, $mc_t(h) = \frac{P_t}{P_{H,t}} \frac{w_t}{a_t}$, and technology, a_t, assumed constant ($a_t = a$).[5] These firms enjoy some monopoly power in producing a differentiated product and therefore face a downward sloping demand curve, but are also subject to Rotemberg (1982) quadratic-adjustment costs in changing prices. That is, in each period, firms pay a cost proportional in real terms to aggregate real income $pac_t\,(i) = \frac{\psi}{2} \left(\frac{P_{H,t}(i)}{\pi P_{H,t-1}(i)} - 1 \right)^2 y_t$ to be able to change their prices and this penalizes large price changes in excess of steady state inflation rates. The dynamic problem of firm h is:

$$\max_{n_t(h), P_{H,t}(h)} E_t \sum_{t=\tau}^{\infty} \beta^t \frac{\lambda_t}{\lambda_\tau} \left[\frac{P_{H,t}(h)}{P_{H,t}} y_t(h) - mc_t y_t(h) - \frac{\psi}{2} \left(\frac{P_{H,t}(h)}{\pi P_{H,t-1}(h)} - 1 \right)^2 y_t \right], \tag{17}$$

subject to:

$$y_t(h) = a n_t(h) = \left(\frac{P_{H,t}(h)}{P_{H,t}} \right)^{-\theta} y_t. \tag{18}$$

[5] Note that we have defined the real wage in terms of the CPI ($w_t = \frac{W_t}{P_t}$), while the real marginal cost is defined in terms of domestic PPI ($mc_t(h) = \frac{MC_t(h)}{P_{H,t}}$).

The first order condition after imposing symmetry across firms is

$$(1-\theta)+\theta mc_t-\psi\left(\frac{\pi_{H,t}}{\pi}-1\right)\frac{\pi_{H,t}}{\pi}+\psi\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\pi_{H,t+1}}{\pi}-1\right)\frac{\pi_{H,t+1}}{\pi}\frac{y_{t+1}}{y_t}\right]=0, \quad (19)$$

which represents the home New Keynesian Phillips curve (NKPC) under Rotemberg pricing.

2.5 Government

The government (of each country member of the union) finances unproductive purchases (g_t) by collecting tax revenue and issuing one-period bonds (B_t). The tax revenue is raised through a distortionary time varying tax rate (τ_t) on labor income. The government sets the distortionary tax according to a tax rule and it faces the following budget constraint:

$$\frac{B_t}{R_t}+\tau_t\left[1-\frac{\psi}{2}\left(\frac{\pi_{H,t}}{\pi}-1\right)^2\right]P_{H,t}y_t=(1-\delta_t)\,B_{t-1}+P_{H,t}g_t. \quad (20)$$

Note that in the case of the home country, where default may happen, the relevant stock of debt is the one net of default ($(1-\delta_t)\,B_{t-1}$). We assume only domestic households may purchase domestic government bonds, that is, there is total home bias on domestic debt.[6] The government's budget constraint can be rewritten in real terms as

$$\frac{b_t}{R_t}+\frac{\tau_t\left[1-\frac{\psi}{2}\left(\frac{\pi_{H,t}}{\pi}-1\right)^2\right]y_t-g_t}{tot_t^{1-\eta}}=\frac{(1-\delta_t)\,b_{t-1}}{\pi_t}, \quad (21)$$

where $b_t=B_t/P_t$ is real government debt.

The distortionary tax is set according to a simple tax rule,[7]

$$\tau_t=\tau+\gamma_b(b_{t-1}-b), \quad (22)$$

where $\gamma_b>0$ is the tax adjustment parameter, so that a larger γ_b means that the government is more willing to retire debt by raising the tax rate, making debt converge back quicker to its long-run steady state. We assume that government purchases follow an AR(1) process

$$\ln\frac{g_t}{g}=\rho^g\ln\frac{g_{t-1}}{g}+\varepsilon_t^g. \quad (23)$$

[6]In our model, the defaulting government is neither forced to reform its policies by dramatically reducing deficits, nor is it locked out of credit markets for some period.

[7]The fiscal rule and the fiscal limit represent different constraints to the accumulation of public debt. The former ensures that the fiscal level is bounded and does not enter into an explosive path. The fiscal limit simply reflects the fact that there may be some level of debt that even when stable, it cannot be financed given the tax system and the political constraints in the country, and that gives rise to a non zero probability of default.

where g is the steady state government purchase at home.

2.6 Monetary Policy

The Central Bank of the Monetary Union sets the gross nominal interest rate to stabilize union-wide inflation and follows a regime-switching process:

$$R_t = \begin{cases} R + \alpha_\pi(\pi_{MU,t} - \pi_{MU}) & \text{if } s_t^R = 1 \\ 1 & \text{if } s_t^R = 2, \end{cases} \tag{24}$$

where α_π is the policy response to union-wide inflation, and $\pi_{MU,t} = s\pi_t + (1-s)\pi_t^*$. In a Taylor rule regime ($s_t^R = 1$), the Central Bank obeys the Taylor principle and in a zero lower bound regime ($s_t^R = 2$), the Central Bank exogenously pegs the gross nominal interest rate at one. Thus, all ZLB events are due to exogenous changes in s_t^R, and the switches between the two monetary policy regimes are similar to large exogenous shocks.[8]

The monetary policy regime index s_t^R evolves according to the transition matrix

$$\begin{pmatrix} p_1 & 1-p_1 \\ 1-p_2 & p_2 \end{pmatrix},$$

where $p_1(p_2)$ is the probability of continuing to stay in the Taylor rule (ZLB) regime each period, calibrated to be persistent.

2.7 Union-Wide Demand and Market Clearing

Union-wide demand for home goods, y_t^D, comes from the producers of home and foreign final consumption goods ($c_{H,t}$, $c_{H,t}^*$), government spending ($g_t = g_{H,t}$),[9] and price adjustment costs,

$$y_t^D(h) = sc_{H,t}(h) + sg_{H,t}(h) + (1-s)c_{H,t}^*(h) + \frac{\psi}{2}\left(\frac{\pi_{H,t}}{\pi} - 1\right)^2 y_t. \tag{25}$$

Substituting the demands from (15) above we get[10]

$$y_t^D(h) = \left(\frac{p_{H,t}(h)}{P_{H,t}}\right)^{-\theta}\left(\frac{tot_t^{1-\eta}}{s}c_{MU,t}^H + g_t\right) + \frac{\psi}{2}\left(\frac{\pi_{H,t}}{\pi} - 1\right)^2 y_t, \tag{26}$$

[8]We impose the ZLB by exogenous regime switching in monetary policy rules, similar to Richter & Throckmorton (2015), to minimize the number of state variables in solving the nonlinear model. In section 5, the responses of the real interest rate from fiscal shocks are qualitatively similar to the ones that would be found under endogenous ZLB events.

[9]We assume absolute home bias in government spending. This is a reasonable assumption since the import content of government spending in the largest Euro Area countries is very small, at around 10%.

[10]We assume the law of one price holds: (i.e.: the price of variety $h(f)$ of the home (foreign) good is equal at home and abroad).

where we define union-wide private consumption of home produced goods as $c^H_{MU,t} = \eta c_t + \eta^* \frac{1-s}{s} c^*_t$.

The real exchange rate, the ratio of relative consumption price levels, can be expressed as the ratio of the home and foreign producer prices

$$RER_t = \frac{P^*_t}{P_t} = \left(\frac{P_{F,t}}{P_{H,t}}\right)^{\eta-\eta^*} = tot_t^{\eta-\eta^*}. \tag{27}$$

To derive the equilibrium in the goods market in the home country we equate the demand for each intermediate good producer of the home product, equation (26), with its production function $y^D_t(h) = y_t(h)$ and aggregate across all home intermediate firms $\int_0^s y_t(h)dh$ to get

$$an_t \left[1 - \frac{\psi}{2}\left(\frac{\pi_{H,t}}{\pi} - 1\right)^2\right] = tot_t^{1-\eta} c^H_{MU,t} + g_t, \tag{28}$$

where we define home aggregate labor as $n_t = \int_0^s n_t(h)\, dh$.

Finally, union-wide output is defined as

$$y_{MU,t} = sy_t + (1-s)y^*_t. \tag{29}$$

3 Calibration

The model is calibrated at a quarterly frequency. In general, the home country is calibrated using data for Spain and the foreign country using data for Germany. However, there are a number of parameters which are common across countries. The household discount rate is 0.99. Preference over consumption is logarithmic, so $\sigma = 1$. The inverse of the Frisch elasticity of labor supply is set to $\varphi = 1$. The productivity levels at the steady state are normalized to 1.

The price elasticity of demand, θ, is assumed to be 11, indicating a steady state markup of 10 percent. Álvarez et al. (2006) and Vermeulen et al. (2012) find that prices in the euro area are sticky and price durations are significantly longer than in the U.S. In addition, survey results show that in the euro area about two-thirds of firms do not change their prices more than once a year (Fabiani et al. 2005). In line with this empirical evidence, we set the Rotemberg adjustment parameter, ψ, to 357.5, which implies that 15 percent of the firms reoptimize prices each quarter.[11]

The two countries are assumed to have the same degree of home bias, $\eta = 1-\eta^* = 0.63$, calibrated from Euro Area's import share. We calibrate the size of the home country by comparing the nominal GDP of the Euro Area periphery (Spain & Italy) vs core (Germany & France), and $s = 0.36$.[12] The fiscal parameters are calibrated to match

[11]See Ascari & Rossi (2012) for the equivalence of the first-order condition on the NK Phillips curve for the Rotemberg and Calvo specifications on price stickiness.

[12]Thus, the relative size of the domestic economy ($s = 0.36$) is meant to encompass a broader group

Spain and German data since the creation of the euro area (1999-2016). In steady state, government purchases are 18.3 and 18.7 percent of GDP, respectively, and the tax rates are 0.3005 and 0.3425, while the steady state debt-to-GDP ratio is 0.6 for both countries and the model implied lump-sum transfers are 9.5 and 13 percent of GDP. The tax adjustment parameter in the fiscal rule γ_b is calibrated to 0.04. The magnitude of fiscal adjustments is kept small, just sufficient to satisfy the transversality condition for government debt.

The shock processes for g_t and g_t^* are calibrated based on the empirical evidence for the euro area and Spain and Germany. For instance, Gadatsch et al. (2015) and Batini et al. (2018) estimate a model of a monetary union with Spain and Germany as members and get the following parameter values for the government spending processes: $\rho^g = \rho^{g^*} = 0.9$ and $\sigma^g = \sigma^{g^*} = 0.01$. These numbers are in line with the theoretical literature (see Schmitt-Grohe & Uribe 2007).

The steady-state inflation rate is assumed to be one. The Taylor rule parameter, α_π, is 1.5.[13] For the transition probability in the regime switching process of monetary policy, we use $p_1 = 0.9917$, which gives an average length of a Taylor rule regime of 30 years, and a less persistent ZLB regime is required to maintain the stationarity of the equilibrium system. We calibrate $p_2 = 0.65$, which implies an average length of a ZLB regime of 2.8 quarters.

Our default scheme assumes a constant haircut rate, δ. Bi (2012) uses the estimated haircut rates of sovereign debt restructures in emerging market economies between 1998 and 2005 from Sturzenegger & Zettelmeyer (2008), and calculates that 90 percent of the annual haircut rates (as a share of all sovereign debt) fall below 0.3. Thus, we assume a constant annual haircut rate of 0.28, implying a quarterly rate of $\delta = 0.07$. Table (1) summarizes the parameter values.

Appendix A lists equations that characterize the equilibrium. We use the monotone mapping method of Coleman II (1991) and Davig (2004) to obtain a fully nonlinear solution. Appendix B describes the numerical solution method.

of countries in the union with comparable debt sustainability problems, so that fiscal responses in this group of countries exert some meaningful effects on the monetary union as a whole.

[13] Bi et al. (2018) show that a higher Taylor rule parameter is needed when the rule is targeting the risky rate and the haircut δ is sizable. On the contrary, this is not the case when the rule is targeting the risk-free rate like in our model.

Table 1: Parameter calibration

parameters	values	
β	0.99	The discount factor.
σ	1	Inverse of the intertemporal elasticity of substitution.
φ	1	Inverse of Frisch elasticity of labor supply.
θ	11	Elasticity of substitution.
ψ	357.5	Rotemberg adjustment parameter.
η	0.63	Home country bias in home goods.
η^*	0.37	Foreign country bias in home goods.
s	0.36	Share of home country.
b/y	0.6	Steady state debt to output ratio (home).
b^*/b^*	0.6	Steady state debt to output ratio (foreign).
g/y	0.183	Steady state gov spending to output ratio (home).
g^*/y^*	0.187	Steady state gov spending to output ratio (foreign).
τ	0.3005	Steady state income tax rate (home),
τ^*	0.3425	Steady state income tax rate (foreign),
γ_b	0.04	Tax response parameter to changes in debt.
ρ^g, ρ^{g*}	0.9	AR(1) coefficient in government spending rules.
σ_g, σ_{g*}	0.01	Standard deviation of government spending shock.
α_π	1.5	Taylor rule parameter to inflation.
p_1	0.9917	Regime-switching parameter for the normal monetary policy regime.
p_2	0.65	Regime-switching parameter for the ZLB regime.
δ	0.07	Quarterly haircut on debt if default occurs.

4 Fiscal Limit Distribution

4.1 Simulating Fiscal Limit Distribution

Following Bi (2012), fiscal limits are defined as the present value of maximum future primary surpluses over an infinite horizon. Government spending, monetary policy regimes, and institutional quality vary with the stochastic shocks hitting the economy, generating a distribution for the maximum debt level that a government is able to service.

When simulating fiscal limits, we set $\delta_t = 0$ for all t, since the fiscal limits are the maximum level of debt that a government can support without default. We derive the intertemporal government budget constraint given the real government budget constraint, (21), the Euler equation, (6), and the transversality condition, (7),

$$b_{t-1} = \pi_t E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \frac{T_{t+j} - g_{t+j} - z}{tot_{t+j}^{1-\eta}}. \tag{30}$$

Following Bi et al. (2016), fiscal limits are simulated based on (30), but all the variables are computed under $\tau_{t+i} = \tau^{\max}$, the maximum income tax rate a government is willing and able to impose, as in (31). We set $\tau^{\max} = 0.435$, the marginal statutory rate for highest income earners in Spain (above $60,000$ € per year) (European Commission

2018).[14]

$$\mathcal{B}^H(\mathcal{S}_t) = \beta_t^p \pi^{\max}(\mathcal{S}_t) E_t \sum_{j=0}^{\infty} \beta^j \frac{1}{(tot^{\max}(\mathcal{S}_{t+j}))^{1-\eta}} \frac{\lambda^{\max}(\mathcal{S}_{t+j})}{\lambda^{\max}(\mathcal{S}_t)} (\mathcal{T}^{\max}(\mathcal{S}_{t+j}) - g_{t+j} - z). \quad (31)$$

The simulated fiscal limits are uncertain and state-dependent, conditional on an initial state of the economy $\mathcal{S}_t = \left\{g_t, g_t^*, tot_{t-1}, s_t^R\right\}$. The fiscal limit also captures the private sector's perception of the limit, as it uses the stochastic discount factor evaluated at the maximum tax rate, $\beta^j \frac{\lambda^{\max}(\mathcal{S}_{t+j})}{\lambda^{\max}(\mathcal{S}_t)}$, and allows for a stochastic political risk (β_t^p) that follows an AR(1) process,

$$\ln \frac{\beta_t^p}{\beta^p} = \rho^{\beta^p} \ln \frac{\beta_{t-1}^p}{\beta^p} + \varepsilon_t^{\beta^p}, \qquad \varepsilon_t^{\beta^p} \sim N(0, (\sigma^{\beta^p})^2). \quad (32)$$

Lower β_t^p indicates higher political risk and hence lower fiscal limits. A possible interpretation for this assumption is that the policy makers have a shorter planning horizon than the private sector (Bi et al. 2018). In the data, risk premia in several European countries start to increase even at lower levels of debt. Setting $\beta^p < 1$ and $\varepsilon_t^{\beta^p} \sim N(0, (\sigma^{\beta^p})^2)$ serves to generates movements in risk premia as observed in the data. In particular, we calibrate the political risk in the home country by using an indicator about the current political situation derived from a Spanish nation-wide sociological survey (see Gil et al. (2017)) to get $\beta^p = 0.37$, $\rho^{\beta^p} = 0.96$, and $\sigma^{\beta^p} = 0.13$. This allows us to match Spain's risk premium (below 100 bps) at its current debt level (97% of GDP).

We simulate the distributions of fiscal limits using Markov Chain Monte Carlo method, which is described in Appendix B.1. As shown in (31), each draw of a fiscal limit from the distribution is conditional on the current state, $\mathcal{S}_t$, and particular sequences of realized shocks in the Markov Chain Monte Carlo simulations. When simulating the fiscal limit, if the state variables of the system ($\mathcal{S}_t = \{g, g^*, tot, 1\}$) start from their steady state we define it as the unconditional fiscal limit, while when they start away from their steady state we name it state-dependent fiscal limit.

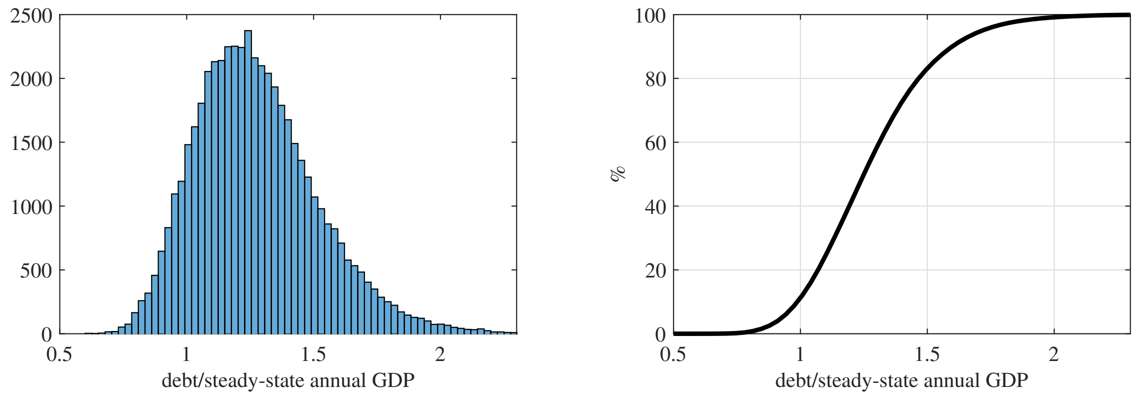
4.2 Unconditional Fiscal Limit:

As a baseline, Figure (2) plots the histogram of the simulated unconditional fiscal limits for the home country and the corresponding cumulative density function (cdf), starting

[14] Another way to quantify the fiscal limit is the Laffer curve. Bi (2012) derives the peak of the Laffer curve analytically in a real business cycle model. In a nominal model, Bi et al. (2018) assume that the Central Bank is able to set the inflation rate equal to its objective, which allows for a simple solution for the main variables determining the maximum of the Laffer curve. However, in a monetary union setting the aggregate inflation at its target does not guarantee that each country's inflation is also equal to its target, and thus it does not allow for an analytical solution of the Laffer Curve. Trabandt & Uhlig (2011) use a model-based approach to simulate the theoretical Laffer Curve and find that for plausible calibration, the peak of the Laffer Curve for Spain is 0.415.

from the steady state and a Taylor rule regime ($\mathcal{S}_t = \{g, g^*, tot, 1\}$). The x-axis plots the ratio of government debt to steady-state annual GDP. The histogram in the left panel indicates that the fiscal limit is centered around a debt-to-GDP ratio of 125 percent ($\frac{\overline{\mathcal{B}^H}}{y} = 1.25$) with a standard deviation ($\sigma^{\frac{\mathcal{B}^H}{y}}$) of about 0.24.[15]

Figure 2: Distribution of unconditional fiscal limit computed
from expression for $\mathcal{B}^H$ in (31)



(a) Histogram

(b) Cumulative density function (cdf)

Although default does not occur when simulating fiscal limits, recall the default mechanism in (3), which makes the role of fiscal limits in default decisions clear,

$$\delta_t = \begin{cases} 0 & \text{if} \quad b_{t-1} < \mathcal{B}^H(\mathcal{S}_t) \\ \delta & \text{if} \quad b_{t-1} \geq \mathcal{B}^H(\mathcal{S}_t). \end{cases}$$

The fiscal limit, $\mathcal{B}^H(\mathcal{S}_t)$, is uncertain and describes the stochastic upper bound on how much debt a government is willing and able to service given the economic and political constraints. Thus, the cumulative distribution function (cdf) of the home fiscal limit in Figure (2b) can be interpreted as the probability of the home government defaulting on its debt, which is nil for debt levels close to 80 percent of GDP, while it is close to 1 for debt levels above 200 percent of GDP. In between those values, the probability of default gradually increases as debt accumulates.

A large literature adopts strategic sovereign default approach in which an optimizing government accounts for some economic costs in making default decisions (Eaton & Gersovitz 1981, Aguiar & Gropinath 2006, Arellano 2008, Yue 2010, Dovis 2018). Another literature incorporates default risk by exogenously specifying fiscal limits (Daniel & Shiamptanis 2012, Corsetti et al. 2013, Batini et al. 2018). Instead, we follow a different approach. Although the government in the model does not optimize over its default decisions, our definition of the fiscal limit captures uncertainty in default risk. Moreover,

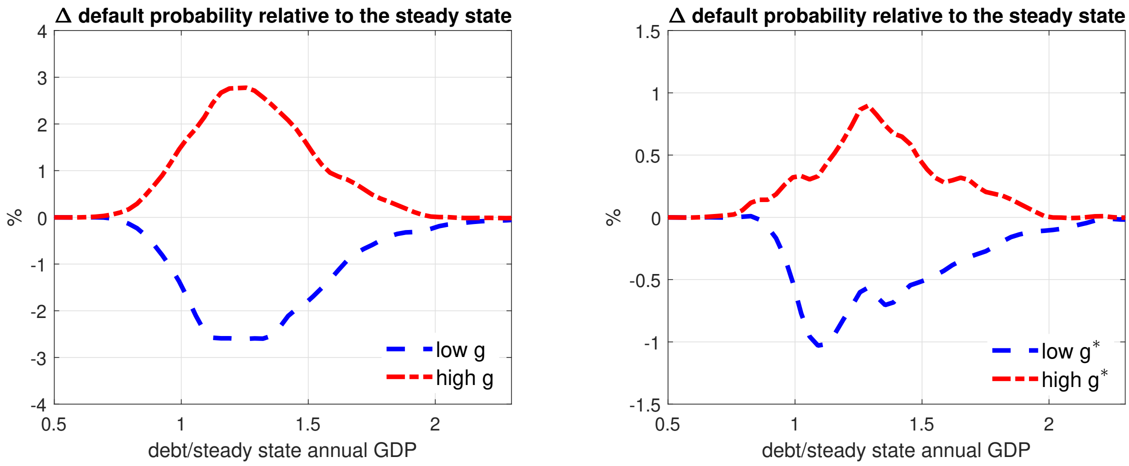
[15]The histogram has a slightly longer right tail. This asymmetry is due to the effect of the stochastic process estimated for the political factor, which is bounded above zero and has a fairly large standard deviation. The simulated distribution of the fiscal limit without the political factor is symmetric.

the fiscal limit, when included in the full model, responds endogenously to economic disturbances, which we will discuss in the next section.

4.3 State-Dependent Fiscal Limit: The Effect of Fiscal and Monetary Policy

In line with the definition of fiscal limit in (31), the state of the economy can have a significant impact upon the default probability in the home country. A change in fiscal or monetary variables changes the household's perception of debt sustainability and thus can shift the fiscal limit. Figure (3a) compares the changes in the home country's default probability conditional on different initial government spending levels, relative to the steady state. In particular, the red dash-dotted (blue dashed) line represents the change in the default probability when home's government spending starts 10 percent above (below) the steady state value.[16]

Figure 3: State-dependent distributions of fiscal limits:
change in government spending



(a) Home gov spending (g) (b) Foreign gov spending (g*)

A fiscal expansion consists of a 10% higher initial level of government spending increases aggregate demand and generates more tax revenues. The fiscal expansion, however, also raises public deficit today and worsens the sustainability perspectives of home government's finances, shifting its fiscal limit to the left and increasing its default probability for debt levels between 80 and 200 percent of GDP (red dash-dotted line in Figure 3a).[17] On the contrary, a fiscal consolidation with a 10% lower initial government spending improves debt sustainability and decreases the default probability. The maximum

[16]In the state-dependent fiscal limits case, with $g_t > g$ ($g_t < g$), spending it is likely to remain above (below) the unconditional case for most simulations since it follows a very persistent process.

[17]The marginal change in the default probability from changes in home government spending is the largest when debt to steady-state annual GDP reaches 120 percent, where the slope of the estimated cdf is the steepest.

impact of a 10 percent increase (reduction) in government spending raises (lowers) the default probability by 3 percentage points, that is from 38 percent to 41 percent (to 35 percent), when debt to steady-state annual GDP is 120 percent.

When the initial government spending changes in the foreign (low debt) country instead, the model generates spillover effects on the home country's fiscal limit. Figure (3b) shows the change in the default probability of home's fiscal limit in response to changes in foreign government spending, while keeping the other state variables at the steady state. A foreign fiscal expansion with a 10% higher initial government spending increases foreign output and inflation. As monetary policy follows the Taylor rule ($s_t^R = 1$), given the greater size of the foreign country, fiscal expansions lead to higher nominal interest rate, which also raise the real interest rate in the home, thus depressing home's demand and increasing its financing cost of debt. This negative real interest rate channel worsens the perspectives of home's public finances, reducing its fiscal limit. In addition, high foreign government spending crowds out foreign private consumption and therefore lowers foreign demand for home goods, which reduces exports in the home country. The negative trade channel reinforces the negative real interest rate channel that increases home's default probability (red dash-dotted line in Figure 3b). In quantitative terms, the negative spillover effect of a fiscal expansion abroad on home's probability of default on its public debt is about a third of the size of the direct effect of its own fiscal expansion.[18]

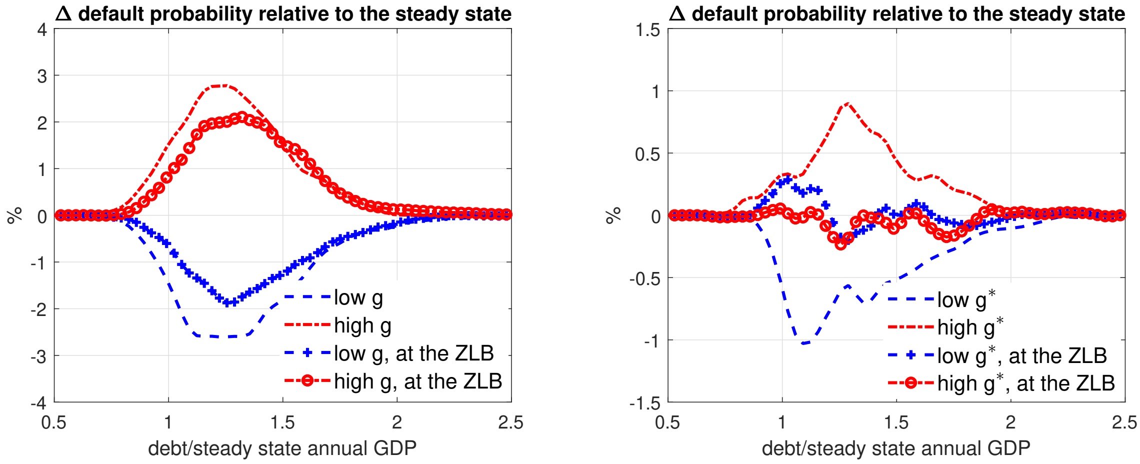
The effects of government spending on fiscal limits also interact with the monetary policy regime. Figure (4a) compares the changes in default probability conditional on home government spending in a Taylor rule regime and in a binding ZLB monetary regime.[19] Changes in the default probability due to home government spending are smaller in the ZLB regime relative to the Taylor rule regime. In a Taylor rule regime, a high home government spending increases the real interest rate, lowering demand and worsening fiscal sustainability. The binding ZLB, on the other hand, counteracts the effects on real interest rates. As the nominal interest rate is constrained, the expectation of higher inflation caused by fiscal expansion lowers the real interest rate, increasing domestic demand and generating more tax revenue, and hence reduces the increase in the default probability. Figure (4b) shows that being in the ZLB regime almost completely mitigates the spillover from foreign government spending to the home default probability. In a Taylor rule regime, as mentioned earlier, a negative trade channel brings up the default probability. When the ZLB is binding, although the trade channel is still present, nominal rates do not rise and the negative trade channel is counteracted by a fall in the

[18]This negative trade channel could be mitigated by allowing public spending to be a basket of home and foreign goods and/or assuming that consumers derive utility from the provision of public goods, which then become complements of private consumption and thus reduce the crowding-in effects. However, the quantitative relevance of the former element is very small in reality, since the weight of foreign goods in government spending is close to 10 per cent for most euro area countries.

[19]Except for the initial period, the monetary policy regime evolves according to the regime-switching process in (24).

real interest rate, making the increase in the default probability smaller.

Figure 4: State-dependent distributions of fiscal limits under the ZLB: government spending



(a) Home gov spending (g) (b) Foreign gov spending (g*)

The impact of the binding ZLB on default probabilities, depends on the persistency of the ZLB regime. If the ZLB regime is expected to last for longer, the response of real interest rates to inflation expectations is augmented and the monetary channel gets stronger. As we calibrate the average duration of the ZLB regime to be relatively short, the quantitative impact of the ZLB regime as shown in Figure (4) can be understood as a lower bound. If the persistency in the ZLB regime were higher, the positive monetary channel could dominate the negative trade channel. A high foreign government spending could even reduce the home default probability.

4.4 Exogenous Fiscal Limit

An emerging literature studies the effects of fiscal limits in a monetary union. In particular, Corsetti et al. (2013), Corsetti et al. (2014), and Batini et al. (2018) allow for the possibility of home government default on public debt and hence pay a risk premium. In these papers, the fiscal limit follows a distribution (normally a logistic or beta) which is a function of the country's debt-to-GDP ratio. As debt is a slow moving predetermined variable, the risk premium generated in the literature does not capture the immediate market perception on the country's debt sustainability in response to policy shocks. To compare our setting to the previous literature, although the fiscal limit is model-based, we map the simulated unconditional fiscal limit (as shown in Figure 2b) to a logistic function, which we name this version of the model the "exogenous fiscal limit" case.[20]

[20]In terms of our default equation, (3), this is equivalent to making $\mathcal{B}^H$ a logistic function of b_{t-1}. This setting of the fiscal limit is not completely exogenous as there exists an endogenous feedback between debt and the risk premium. We name it "exogenous fiscal limit" to differentiate the model consistent state-dependent fiscal limit described in Section (4.3).

Finally, to compare our results to those obtained from a model without risk premium, we set $\delta = 0$ in equation (3), which we name "no default" case.

5 Fiscal Policy in a Monetary Union

We now present several fiscal issues in the current policy debate with the full non-linear model incorporating the state-dependent fiscal limit discussed in the previous section. First, we analyze the long-term process of public deleveraging required for high-debt countries to converge back to the target levels, and how the speed of fiscal adjustments determines its cost. Second, we look at the effect of discretionary policy measures along this process of convergence. In particular, we will show the effect on the economy of a transitory fiscal consolidation in a member of the euro area with high debt, and a fiscal policy coordination between both countries. These cases are analyzed under two monetary policy regimes: the Taylor rule and the ZLB.

5.1 Long-Run Fiscal Consolidation at Home

One of the current main challenges in the euro area is for high debt countries to converge back towards more sustainable debt levels. In fact, the Stability and Growth Pact (SGP) sets a limit of 60 percent of GDP for public debt, beyond which the debt rule is active.[21] Given the high level of government debt in many countries, this implies a long-term process of consolidation, which could take several decades, and affect the area as a whole. This process is the focus of this section.

As shown in the previous section, the state-dependent fiscal limit becomes relevant when debt-to-GDP ratio is around 100 percent. We consider a monetary union of two countries with different levels of debt. The home country is highly-indebted, while the foreign country maintains its debt at the steady state. In particular, to achieve this we set the initial stock of home debt (b_{t-1}) to 100 percent of GDP at the beginning of the scenario ($t = 0$), which generates a risk premium of almost 80 basis points, and then, we let the fiscal and monetary rules bring the economy back to its steady state.[22] This initial state is in line with the situation in Spain and Germany at the end of 2018, with debt-to-GDP ratios of 97.1 percent and 60.9 percent, respectively, and a spread between the Spanish and German 10-year bonds around 100 bps.

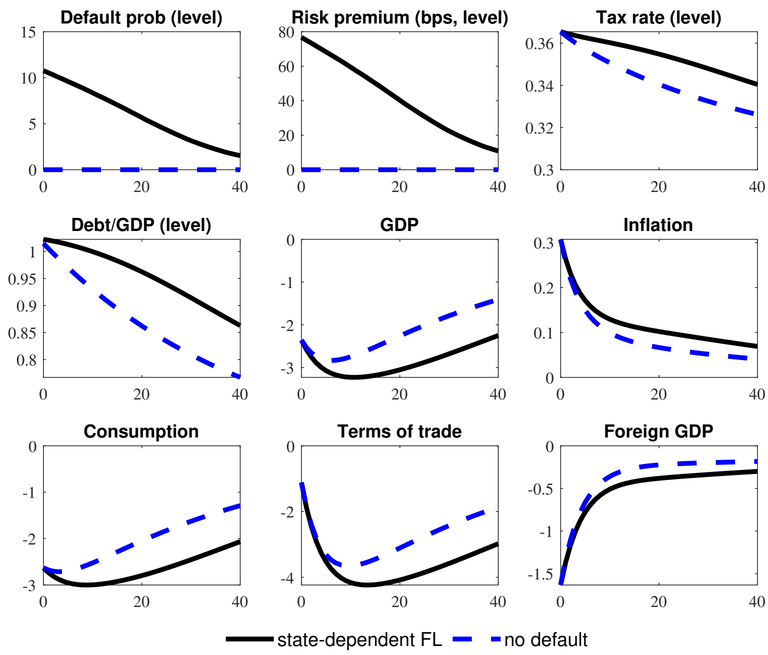
To highlight the risk premium channel, Figure (5) compares the long-run fiscal consolidation in an economy with risk premium generated by the state-dependent fiscal limits (solid lines) and an economy without default risk (dashed lines). In both cases, the con-

[21] The debt rule of the SGP requires a reduction of 1/20th of the distance with 60 percent each period.

[22] This approach provides a good approximation to the true high debt scenario, since although in our model terms of trade in the previous period (tot_{t-1}) is also a state variable, endogenizing it would have only a negligible effect.

solidation process is characterized by an increase in income tax rates as implied by the fiscal rule, to slowly reduce debt, which may take more than a decade to reach its steady state level. When the debt-to-GDP ratio is 100 percent, the income tax rate increases to 0.37, compared to the steady state value of 0.30. Home GDP and consumption are 2.4 percent and 2.7 percent below their state state values, respectively. In our baseline model with state-dependent fiscal limits, the default probability is 11 percent at $t = 0$, producing a risk premium of 77 bps. Although the initial states for the real macroeconomic variables are the same between the two economies, higher risk premium increases the interest burden on debt and produces significant differences in the long run. After ten years, the negative output gap in the economy with risk premium is almost twice as large as the economy without default risk. In the case with risk premium, debt-to-GDP ratio and tax rates are higher for all periods while consumption is lower. This worsens the terms of trade and further deprives activity. The fiscal adjustment in the high-debt economy also spills over to the rest of the euro area, where foreign output falls persistently.

Figure 5: Convergence back to the steady state from a high debt scenario at home



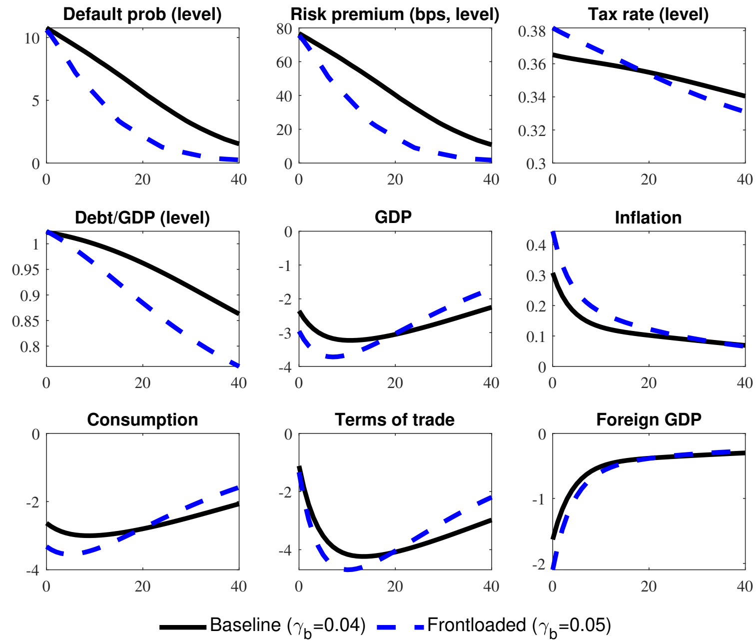
Notes: The responses (for those without a parenthesis) are plotted as the differences in percent of stochastic steady-state. Default probability, risk premium, tax rate, debt-to-annual GDP ratio are plotted as levels.

The long-term convergence back towards the 60 percent debt-to-GDP ratio depends on the intensity of the consolidation process, captured by the fiscal adjustment parameter (γ_b) in (22). The presence of a risk premium at high levels of public debt, speaks in favor of a quicker consolidation to reduce the effective borrowing rate, nevertheless, it does not imply that fast consolidations are going to be less painful. As Figure (6a) shows, with a frontloaded tax-based consolidation ($\gamma_b = 0.05$), the GDP loss is larger within five years relative to the baseline ($\gamma_b = 0.04$), despite that the risk premium reduces much faster. The debt-to-output ratio falls quicker and the risk premium returns to zero after roughly seven years, two-thirds of the time it needs to do so in the baseline. As the income tax rate is higher to retire debt sooner, in the frontloaded scenario it has a more negative effect on consumption and GDP, at least in the short to medium run. This is reversed after five years, but the burden of the short-run cost carries a high weight.

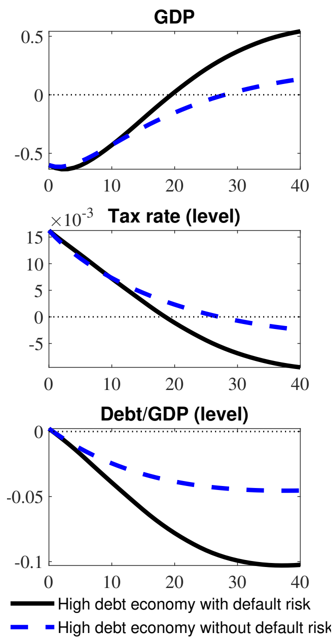
Nevertheless, the presence of the fiscal limit implies that increasing the speed of consolidation is less painful in an economy close to its fiscal limit, than in a similar economy operating further away from the fiscal limit.[23] Figure (6b) shows that in the latter case (dashed lines) speeding up the consolidation process induces a larger and more persistent GDP fall relative to the more gradual consolidation, than it does in the economy operating close to the state-dependent fiscal limit. In particular, in the high debt country without default risk, the frontloaded consolidation induces an additional cost in terms of GDP loss that lasts 2 years longer (6.5 vs 4.5 years) than the one with nontrivial default risk. This is because in the high debt country with default risk, more frontloaded consolidation reduces the risk premium more effectively, which allows for a quicker reduction in the debt-to-GDP ratio and tax rates.

[23] For simplicity, we model the latter case as an economy where there is no default, so that its risk-premium is not affected by the level or the speed of debt reduction. This would be equivalent to a high-debt economy for which the fiscal limit is high enough so that reaching a debt level of 100% of GDP does not to increase significantly the risk-premium.

Figure 6: Increasing the speed of convergence back to the steady state from a high debt



(a) Frontloaded vs baseline



(b) Differences between the frontloaded consolidation ($\gamma_b = 0.05$) and the baseline ($\gamma_b = 0.04$)

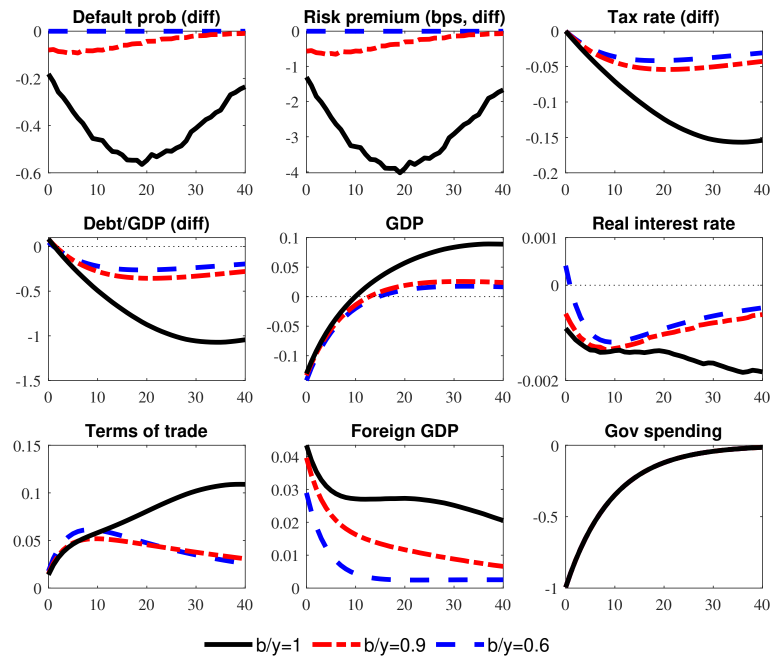
Notes: LHS panel (a): see Figure (5) for units of y-axes. RHS panel (b): GDP is plotted as the differences in percent of of stochastic steady-state between the paths with frontloaded consolidation and the baseline. Tax rate and debt-to-annual GDP ratio are plotted as the level difference between the paths with frontloaded consolidation and the baseline.

5.2 Discretionary Fiscal Consolidation at Home

To see how government indebtedness matters for discretionary fiscal consolidation effects in the home country, we examine an exogenous government spending cut for different levels of debt, first in the normal (Taylor rule) monetary regimFrontloaded vs baselinee and then in the ZLB regime. The impulse responses shown in this section represent marginal effects. That is, the differential impact when we add a transitory spending cut to the baseline long-term consolidation process described in the previous section.

Figure (7) shows the macroeconomic responses to a 1 percent transitory government spending cut in a high-debt member of a monetary union with an initial debt-to-GDP ratio 100 percent (solid lines). Discretionary fiscal consolidation reduces output and inflation on impact due to lower demand for domestic goods. The real interest rate falls immediately by 10 bps and then slowly gets more negative as output gap closes and inflation expectation rises. On the fiscal side, lower spending reduces public deficit and debt-to-GDP ratio, while at the same time, the fiscal rule sets a slightly lower tax rate. The public deficit reduction in the short term leads to an improvement of expected medium and long-term debt sustainability, increasing home's fiscal limit (see Figure 3a). The increase in the fiscal limit generates an immediate and persistent fall in home's default probability and risk premium. By the end of the fourth year, the risk premium on home government bonds falls by almost 4 bps. Thanks to the reduction in the home's cost of financing, the improvement in the terms of trade, together with gradual reduction in the tax rate, home GDP starts to recover after ten quarters. In addition, home spending cut produces a positive spillover to the rest of the union, mainly through two channels. On the one hand, the initial fall in activity and inflation at home slightly pushes down the union's nominal interest rate and fosters economic activity in the rest of the union. On the other hand, the increase in home's consumption fosters exports from the rest of the union.

Figure 7: Transitory government spending cut at Home: The effect of initial debt level



Notes: The responses (for those without a parenthesis) are plotted as the differences in percent of stochastic steady-state levels between the paths with and without a 1% government spending cut. The default probability, tax rate, and debt-to-annual GDP ratio are the level difference in percent, and risk premium is the level difference in basis points between the paths with and without the shock.

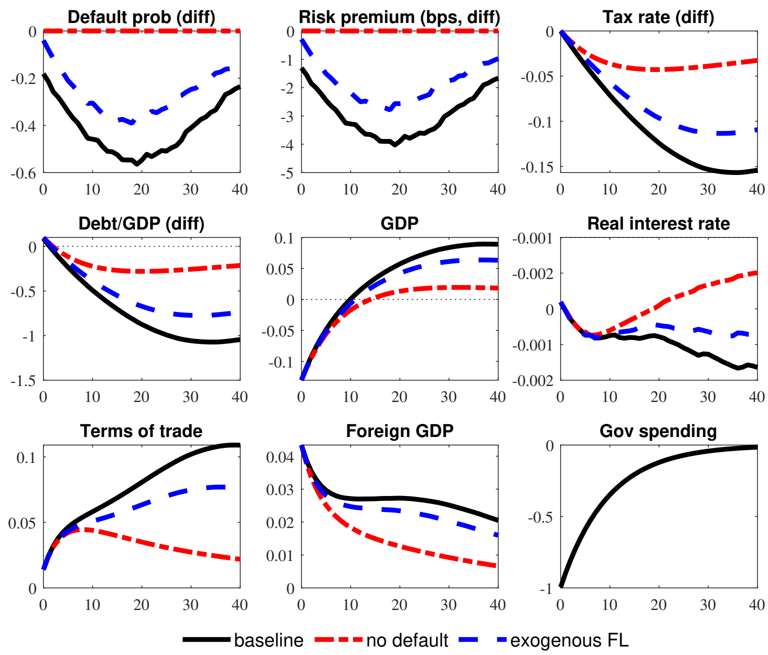
Our nonlinear model shows that the benefits from fiscal consolidation are greater when an economy is in a high debt situation than when its public finances are in better shape. The blue dashed lines of Figure (7) depict the effect of the same discretionary spending cut starting from a low level of debt (60 percent of GDP, the stochastic steady state). As debt in the home is very far from the fiscal limit, risk premium is close to zero and unresponsive to spending shocks. Without the reduction in the risk premium, output recovery in the home is much weaker, and the positive spillover effect to the rest of the union, is also smaller. As the initial level of debt increases to 90 percent debt-GDP ratio (red dashed-dotted lines in Figure 7), the impact of a spending cut on the risk premium starts to have more significant effects. Comparing the three cases shows that the effect of the fiscal limit on real activity is nonlinear, increasing quickly as the country approaches the fiscal limit. It is worth reminding that the nonlinearity can play an important role, even when the probability of reaching the fiscal limit is small. With a debt-to-GDP ratio of 100 percent, the economy has barely a 10 percent probability of default on the service of debt and yet generates very different dynamics compared to the economy with low debt.

To study more closely the role of risk premium in the transmission channel of fiscal shocks, Figure (8) compares the baseline model with the two alternative ones previously used in the literature and explained in the previous section: a standard model without default risk (red dashed-dotted lines), and a model with an exogenous fiscal limit (blue dashed lines).[24] In the model without default risk, reducing public debt does not affect the cost of financing and therefore, the impulse responses look very similar to the ones under a scenario of low debt. In the model with exogenous fiscal limits, the spending cut increases government primary surplus and lowers debt gradually. As public debt adjusts back to its steady state level, the default probability and risk premium decrease slowly. In our model with state-dependent fiscal limits, as explained before, the spending cut improves the home government's fiscal outlook and immediately pushes the fiscal limit distribution to the right. The decrease in risk premium is almost 2 bps more than that with exogenous fiscal limits throughout the horizon. The small variations in risk premium can produce significant differences in debt dynamics. After ten years, annual debt-to-GDP ratio decreases by 1.08 percentage points with state-dependent fiscal limits, compared to 0.76 with exogenous fiscal limits. In addition, the more powerful risk premium channel in our model increases the positive spillover effect to the rest of the union.

Key in the outcome of these consolidation scenarios is the monetary regime the union finds itself in. Figure (9) compares our baseline simulation with an alternative monetary policy regime at the ZLB. On impact, the reduction in default probability and risk premium under the ZLB regime are smaller than that under the Taylor rule regime, consistent with the fiscal limit distribution in Figure (4a). For the longer horizon, a persistent fiscal consolidation generates deflation expectations. When the nominal interest rate is constrained at zero, the real interest rate is consistently higher than that under the Taylor rule regime. Higher real interest rates lower demand and increase the cost of servicing the debt, mitigating the risk premium channel explained before. Therefore, under the ZLB, the presence of state-dependent fiscal limits does not make fiscal consolidations as beneficial as they are when monetary policy operates under a Taylor rule.

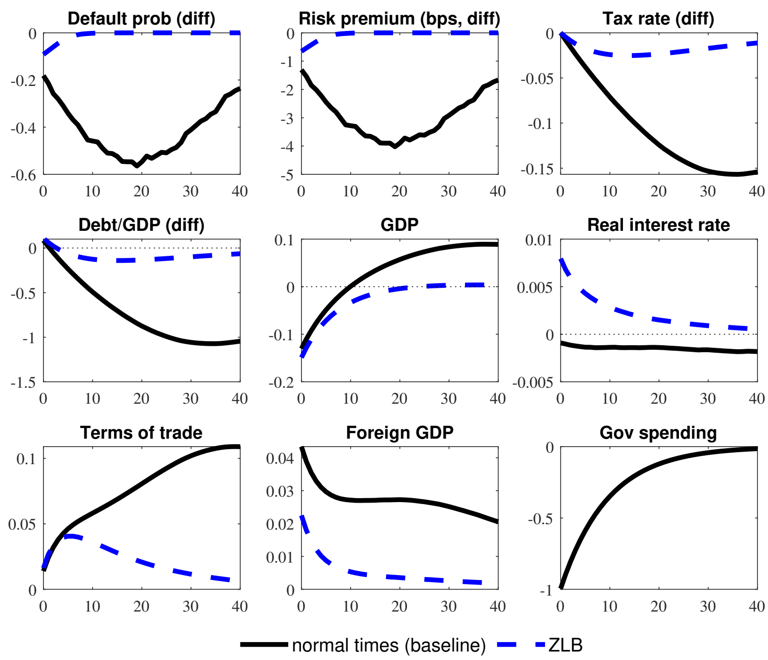
[24] See Section (4.4) for detailed explanations of the two alternative models.

Figure 8: Transitory government spending cut at Home: The effect of a state-dependent risk premium



Notes: See Figure (7) for units of y-axes.

Figure 9: Transitory government spending cut at Home: The effect of binding ZLB



Notes: See Figure (7) for units of y-axes.

Table (2) reports the cumulative government spending multipliers for output in the home country, foreign country, and the Euro area for various models, computed as

$$\frac{\sum_{i=0}^{k}\left(\prod_{j=0}^{i} r_{t+j}{}^{-1}\right)\triangle x_{t+i}}{\sum_{i=0}^{k}\left(\prod_{j=0}^{i} r_{t+j}{}^{-1}\right)\triangle g_{t+i}}, \qquad x \in \{y, y^{*}, y_{MU}\}, \tag{33}$$

where $\triangle$ denotes level changes relative to a path without government spending changes. To keep the comparison consistent among different models, we use $r_t = \beta^{-1}$ as the real interest rate for all t.

Table 2: Output multipliers from a discretionary government spending cut in the home country

Multiplier	Periphery (home) $\frac{PV(\Delta y)}{PV(\Delta g)}$				Spillover to the core (foreign) $\frac{PV(\Delta y^{*})}{PV(\Delta g)}$				Euro area $\frac{PV(\Delta y_{MU})}{PV(\Delta g)}$			
Models	Impact	1 yr	5 yr	10 yr	Impact	1 yr	5 yr	10 yr	Impact	1 yr	5 yr	10 yr
No default	0.71	0.66	0.44	0.24	-0.18	-0.17	-0.21	-0.25	0.14	0.13	0.03	-0.08
Exogenous FL	0.71	0.66	0.31	-0.24	-0.18	-0.18	-0.26	-0.37	0.14	0.12	-0.05	-0.32
State-dependent FL	0.71	0.65	0.23	-0.50	-0.18	-0.18	-0.28	-0.42	0.14	0.12	-0.10	-0.45
State-dependent FL, ZLB	0.82	0.77	0.63	0.56	-0.09	-0.08	-0.07	-0.09	0.23	0.22	0.18	0.15

The fiscal multiplier, measuring the changes in output due to a 1 euro fall in government spending, is similar in all four models on impact, but the difference can be sizable in the medium to long run. The ten-year cumulative fiscal multiplier in a model without default risk is 0.24, and it changes its sign, becoming -0.24, when default risk is introduced through exogenous fiscal limits. When the state-dependent fiscal limit is considered, the fiscal consolidation becomes more expansionary (-0.5), consistent with the theory on expansionary fiscal consolidation (Giavazzi & Pagano 1990, Alesina et al. 1998, Alesina & Ardagna 2010). Having fiscal limits respond directly to the economic states amplifies the risk premium channel and makes fiscal consolidation more expansionary for the home country in the long run. For the Euro area, consolidation in the highly-indebted country also results in long-run gains. The ten-year multiplier for the Euro area is -0.45 with state-dependent fiscal limits, compared to -0.08 in the model without default. A binding ZLB, on the other hand, mitigates the risk premium channel by weakening demand through a higher real interest rate. The real interest rate channel can be sizeable, in which the ten-year multiplier for the home country reverses its sign, from -0.50 to 0.56.

5.3 Discretionary Fiscal Coordination

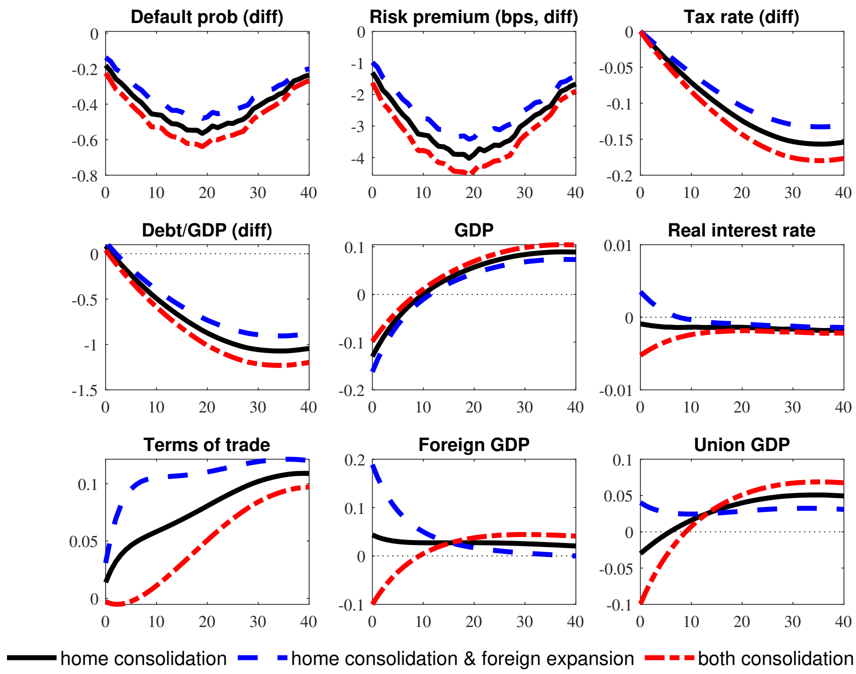
Following the sovereign debt crises in the euro area, a consensus has emerged about the need of a coherent and integrated fiscal policy strategy to ensure growth and debt sustainability. A pertinent question is whether the various fiscal policies at the national levels are complements or substitutes. In this section, we consider two fiscal policy coordination plans and compare them with the discretionary home consolidation in the previous section.

Figure (10) plots the macroeconomic effects of three fiscal policy combinations: A fiscal consolidation in the high debt home country (black solid lines), a fiscal consolidation in the high debt home country and a fiscal expansion in the low debt foreign country (blue dashed lines), and fiscal consolidations in both countries (red dashed-dotted lines). The fiscal consolidation (expansion) consists of a one percent decrease (increase) in government spending. In the short run, a fiscal expansion in the foreign country increases foreign GDP and union-wide inflation rate, pushing up real interest rates in both countries. As a consequence, home GDP decreases, but given the large size of the foreign country, the union GDP improves. Moreover, the higher real interest rates worsen debt sustainability in the home country, increasing default probabilities and the risk premium. This effect partially offsets the reduction of risk premium from home consolidation and lowers home GDP relative to the case without foreign fiscal expansions. After year three, when the initial positive effects on the foreign GDP fade away, the negative effects on home GDP start to dominate, and the union-wide GDP becomes lower with a foreign expansion than otherwise without one.[25] Fiscal consolidations in both countries, on the other hand, lower foreign GDP in the short run but yield the highest union GDP in the long run among the three policy strategies. Although a foreign fiscal consolidation lowers demand in the foreign country and hence its GDP, it also reduces the real interest rate in the home country. Given the existing high levels of debt in the home economy, lower real interest rate due to fiscal consolidation, and lower risk premium due to the increase in fiscal limits reduce real debt service significantly. In the long run, both the home and foreign country benefit from the policy coordination of consolidation, at a cost of lower foreign GDP in the short run.

When the ZLB is binding, the implications from policy coordination change significantly (see Figure 11). The real interest rate channel at the ZLB, as explained earlier, works against the risk premium channel and almost offsets lower risk premium from fiscal consolidation. The maximum changes in risk premium are between 0.6 to 0.8 bps at the ZLB regime compared to 3 to 5 bps at the Taylor rule regime, given different policy combinations. The policy effects, as a result, are dominated by the real interest rate channel. With a foreign fiscal expansion at the ZLB, the increase in the real interest rate is most muted and converges to zero after five quarters, producing the largest expansionary effect union wide. This result is in line with previous findings in the literature. Blanchard et al. (2017) show that a fiscal expansion by the core economies of the euro area would have a

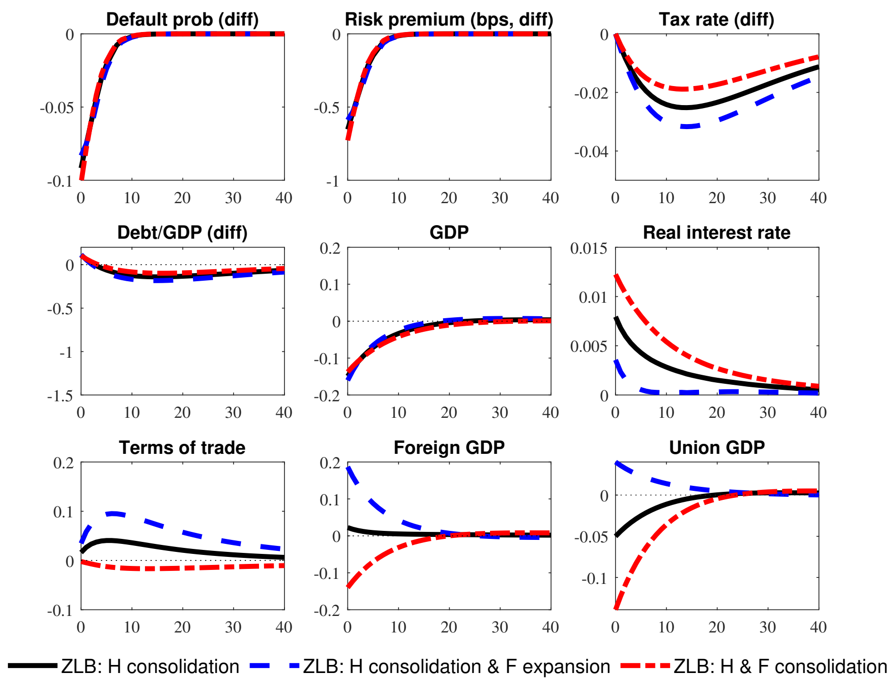
[25]The little spillover on home activity from a foreign fiscal expansion is partly due to the weak trade channel present in this model. This could be mitigated by allowing public spending to be a basket of home and foreign goods and/or assuming that consumers derive utility from the provision of public goods, which then become complements of private consumption and thus reduce the crowding-in effects.

Figure 10: Fiscal policy coordination: Taylor rule regime



Notes: See Figure (7) for units of y-axes.

Figure 11: Fiscal policy coordination: the ZLB regime



Notes: See Figure (7) for units of y-axes.

large and positive impact on periphery GDP assuming that policy rates remain low for a prolonged period. Arce et al. (2016) find that a fiscal expansion in the core aggravates the recession in the periphery, due to higher real rates, but the cross-country spillovers are reversed in a liquidity trap. Therefore, our results extend the previous findings to show that even in the presence of a fiscal limit the fiscal policy spillovers depend critically on the monetary policy regimes.

6 Concluding Remarks

This paper studies the effects of fiscal policy and its spillovers for countries in a monetary union with different levels of public debt. We develop a DSGE model of a two-country monetary union in which country members have different debt-to-GDP ratios. The high-debt country faces default risk driven by the fiscal limits, defined as the expected discounted sum of maximum primary surplus that can be generated in the future, given the current fiscal strategy and the expected evolution of the economy.

We find that the long-run fiscal adjustment necessary to bring a high-debt country back to more sustainable debt levels is costly in terms of output loss. When monetary policy follows a standard Taylor rule, transitory government spending cuts in the high-debt country enlarge its fiscal space, reducing risk premium and facilitating a faster recovery of output, compared with a low-debt economy. The effect of the risk premium channel on fiscal multipliers is highly nonlinear. It is hardly significant when the debt-to-GDP ratio is below 90 percent, and increases rapidly beyond 100 percent. In terms of fiscal coordination within the monetary union, the best strategy in a Taylor rule regime is for both members to consolidate simultaneously. It decreases real interest rates and amplifies the reduction in risk premium in the high-debt country, improving union-wide output in the long run, but at the cost of lower output in the low-debt country in the short term.

On the contrary, when monetary policy is constrained at the ZLB, the risk premium channel arising from fiscal limits becomes muted, making the costs of consolidation similar across the high and low debt economies within the monetary union. At the ZLB, the real interest rate moves in the opposite direction to the inflation rate, which may offset the response of the risk premium. In this context, the best coordination strategy is for the low-debt country to expand government spending and for the high-debt country to consolidate.

In future work we would like to explore two different environments. First, what happens when risk sharing mechanisms are available within a monetary union, currently under discussion in the policy debate. In particular, it would be interesting to extend the model to allow for cross border purchases of sovereign debt, the introduction of a Eurobond or the creation of a fiscal capacity at the EMU level to smooth the effects of asymmetric shocks. Second, what happens when government expenditure is productive and thus affects positively the supply-side of the economy. In this case, the impact on the fiscal limit from the worsening of public finances due to the fiscal expansion may be partially compensated by the improvement in productivity. This trade-off may change the implications of policy coordination and is a policy option currently very prominent in the debate of fiscal coordination within EMU.

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Appendix A Equilibrium

The equilibrium consists of 35 equations (16 for the home country, 14 for the foreign country and 5 are common), to solve for 35 variables (λ_t, c_t, n_t, w_t, δ_t, R_t, τ_t, π_t, $\pi_{H,t}$, mc_t, y_t, b_t, T_t, $c^H_{MU,t}$, Υ_t, g_t for the home country, λ^*_t, c^*_t, n^*_t,w^*_t, τ^*_t, π^*_t, $\pi_{F,t}$, mc^*_t, y^*_t, b^*_t, T^*_t, $c^F_{MU,t}$, Υ^*_t, g^*_t for the foreign country and tot_t, RER_t, $\pi_{MU,t}$, $y_{MU,t}$, R^f_t) for the union.

Home equations

$$\lambda_t = c_t^{-\sigma}, \tag{A.1}$$

$$n_t^{\varphi} = \lambda_t(1-\tau_t)w_t. \tag{A.2}$$

Since we have default, we need both the Euler equation under default and under no default to evaluate the expectation about future defaults

$$\lambda_t = \beta R^f_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{A.3}$$

$$\lambda_t = \beta R_t E_t \frac{(1-\delta_{t+1})\lambda_{t+1}}{\pi_{t+1}}, \tag{A.4}$$

$$\delta_t \;= \begin{cases} 0 & \text{if} \quad b_{t-1} < \mathcal{B}^H(\mathcal{S}_t) \\ \delta & \text{if} \quad b_{t-1} \geq \mathcal{B}^H(\mathcal{S}_t), \end{cases} \tag{A.5}$$

$$\pi_t = \pi_{H,t}^{\eta}\pi_{F,t}^{1-\eta}, \tag{A.6}$$

$$mc_t = tot_t^{1-\eta}\frac{w_t}{a}, \tag{A.7}$$

$$\psi\left(\frac{\pi_{H,t}}{\pi}-1\right)\frac{\pi_{H,t}}{\pi} = (1-\theta)+\theta mc_t + \psi\beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\left[\frac{\pi_{H,t+1}}{\pi}-1\right]\left(\frac{y_{t+1}\pi_{H,t+1}}{y_t\pi}\right)\right), \tag{A.8}$$

$$\left[1-\frac{\psi}{2}\left(\frac{\pi_{H,t}}{\pi}-1\right)^2\right]y_t = tot_t^{1-\eta}c^H_{MU,t}+g_t, \tag{A.9}$$

$$c^H_{MU,t} = \eta c_t + \eta^*\frac{1-s}{s}c^*_t, \tag{A.10}$$

$$y_t = an_t, \tag{A.11}$$

$$\Upsilon_t = \left[1-mc_t-\frac{\psi}{2}\left(\frac{\pi_{H,t}}{\pi}-1\right)^2\right]y_t, \tag{A.12}$$

$$\frac{b_t}{R_t}+\frac{T_t-g_t}{tot_t^{1-\eta}} = \frac{(1-\delta_t)\,b_{t-1}}{\pi_t}, \tag{A.13}$$

$$T_t = \left[1-\frac{\psi}{2}\left(\frac{\pi_{H,t}}{\pi}-1\right)^2\right]\tau_t y_t, \tag{A.14}$$

$$\tau_t = \tau + \gamma_b(b_{t-1}-b), \tag{A.15}$$

$$\ln \frac{g_t}{g} = \rho^g \ln \frac{g_{t-1}}{g} + \varepsilon_t^g. \tag{A.16}$$

Foreign equations

$$\lambda_t^* = (c_t^*)^{-\sigma}, \tag{A.17}$$

$$(n_t^*)^{\varphi} = \lambda_t^*(1-\tau_t^*)w_t^*. \tag{A.18}$$

In the full model, if there is no default in either country, the risky and risk free rates are identical and we can use the RER_t expression for foreign consumption. If instead home country sovereign debt is subject to default, while foreign country's sovereign debt is not, then the rate of return in foreign is equal to the risk-free rate, but we have to use the foreign Euler equation to solve the model.

$$\text{no default in the model (both home and foreign):} \left(\frac{c_t}{c_t^*}\right)^{\sigma} = RER_t = tot_t^{\eta-\eta^*}, \tag{A.19}$$

$$\text{default in home but no default in foreign:} \ \lambda_t^* = \beta R_t^f E_t \frac{\lambda_{t+1}^*}{\pi_{t+1}^*}, \tag{A.20}$$

$$\pi_t^* = \pi_{H,t}^{\eta^*} \pi_{F,t}^{1-\eta^*}, \tag{A.21}$$

$$mc_t^* = tot_t^{-\eta^*} \frac{w_t^*}{a^*}, \tag{A.22}$$

$$\psi \left(\frac{\pi_{F,t}}{\pi^*} - 1\right) \frac{\pi_{F,t}}{\pi^*} = (1-\theta) + \theta mc_t^* + \psi\beta E_t \left(\frac{\lambda_{t+1}^*}{\lambda_t^*} \left(\frac{\pi_{F,t+1}}{\pi^*} - 1\right) \frac{y_{t+1}^* \pi_{F,t+1}}{y_t^* \pi^*}\right), \tag{A.23}$$

$$\left[1 - \frac{\psi}{2}\left(\frac{\pi_{F,t}}{\pi^*} - 1\right)^2\right] y_t^* = tot_t^{-\eta^*} c_{MU,t}^F + g_t^*, \tag{A.24}$$

$$c_{MU,t}^F = \frac{s\,(1-\eta)}{1-s} c_t + (1-\eta^*)\, c_t^*, \tag{A.25}$$

$$y_t^* = a^* n_t^*, \tag{A.26}$$

$$\Upsilon_t^* = \left[1 - mc_t^* - \frac{\psi}{2}\left(\frac{\pi_{F,t}}{\pi^*} - 1\right)^2\right] y_t^*, \tag{A.27}$$

$$\frac{b_t^*}{R_t^f} + tot_t^{\eta^*}(T_t^* - g_t^*) = \frac{b_{t-1}^*}{\pi_t^*}, \tag{A.28}$$

$$T_t^* = \left[1 - \frac{\psi}{2}\left(\frac{\pi_{F,t}}{\pi^*} - 1\right)^2\right] \tau_t^* y_t^*, \tag{A.29}$$

$$\tau_t^* = \tau^* + \gamma_b(b_{t-1}^* - b^*), \tag{A.30}$$

$$\ln \frac{g_t^*}{g^*} = \rho^g \ln \frac{g_{t-1}^*}{g^*} + \varepsilon_t^{g^*}. \tag{A.31}$$

Union wide equations

$$tot_t = tot_{t-1}\frac{\pi_{F,t}}{\pi_{H,t}}, \tag{A.32}$$

$$R_t = \begin{cases} R + \alpha_\pi(\pi_{MU,t} - \pi_{MU}), & \text{if } s_t^R = 1 \\ 1, & \text{if } s_t^R = 2, \end{cases} \tag{A.33}$$

$$y_{MU,t} = sy_t + (1-s)y_t^*, \tag{A.34}$$

$$\pi_{MU,t} = s\pi_t + (1-s)\pi_t^*, \tag{A.35}$$

$$RER_t = tot_t^{\eta-\eta^*}. \tag{A.36}$$

Appendix B The Numerical Solution Method

Appendix B.1 Solving the Fiscal Limit

This appendix describes procedures in simulating fiscal limit distributions. First, derive the expression of the fiscal limit as (31). Second, replace the fiscal rules (A.15) and (A.30) with the maximum tax rate $\tau^{\max}$ and $\tau^{*\max}$ and solve the full model (without default) non-linearly evaluated at the maximum tax rate.

When solving the nonlinear model without default, the state space is $\mathcal{S}_t = \Big\{g_t, g_t^*, tot_{t-1}$ $,s_t^R\Big\}$,depending on the number of exogenous shocks we consider to build the fiscal limit. Since the model without default has three expectation terms in equations (Euler equation of home country (A.3),[26] Phillips curve of home (A.8) and foreign (A.23) we need three decision rules. It can be shown that these three equations can be written as a function of only three variables: n_t, $\pi_{H,t}$ and $\pi_{F,t}$. Therefore, the decision rule for labor in home country is $n_t^{max} = f^n(\mathcal{S}_t)$, the rule for inflation of the home goods is $\pi_{H,t}^{max} = f^{\pi_H}(\mathcal{S}_t)$, and the rule for inflation of the foreign goods is $\pi_{F,t}^{max} = f^{\pi_F}(\mathcal{S}_t)$.

From the converged rules for $f^n(\cdot)$, $f^{\pi_H}(\cdot)$, and $f^{\pi_F}(\cdot)$, we derive the rules for the remaining variables determining the fiscal limit $T_t^{max} = f^{\mathcal{T}}(\mathcal{S}_t)$, $T_t^{*max} = f^{\mathcal{T}*}(\mathcal{S}_t)$, $\lambda_t^{max} = f^{\lambda}(\mathcal{S}_t)$, $\lambda_t^{*max} = f^{\lambda*}(\mathcal{S}_t)$, and $tot_t^{max} = f^{tot}(\mathcal{S}_t)$, which are consistent with the optimization conditions from the household's and the firms' problems

To solve the model we proceed as follows:

1. Define the grid points by discretizing the state space (over the 4 dimensions). Make initial guesses for f_0^n, $f_0^{\pi_H}$, and $f_0^{\pi_F}$ over the state space.

2. Under the maximum tax rates $(\tau^{max}, \tau^{*max})$, at each grid point, solve the nonlinear model using the given rules f_{i-1}^n, $f_{i-1}^{\pi_H}$, and $f_{i-1}^{\pi_F}$, and obtain the updated rules f_i^n, $f_i^{\pi_H}$, and $f_i^{\pi_F}$. Specifically:

[26] We do not have an expectation term in the foreign Euler equation because in the model without risk we can use instead equation (A.20).

(a) derive π_t and tot_t in terms of $\pi_{H,t}$ and $\pi_{F,t}$ using (A.6) and (A.32). Derive y_t in terms of a and n_t using (A.11).

(b) Compute $c^H_{MU,t}$ from (A.9). Given (A.20) and (A.10), we have $c_t = \frac{c^H_{MU,t}}{\eta+\eta^*\frac{1-s}{s}tot_t^{\eta^*-\eta}}$ and $c^*_t = c_t tot_t^{\eta^*-\eta}$. Then $\lambda_t = (c_t)^{-\sigma}$ and $\lambda^*_t = (c^*_t)^{-\sigma}$

(c) Compute w_t, mc_t, and T_t using (A.2), (A.7), and (A.14).

(d) From (A.24), (A.26), (A.29), (A.18), and (A.22), we can derive $c^F_{MU,t}$, y^*_t, n^*_t, T^*_t, w^*_t, and mc^*_t.

(e) Derive π^*_t using (A.21). Given π_t, π^*_t, y_t, y^*_t, s^R_t, and (A.34), (A.35), obtain the nominal interest rate R_t from equation (A.33).

(f) Use linear interpolation to obtain $f^n_{i-1}(\mathcal{S}_{t+1})$, $f^{\pi_H}_{i-1}(\mathcal{S}_{t+1})$, and $f^{\pi_F}_{i-1}(\mathcal{S}_{t+1})$, where the state vector is $\mathcal{S}_{t+1} = (g_{t+1}, g^*_{t+1}, tot_t, s^R_{t+1})$. This is necessary because the policy function at time t is a mapping from a value of the state variables on the grid points $(g_t, g^*_t, tot_{t-1}, s^R_t)$ to endogenous variables $n_t, \pi_{H,t}, \pi_{F,t}$, but the policy function at time $t+1$ that evaluates $n_{t+1}, \pi_{H,t+1}, \pi_{F,t+1}$ may correspond to a value of the state variables in between two grid points, and therefore, to calculate it we have to linearly interpolate those two points. Then, follow the above steps to solve λ_{t+1}, λ^*_{t+1}, y_{t+1}, y^*_{t+1}, π_{t+1}, π^*_{t+1}.

(g) Update the decision rules f^n_i, $f^{\pi_H}_i$, and $f^{\pi_F}_i$, using (A.3), (A.8), and (A.23). The integral in expectation terms is evaluated using numerical quadrature.

3. Check convergence of the decision rules. If $|f^n_i - f^n_{i-1}|$, $|f^{\pi_H}_i - f^{\pi_H}_{i-1}|$, or $|f^{\pi_F}_i - f^{\pi_F}_{i-1}|$ is above the desired tolerance (set to $1e-6$), go back to step 2. Otherwise, f^n_i, $f^{\pi_H}_i$, and $f^{\pi_F}_i$ are the decision rules.

4. Use the converged rules—f^n, f^{π_H}, and f^{π_H}—to compute the decision rules for $f^{\mathcal{T}}_i$, $f^{\mathcal{T}*}_i$, f^{λ}_i, $f^{\lambda*}_i$, and f^{tot}_i.

Since the maximum tax rate is quite far away from the average tax rate, we may need to solve the non-linear model increasing the tax rate gradually from the calibration until we reach the maximum level.

Using the maximum tax revenue $f^{\mathcal{T}}(\cdot)$, $f^{\mathcal{T}*}(\cdot)$, $f^{\lambda}(\cdot)$, $f^{\lambda*}(\cdot)$, and $f^{tot}(\cdot)$, the distribution of fiscal limits is obtained using Markov Chain Monte Carlo simulations. Now, since we want to obtain the whole distribution, we evaluate expressions (31) without taking expectations. To proceed,

1. For each simulation j, we randomly draw sequences of the exogenous shocks for government spending shocks in the home country ($\varepsilon^{g,j}_{t+i}$), and government spending

shocks in the foreign country ($\varepsilon_{t+i}^{g*,j}$) for 1000 periods, $i = \{1, 2, 3, ...1000\}$, conditional on the starting state $\mathcal{S}_t = \left\{g_t, g_t^*, tot_{t-1}, s_t^R\right\}$. If the simulation starts from the steady state, we call it the unconditional fiscal limit, otherwise it is the conditional one. At each period i, we obtain $T_{t+i}^{max,j}$, $T_{t+i}^{*max,j}$, $\lambda_{t+i}^{max,j}$, $\lambda_{t+i}^{*max,j}$, and $tot_{t+i}^{max,j}$, ($i = 1, ..., 1000$) by interpolating on the decision rules $f^{\mathcal{T}}(\cdot)$, $f^{\mathcal{T}*}(\cdot)$, $f^{\lambda}(\cdot)$, $f^{\lambda*}(\cdot)$, and $f^{tot}(\cdot)$. This is necessary because the policy function is a mapping from a value of the state variables on the grid points ($g_t, g_t^*, tot_{t-1}, s_t^R$) to endogenous variables $\mathcal{T}_t$, $\mathcal{T}_t^*$, λ_t, λ_t^*, and tot_t, but following the stochastic processes, the realizations of the state variables may fall in between two grid points, and therefore, to calculate it we have to linearly interpolate those two points. Then, the fiscal limit for simulation j is computed (without taking expectations), conditional on $\mathcal{S}_t$ and particular sequences of shocks,

2. Repeat the simulation $50,000$ times ($j = \{1, ..., 50,000\}$) to have $\{\mathcal{B}^{max,j}(\mathcal{S}_t)\}_{j=1}^{50000}$, which form the distribution of $\mathcal{B}^H(\mathcal{S}_t)$.

Appendix B.2 Solving Full Model

When solving the nonlinear model with default, the state space is $\mathcal{S}_t = \Big\{(1-\delta_t)b_{t-1}, b_{t-1}^*, g_t, g_t^*, tot_{t-1}, s_t^R\Big\}$. In this model there are 5 expectation terms in equations (Euler equation of home country (A.3) and (A.4), Euler equation of the foreign country (A.20)[27], Phillips curve of home (A.8) and foreign (A.23)) and thus we need five decision rules.[28] It can be shown that these five equations can be written as a function of only five variables: b_t, c_t, c_t^*, $\pi_{H,t}$ and $\pi_{F,t}$. Define the decision rules for the end-of-period home government bond as $b_t = f^b(\mathcal{S}_t)$, consumption in home country as $c_t = f^c(\mathcal{S}_t)$, consumption in foreign country as $c_t^* = f^{c^*}(\mathcal{S}_t)$, inflation of the home goods is $\pi_{H,t} = f^{\pi_H}(\mathcal{S}_t)$, and inflation of the foreign goods is $\pi_{F,t} = f^{\pi_F}(\mathcal{S}_t)$. The decision rules are solved as follows:

1. Define the grid points by discretizing the state space (over the 6 dimensions). Make initial guesses for f_0^b, f_0^c, $f_0^{c^*}$, $f_0^{\pi_H}$, and $f_0^{\pi_F}$ over the state space.

2. At each grid point, solve the nonlinear model and obtain the updated rules f_i^b, f_i^c, $f_i^{c^*}$, $f_i^{\pi_H}$,and $f_i^{\pi_F}$ using the given rules f_{i-1}^b, f_{i-1}^c, $f_{i-1}^{c^*}$, $f_{i-1}^{\pi_H}$, and $f_{i-1}^{\pi_F}$:

   (a) Derive π_t and tot_t in terms of $\pi_{H,t}$ and $\pi_{F,t}$ using (A.6) and (A.32). Derive RER_t and τ_t using (A.36) and (A.15).

[27] Since we have assumed that the foreign country is Germany, we do not need to have default in that economy, and we have replaced the Foreign Euler equation with (A.20).

[28] If we consider two countries which are potentially close to their fiscal limits, then we would need to include the Foreign Euler equation and the interest rate on foreign sovereign bonds (R_t^*) and we would have an additional expectation term and an additional decision rule for n_t^*.

(b) Compute $c^H_{MU,t}$ from (A.10). Then y_t can be obtained from (A.9), and n_t is given by (A.11).

(c) Compute w_t, mc_t, and T_t using (A.2), (A.7), and (A.14).

(d) From (A.24), (A.26), (A.30), (A.29), (A.18), and (A.22), we can derive $c^F_{MU,t}$, y^*_t, n^*_t, τ^*_t, T^*_t, w^*_t, and mc^*_t.

(e) Derive π^*_t using (A.21). Given π_t, π^*_t, y_t, y^*_t, s^R_t, and (A.34), (A.35), obtain the nominal risk free interest rate R^f_t from equation (A.33).

(f) Compute b^*_t using (A.28) and the risky rate R_t using (A.13).

(g) Use linear interpolation to obtain $f^b_{i-1}(\mathcal{S}_{t+1})$, $f^c_{i-1}(\mathcal{S}_{t+1})$, $f^{c^*}_{i-1}(\mathcal{S}_{t+1})$, $f^{\pi_H}_{i-1}(\mathcal{S}_{t+1})$ and $f^{\pi_F}_{i-1}(\mathcal{S}_{t+1})$, where $\mathcal{S}_{t+1} = ((1-\delta_{t+1})b_t, b^*_t, g_{t+1}, g^*_{t+1}, tot_t, s^R_{t+1})$. Then follow the above steps to solve λ_{t+1}, λ^*_{t+1}, y_{t+1}, y^*_{t+1}, π_{t+1}, π^*_{t+1}.

(h) Update the decision rules f^b_i, f^c_i, $f^{c^*}_i$, $f^{\pi_H}_i$, and $f^{\pi_F}_i$, using (A.3), (A.4), (A.8), (A.20), and (A.23).

3. Check convergence of the decision rules. If $|f^b_i - f^b_{i-1}|$, or $|f^c_i - f^c_{i-1}|$, or $|f^{c^*}_i - f^{c^*}_{i-1}|$, or $|f^{\pi_H}_i - f^{\pi_H}_{i-1}|$, or $|f^{\pi_F}_i - f^{\pi_F}_{i-1}|$ are above the desired tolerance (set to $1e-6$), go back to step 2; otherwise, f^b_i, f^c_i, $f^{c^*}_i$, $f^{\pi_H}_i$, and $f^{\pi_F}_i$ are the decision rules.