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Modeling Hyperinflation Phenomenon: A Bayesian Approach

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ABSTRACT

Hyperinflations are short-lived episodes of economic instability in prices which characteristically last twenty months or less. Classical statistical techniques applied to these small samples could lead to an incorrect inference problem. This paper describes a Bayesian approach for modeling hyperinflations which improves the modeling accuracy using small-sample inference based on specific parametric assumptions. A theory-congruent model for the Bolivian hyperinflation was estimated as a case study.

JEL codes: *E31, C11*
Keywords: *Hyperinflation, Bayesian methods*

1. INTRODUCTION

A hyperinflation is a short-lived episode of economic instability in prices, which characteristically lasts twenty months or less (Mladenovic and Petrovic, 2010). The estimation of hyperinflation models with classical asymptotic theory applied to these extremely small samples could lead to misleading conclusions. Thus, as the precision of any hyperinflation study is constrained by the small sample of these phenomena, policy-makers face the dilemma of basing their decisions on inaccurate empirical results.

For example, in the Cagan (1956) model of hyperinflation, a negative inflation elasticity (α) is predicted, which can be used to find the optimal seigniorage ($1/\alpha$) that central banks could achieved by printing money. Cagan found that the average inflation rate in many hyperinflations was beyond this optimal rate, thus concluding that money supply was the cause of hyperinflations. Nevertheless, Sargent (1977) argues that estimating α with classical (asymptotic) statistical techniques leads to imprecise values and thus the conclusions not convincingly conform to the predictions from the Cagan's model. Bayesian methods can be used to solve this problem: these methods provide techniques for handling uncertainty in finite samples, because Bayesian finite-sample inference based on specific parametric assumptions is approximately correct when the parametric assumptions are approximately correct (Sims, 2007).[1]

The aim of this paper is to describe a complete Bayesian specification approach for modeling hyperinflation phenomenon. Bayesian methods for *i)* the estimation of a set of models with different prior densities, *ii)* the empirical comparison of these models with data evidence, and *iii)* the statistical evaluation of the estimated models are describe in Section 2. In Section 3 the hyperinflation model of Cagan-Sargent-Wallace (henceforth, CSW) is estimated and evaluated with Bayesian methods using data of the Bolivian hyperinflation as a case study. Section 4 discusses the results.

2. METHODS

This section outlines the CSW model of hyperinflation and describes the estimation, comparison and testing of this model with Bayesian methods.

2.1 THE CSW MODEL OF HYPERINFLATION

The Cagan model is an equation of money demand of real balances as a function of expected future inflation (Δp_t^e) that can be derived from an intertemporal utility maximizing framework (see Appendix 1),

$$m_t - p_t = -\alpha \Delta p_t^e + u_t,$$

where m_t and p_t are logs to nominal money and prices, respectively, u_t is a stochastic disturbance term of money demand shocks, and α is the semi-elasticity of the demand for

[1] See Bolstad (2004), Gosh et al. (2006) and Greenberg (2008) for an introduction to Bayesian statistics, and Koop (2003), Geweke (2005), Gill (2007) for applications in economics and recent developments in Bayesian methods.

real balances with respect to expected inflation, Δp_t^e. The unobservable expectation Δp_t^e is a distributed lag of current and past actual rates of inflation, with geometrically declining weights,

$$\Delta p_t^e = (1-\lambda)\sum_{i=0}^{\infty}\lambda^i log\frac{p_{t-i}}{p_{t-i-1}}.$$

Letting $x_t = \log(p_{t-i} / p_{t-i-1})$ and using the backwards operator $L^i z_t = z_{t-i}$,

$$\Delta p_t^e = (1-\lambda)\sum_{i=0}^{\infty}\lambda^i log\frac{p_{t-i}}{p_{t-i-1}}$$
$$= \frac{1-\lambda}{1-\lambda L}x_t.$$

This expectation is not rational in the sense of Muth (1961). To provide a rationalization of Cagan's model, Sargent and Wallace (1973) assumed that u_t follows the Markov process $u_t = u_{t-1} + \eta_t$, with a rate of money creation μ_t governed by,

$$\mu_t = \left(\frac{1-\lambda}{1-\lambda L}\right)x_t + \varepsilon_t,$$

$$\underset{t-1}{\mathbb{E}}\, x_t = \left(\frac{1-\lambda}{1-\lambda L}\right)x_{t-1},$$

which is Cagan's adaptive expectations scheme, and on the hypothesis that expectations are rational,[2]

$$\mu_t = \underset{t}{\mathbb{E}}\, x_{t+1} + \varepsilon_t.$$

This equation captures the feedback from expected inflation to money creation that will occur if the government is financing a roughly fixed rate of real expenditures by money creation. Under the previous assumptions, inflation and money creation form a bivariate stochastic process given by (Sargent, 1977)

$$\mu_t - x_t = \alpha(1-L)\left(\frac{1-\lambda}{1-\lambda L}\right)x_t + \eta_t,$$
$$\mu_t = \left(\frac{1-\lambda}{1-\lambda L}\right)x_t + \varepsilon_t.$$

2.2 BAYESIAN ESTIMATION

The previous model can be written as a first-order vector auto-regression, first-order moving average process (see Appendix 2):[3]

[2] The Sargent-Wallace assumption that expectations are rational is imposed by requiring that, $\pi_t = \mathbb{E}_t x_{t+1}$, for $\mathbb{E}_t$ the conditional expectations operator of x_{t+1} formed using the model and available information as of time t.

[3] This rational expectations system can be viewed too as a state space model where the state vector $a_t = \begin{bmatrix} a_{1t} & a_{2t} \end{bmatrix}'$ marks the state transition.

$$\mathcal{M}: \begin{cases} x_t\mu_t = \begin{pmatrix} 1 & 0 \\ (1-\lambda) & \lambda \end{pmatrix} x_{t-1}\mu_{t-1} + a_{1t}a_{2t} - \lambda \mathbf{I} a_{1t-1}a_{2t-1} \\ a_{1t}a_{2t} = \begin{bmatrix} 1 & -1 \\ 1+\alpha(1-\lambda) & -(1-\lambda) \end{bmatrix} / \left(\lambda + \alpha(1-\lambda)\right)\varepsilon_t \eta_t \end{cases}$$

being the random variables a_{1t}, a_{2t} the innovations in the x and μ, processes, respectively. If,

$$\mathbf{a}_t = a_{1t}a_{2t}$$

and D_a is the covariance matrix of a_t,

$$D_a = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \mathbb{E}\left(\mathbf{a}_t \mathbf{a}_t^{'}\right),$$

the likelihood function of the sample $t = 1,\ldots,T$ would be,[4]

$$\mathcal{L}\left(\lambda, \sigma_{11}, \sigma_{12}, \sigma_{22} \mid \mu_t, x_t\right) = \left(2\pi^{-T}\right)\left|\mathbf{D}_a\right|^{-T/2} \exp\left(-\frac{1}{2}\sum_{t=1}^{T} \mathbf{a}_t^{'} \mathbf{D}_a^{-1} \mathbf{a}_t\right).$$

The Bayesian estimation of the vector of parameter $\boldsymbol{\theta}_{\mathcal{M}} = \left(\alpha, \lambda, \sigma_\varepsilon^2, \sigma_\eta^2\right)$ of a model $\mathcal{M}$ with $\mathbf{Y}_T = (\mu_t, x_t)$ comes from the posterior density $\mathbb{P}(\boldsymbol{\theta}_{\mathcal{M}} \mid \mathbf{Y}_T, \mathcal{M})$,

$$\mathbb{P}\left(\boldsymbol{\theta}_{\mathcal{M}} \mid \mathbf{Y}_T, \mathcal{M}\right) = \frac{\mathbb{P}\left(\mathbf{Y}_T \mid \boldsymbol{\theta}_{\mathcal{M}}, \mathcal{M}\right)\mathbb{P}\left(\boldsymbol{\theta}_{\mathcal{M}} \mid \mathcal{M}\right)}{\mathbb{P}\left(\mathbf{Y}_T \mid \mathcal{M}\right)},$$

or

$$\mathbb{P}\left(\boldsymbol{\theta}_{\mathcal{M}} \mid \mathbf{Y}_T, \mathcal{M}\right) \propto \mathbb{P}\left(\mathbf{Y}_T \mid \boldsymbol{\theta}_{\mathcal{M}}, \mathcal{M}\right)\mathbb{P}\left(\boldsymbol{\theta}_{\mathcal{M}} \mid \mathcal{M}\right),$$

which is the un-normalized posterior density, where $\mathbb{P}\left(\boldsymbol{\theta}_{\mathcal{M}} \mid \mathcal{M}\right)$ is the Bayesian prior of the parameters in $\boldsymbol{\theta}_{\mathcal{M}}$ and $\mathbb{P}\left(\mathbf{Y}_T \mid \boldsymbol{\theta}_{\mathcal{M}}, \mathcal{M}\right)$ is the likelihood function,[5]

$$\mathbb{P}\left(\mathbf{Y}_T \mid \boldsymbol{\theta}_{\mathcal{M}}, \mathcal{M}\right) = \mathbb{P}\left(y_0 \mid \boldsymbol{\theta}_{\mathcal{M}}, \mathcal{M}\right)\prod_{t=1}^{T} \mathbb{P}\left(y_t \mid \mathbf{Y}_{T-1}, \boldsymbol{\theta}_{\mathcal{M}}, \mathcal{M}\right).$$

2.3 BAYESIAN COMPARISON OF CSW MODELS

Marginal likelihoods and Bayes factors can be used to compare CSW models with different priors. Let $\mathbb{P}(\mathcal{M}_1)$ and $\mathbb{P}(\mathcal{M}_2)$ be the prior probability of the validity of two competing models $\mathcal{M}_1$ an $\mathcal{M}_2$. The posterior odds ratio,

$$\frac{\mathbb{P}(\mathcal{M}_1 \mid \mathbf{Y}_T)}{\mathbb{P}(\mathcal{M}_2 \mid \mathbf{Y}_T)} = \frac{\mathbb{P}(\mathcal{M}_1)\mathbb{P}(\mathbf{Y}_T \mid \mathcal{M}_1)}{\mathbb{P}(\mathcal{M}_2)\mathbb{P}(\mathbf{Y}_T \mid \mathcal{M}_2)},$$

is an aid to balance $\mathcal{M}_1$ an $\mathcal{M}_2$ because this ratio provides evidence of the quality of one

[4] Sargent (1977) showed that, even if α and $\sigma_{\varepsilon\eta}$ are not identified (α does not appear explicitly in $\mathcal{L}$ but indirectly by way of the elements of $\mathcal{D}_a$), it is possible to obtain maximum likelihood estimators of the structural parameters $\alpha, \lambda, \sigma_\varepsilon^2, \sigma_\eta^2, \sigma_{\varepsilon\eta}$, on the basis of the four parameters $\lambda, \sigma_{11}, \sigma_{12}, \sigma_{22}$, imposing $\sigma_{\varepsilon\eta} = 0$, i.e., zero covariance between money demand and supply shocks.

[5] According to Canova (2007), these prior densities are *1)* the subjective beliefs that a researcher has in the occurrence of an event, *2)* an objective evaluation based on recorded information, or *3)* the outcomes of previous experiments.

model specification over the other. With prior equiprobabilities, $\mathbb{P}(\mathcal{M}_1)=\mathbb{P}(\mathcal{M}_2)$, the posterior odds ratio becomes a ratio of marginal likelihood $\mathbb{P}(\mathbf{Y}_T \mid \mathcal{M}_i)$,[6] i.e., the Bayes factor $\mathcal{B}_{\mathcal{M}_1,\mathcal{M}_2} \in [0,\infty)$,

$$\mathcal{B}_{\mathcal{M}_1,\mathcal{M}_2} = \frac{\mathbb{P}(\mathbf{Y}_T \mid \mathcal{M}_1)}{\mathbb{P}(\mathbf{Y}_T \mid \mathcal{M}_2)},$$

and Jeffrey's rules can be used to compare model 1 versus model 2 based on the weight of evidence contained in the data (see Gill, 2007):

$\mathcal{B}_{\mathcal{M}_1,\mathcal{M}_2} > 1$	model 1 supported,
$1 > \mathcal{B}_{\mathcal{M}_1,\mathcal{M}_2} \geq 10^{-\frac{1}{2}}$	minimal evidence against model 1,
$10^{-\frac{1}{2}} > \mathcal{B}_{\mathcal{M}_1,\mathcal{M}_2} \geq 10^{-1}$	substantial evidence against model 1,
$10^{-1} > \mathcal{B}_{\mathcal{M}_1,\mathcal{M}_2} \geq 10^{-2}$	strong evidence against model 1,

2.4 Testable Implications of the CSW Model

The implications of the CSW model can be tested in order to provide evidence in favor or against the adequacy of this specification for modeling hyperinflation phenomena:

Inherent Cointegration

A necessary condition for the CSW model to hold is that real money balances and inflation be integrated of the same order and cointegrate –after normalization on real money balances– with a cointegration parameter equal to α, the semi-elasticity of real money demand with respect to expected inflation in the hyperinflation model (Taylor, 1991). Let ϵ_{t+1} be the stationary rational expectations error $\epsilon_{t+1} = \left(\Delta p_{t+1} - \Delta p_{t+1}^{e}\right)$, then the portfolio balance schedule, $m_t - p_t = -\alpha \Delta p_t^e + u_t$ becomes,

$$\Delta p_{t+1} = \alpha^{-1}\left(m_t - p_t\right) + \zeta_{t+1},$$

wit $\zeta_{t+1} = \left(\epsilon_{t+1} - \alpha^{-1} u_t\right)$. If during hyperinflations real money balances and inflation are non-stationary, first-difference stationary processes, $\left(m_t - p_t\right) \sim I(1)$, $\Delta p_t \sim I(1)$, the omitted real-side variables contained in u_t admit a Wold representation, and subtracting Δp_{t+1} from both sides of $\Delta p_{t+1} = \alpha^{-1}\left(m_t - p_t\right) + \zeta_{t+1}$,

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[6] The marginal density of the data, conditional on a t-model, can be calculated using information from the B-runs of the Metropolis-Hastings algorithm with the harmonic mean estimator of Geweke (1999),

$$\hat{\mathbb{P}}(\mathbf{Y}_T \mid \mathcal{M}_i) = \left[\frac{1}{B}\sum_{b=1}^{B} \frac{f\left(\boldsymbol{\theta}_{\mathcal{M}_i}^{(b)}\right)}{\mathbb{P}\left(\boldsymbol{\theta}_{\mathcal{M}_i} \mid \mathcal{M}_i\right)\mathbb{P}\left(\mathbf{Y}_T \mid \boldsymbol{\theta}_{\mathcal{M}_i}^{(b)}, \%\mathcal{M}_i\right)}\right]^{-1}$$

where $f(\cdot)$ is a probability density function.

$$\Delta^2 p_{t+1} = \alpha^{-1}(m_t - p_t) - \Delta p_t + \zeta_{t+1}.$$

Since $\Delta^2 p_{t+1}$ and ζ_{t+1} are stationary, this equation implies that the linear combination $\left[\alpha^{-1}(m_t - p_t) - \Delta p_t\right]$ must be stationary if the CSW model holds.

Rational Expectations Restriction

If inflation is $I(1)$ and is cointegrated with real money balances, then $e_t = (m_t - p_t) - \alpha \Delta p_t$ would be stationary, and taking expectations of $\Delta^2 p_{t+1} = \alpha^{-1}(m_t - p_t) - \Delta p_t + \zeta_{t+1}$ conditional to information at time t, $\mathbf{\Omega}_t$,

$$\mathbb{E}\left(\alpha \Delta^2 p_{t+1} - e_t \mid \mathbf{\Omega}_t\right) = \mathbb{E}\left(\iota_{t+1} \mid \mathbf{\Omega}_t\right),$$

where $\iota_{t+1} = (\alpha \epsilon_{t+1} - u_t)$. With an information set of the current and n-lagged values of $\Delta^2 p_t$ and e_t, $\mathbf{H}_t^n = \{\Delta^2 p_t, \Delta^2 p_{t-1}, \ldots, \Delta^2 p_{t-n}, e_t, e_{t-1}, \ldots, e_{t-n}\}$, $\mathbf{H}_t^n \subseteq \mathbf{\Omega}_t$, and under the assumption that the CSW model is exact $(\mathbb{E}\left(u_t \mid \mathbf{H}_t^n\right) = 0)$,

$$\mathbb{E}\left(\alpha \Delta^2 p_{t+1} - e_t \mid \mathbf{H}_t^n\right) = 0.$$

Then, testing for zero coefficients in a least square projection of $\left(\alpha \Delta^2 p_{t+1} - e_t\right)$ onto elements of $\mathbf{H}_t^n$ allows to evaluate the orthogonality condition $\mathbb{E}\left(\alpha \Delta^2 p_{t+1} - e_t \mid \mathbf{H}_t^n\right) = 0$, i.e. the rational expectations hypothesis that agents' expectation of next period's inflation rate should be the true conditional mathematical expectation.

Co-explosiveness

Recently, Nielsen (2008, 2010) pointed out that the CSW model is not adequate if inflation accelerates during hyperinflations, i.e., when $(m_t - p_t) \sim I(1)$ but Δp_t is explosive, thus the cointegration analysis cannot be applied directly. This would be the case if the roots of a vector auto-regression are explosive (within the class of vector autoregressive models, accelerating inflations can be captured by allowing for an explosive characteristic root generating a common explosive trend). Let $\mathbf{Z}_t = \left[p_t \quad m_t\right]'$ be the vector of log prices and money during hyperinflations, a vector autoregressive (VAR) model with deterministic exogenous variables for $\mathbf{Z}_t$ would be,

$$\begin{aligned}\mathbf{Z}_t = \mathbf{\Phi}_1 \mathbf{Z}_{t-1} + \cdots + \mathbf{\Phi}_{k-1} \mathbf{Z}_{t-k-1} + \mathbf{\Phi}_k \mathbf{Z}_{t-k} \\ + \mathbf{\Xi}_1 \mathbf{X}_{t-1} + \cdots + \mathbf{\Xi}_q \mathbf{X}_{t-q} + \varpi_t\end{aligned},$$

with ϖ_t a multivariate white noise process and $\mathbf{\Xi}_q, \mathbf{\Phi}_k$ matrixes of coefficients for $\mathbf{Z}_t$ and $\mathbf{X}_t$. Let $\mathbf{D}$ be the companion matrix of this model,

$$\mathbf{D} = \begin{bmatrix} \hat{\boldsymbol{\Phi}}_1 & \hat{\boldsymbol{\Phi}}_2 & \cdots & \hat{\boldsymbol{\Phi}}_{k-1} & \hat{\boldsymbol{\Phi}}_k \\ \mathbf{I}_k & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_k & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I}_k & \mathbf{0} \end{bmatrix},$$

if any of the eigenvalues or the eigenvalues' modulus from the secular equation $\det\left(\mathbf{D} - \kappa\, \mathbf{I}_k\right)$ is larger than one, then the dynamics of prices and money is co-explosive and the CSW model would not be adequate to describe the hyperinflation episode under study.

2.5 Bayesian Testing of the CSW Implications

Bayesian Testing of Unit Roots

The *inherent cointegration* of real money balances and inflation can be tested with the Bayesian unit root test of Sims (1988). Let $\tilde{\alpha}$ be the Bayesian estimation of α, a necessary conditions for inherent cointegration is that both $\left(m_t - p_t\right)$ and Δp_t be $I(1)$ and $\left[\left(m_t - p_t\right) - \tilde{\alpha}\Delta p_t\right] \sim I(0)$, i.e. $\left[\left(m_t - p_t\right) - \tilde{\alpha}\Delta p_t\right]$ should not have a unit root.[7]

According to Sims, the unit root hypothesis can be tested comparing the statistic $\tau^2 = \left[\left(1-\hat{\rho}\right)/\sigma_\rho\right]^2$ with two *critical values*: an asymptotic Schwarz limit $-log\left(\sigma_\rho^2\right)$ and a small-sample limit (assuming a flat prior distribution and a Normal-inverse Gamma likelihood function),[8]

$$2log\left(\frac{1-\mathbb{P}(\rho)}{\mathbb{P}(\rho)}\right) - log(\sigma_\rho^2) - log(2\pi) + 2log(1-\iota),$$

where $1-\mathbb{P}(\rho)$ is the prior probability of $\rho = 1$ and ι is the low bound of the region $(\iota,1)$ where the prior is concentrated.

Bayesian Testing of Rational Expectations Restrictions

Let $\tilde{\beta}_c$ and $\tilde{\boldsymbol{\beta}}_{\mathbf{H}}$ be two Bayesian-conjugate estimators from two different models. In the first model, the variable $y := (\alpha\Delta^2 p_{t+1} - e_t)$ is a function of a constant term only, and in the second model y is a function of $\mathbf{H} := \mathbf{H}_t^n$, [9]

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[7] Note that this procedure is similar to the frequentist two-step testing methodology of Engle and Granger (1987).

[8] With ρ the ordinary least square estimator of the autoregresive parameter and σ_ρ^2, the estimated variance of this parameter. Sims (1988) questioned the use of traditional unit root tests such as the Dickey-Fuller or the Phillips-Perron test, as the *disconcerting topology* (as termed by Sims) of the discontinuous confidence regions encourages unreasonable inference. Also, unlike the traditional unit root tests, the Sims' test allows to incorporate the empirical evidence that a large value of σ_ρ^2 (i.e. the scale parameter of the marginal Student's t distribution of ρ, with $T-1$ degrees of freedom) provides evidence against the unit root hypothesis, even if the value of τ^2 statistic for $\rho = 1$ is relatively small.

[9] See Appendix 3 for details.

$$\tilde{\beta}_c = \left(\boldsymbol{\Sigma}_c^{-1} + \mathbf{1}'\mathbf{1}\right)^{-1}\left(\boldsymbol{\Sigma}_c^{-1}\mathbb{B}_c + \mathbf{1}'\mathbf{1}\hat{b}_c\right),$$

$$\tilde{\boldsymbol{\beta}}_{\mathbf{H}} = \left(\boldsymbol{\Sigma}_{\mathbf{H}}^{-1} + \mathbf{H}'\mathbf{H}\right)^{-1}\left(\boldsymbol{\Sigma}_{\mathbf{H}}^{-1}\mathbb{B}_{\mathbf{H}} + \mathbf{H}'\mathbf{H}\,\hat{b}_{\mathbf{H}}\right).$$

Model 1 involves the rational expectations restriction of zero coefficients for $\Delta^2 p_t, \Delta^2 p_{t-1}, \ldots, \Delta^2 p_{t-n}, e_t, e_{t-1}, \ldots, e_{t-n}$. In model 2 the restriction is not met because the parameters contained in $\tilde{\boldsymbol{\beta}}_{\mathbf{H}}$ are presumed to be statistically different from zero. These two alternatives translate to $\mathbb{P}(\mathbb{H}_0 \mid data) + \mathbb{P}(\mathbb{H}_1 \mid data) = 1$, where model 1 represents the null hypothesis of rational expectations ($\mathbb{H}_0$). This null can be tested with the posterior probability of $\mathbb{H}_0$ as a function of the Bayes factor of model 1 against model 2 $\left(\mathcal{B}_r\right)$, scaled by the ratio of the prior probability of rational expectations $\mathbb{P}\left(\mathbb{H}_0\right)$ against the prior of no rational expectations $\mathbb{P}\left(\mathbb{H}_1\right)$:[10]

$$\mathbb{P}\left(\mathbb{H}_0 \mid data\right) = \left[1 + \frac{1}{\mathcal{B}_r}\frac{\mathbb{P}\left(\mathbb{H}_1\right)}{\mathbb{P}\left(\mathbb{H}_0\right)}\right]^{-1}.$$

Bayesian Stability Evaluation

Since the number of parameters in a VAR(k) model can be quite large relative to the available data, as the number of lags k increases there is an increasing possibility of getting imprecise estimates of $\boldsymbol{\Phi}_1, \ldots, \boldsymbol{\Phi}_k$ and thus an imprecise stability evaluation. As Robertson and Tallman (1999) suggested, in small samples, it is necessary to put constraints on the values of the model's coefficients in order to require less information from the data when determining the coefficient values; with Bayesian methods, these constraints adopt the

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[10] Gill (2007). Because $\mathbb{P}(\mathbb{H}_0 \mid data) + \mathbb{P}(\mathbb{H}_1 \mid data) = 1$, then $\mathbb{P}\left(\mathbb{H}_0 \mid data\right) = 1 - \mathbb{P}\left(\mathbb{H}_1 \mid data\right)$. Using the Bayes law and the definition of the Bayes factor:

$$\begin{aligned}
\mathbb{P}\left(\mathbb{H}_0 \mid data\right) &= 1 - \mathbb{P}\left(data \mid \mathbb{H}_1\right)\frac{\mathbb{P}\left(\mathbb{H}_1\right)}{\mathbb{P}\left(data\right)} \\
&= 1 - \frac{\mathbb{P}\left(data \mid \mathbb{H}_0\right)}{\mathcal{B}_r}\frac{\mathbb{P}\left(\mathbb{H}_1\right)}{\mathbb{P}\left(data\right)}, \\
&= 1 - \frac{1}{\mathcal{B}_r}\left[\frac{\mathbb{P}\left(data\right)}{\mathbb{P}\left(\mathbb{H}_0\right)}\mathbb{P}\left(\mathbb{H}_0 \mid data\right)\right]\frac{\mathbb{P}\left(\mathbb{H}_1\right)}{\mathbb{P}\left(data\right)} \\
&= \left[1 + \frac{1}{\mathcal{B}_r}\frac{\mathbb{P}\left(\mathbb{H}_1\right)}{\mathbb{P}\left(\mathbb{H}_0\right)}\right]^{-1}.
\end{aligned}$$

In the normal-linear model with natural conjugate priors, the integrals of the marginal likelihood of the Bayes factor $\mathcal{B}_r$ can be calculated analytically (see Koop, 2006, pp. 24-25 for details).

In a traditional frequentist approach, the rational expectations restriction is tested running a least squares regression of $(\alpha\Delta^2 p_{t+1} - e_t)$ against $\mathbf{H}_t^n$ and testing the null that the coefficients that correspond to $\mathbf{H}_t^n$ are equal to zero. Testing rational expectations restrictions with Bayesian methods is not as straightforward as in the frequentist analysis, being the two-sided hypothesis $\beta = 0$ antithetical to the Bayesian philosophy: Bayesians are uncomfortable placing prior mass on a point null hypothesis. In testing $\beta = 0$ against $\beta \neq 0$, a continuous prior distribution for β cannot be assigned since this would imply zero mass at the null point, providing an infinite bias against the nesting (See Gill, 2007, page 238, for a detailed discussion).

form of prior densities for $\Phi_1,\ldots,\Phi_k$.

More specifically, with the Sims-Zha Normal-Wishart prior (Sims and Zha, 1998), the traditional OLS estimator of a VAR model,

$$\hat{\Phi}_i^{OLS} = \left(\mathbf{R}'\mathbf{R}\right)^{-1}\mathbf{R}'\mathbf{y}, i = 1,\ldots,k \ ,$$

is replaced with the Bayesian estimator,

$$\Phi_i^B = \left(\overline{\mathbf{H}}^{-1} + \mathbf{R}'\mathbf{R}\right)^{-1}\left(\overline{\mathbf{H}}^{-1}\overline{\mathbf{B}} + \mathbf{R}'\mathbf{y}\right), i = 1,\ldots,k,$$

where $\mathbf{y}$ is the vector of endogenous variables, $\mathbf{R}$ is the matrix of right-hand side regressors in the VAR model (lags of $\mathbf{y}$ and exogenous variables), $\overline{\mathbf{B}}$ is the prior mean of coefficients and $\overline{\mathbf{H}}$ is a diagonal, positive-definite matrix, defined with the vector of hyperparameters $\varkappa = \left[\varkappa_1 \cdots \varkappa_6\right]$, where $\varkappa_1$ to $\varkappa_4$ govern the lag decay and the tightness around the parameters, and $\varkappa_5, \varkappa_6$ control the possibility of cointegration and unit roots.[11] Then, the Bayesian stability evaluation of a hyperinflation episode is based on the eigenvalues of the companion matrix $\mathbf{D}^B$ of a Bayesian vector auto-regression,

$$\mathbf{D}^B = \begin{bmatrix} \hat{\Phi}_1^B & \hat{\Phi}_2^B & \cdots & \hat{\Phi}_{k-1}^B & \hat{\Phi}_k^B \\ \mathbf{I}_k & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_k & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I}_k & \mathbf{0}, \end{bmatrix},$$

with $\hat{\Phi}_i^B$, $i = 1,\ldots,k$ the Bayesian estimators defined previously.

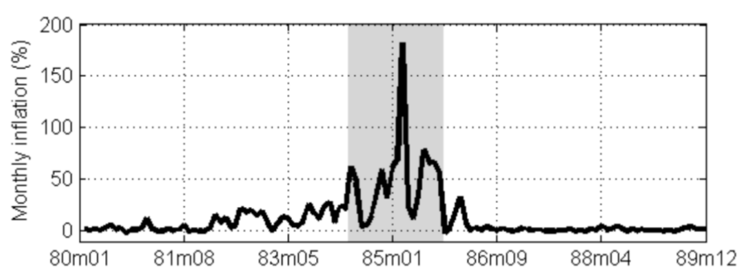
[11] Sims and Zha assume that the prior conditional covariance matrix of this coefficients follow the same pattern that Litterman (1986) gave to the prior covariance matrix on reduced form coefficients, i.e., the conditional prior is independent across elements and the conditional standard deviation of the coefficient on lag l of variable j in equation i is given by $\varkappa_0\varkappa_1 / \sigma_j l^{\varkappa_3}$ .The hyperparameter $\varkappa_0$ controls the tightness of beliefs, $\varkappa_1$ controls the overall tightness around the random walk prior and $\varkappa_3$ controls the rate at which prior variance shrinks with increasing lag length. The parameters $\sigma_1,\ldots,\sigma_m$ are scale factors, allowing for the fact that the units of measurement or scale of variation may not be uniform across variables. Sims and Zha choose these as the sample standard deviations of residuals from univariate autoregressive models fitted to the individual series in the sample. The constant term has a conditional prior mean of zero and a standard deviation controlled by $\varkappa_0\varkappa_4$, where $\varkappa_4$ is a separate hyperparameter. Sims and Zha also introduce dummy observations in the posterior p.d.f. to favor the presence of unit roots and cointegration, with the scale hyperparameters $\varkappa_5$ and $\varkappa_6$. When $\varkappa_5 \to \infty$, the model tends to a form that can be expressed entirely in terms of differenced data, i.e. there are as many unit roots as variables and there is no cointegration. As $\varkappa_6 \to \infty$ the model tends to a form in which either all the variables are stationary, or there are unit root components without drift (linear trend) terms, without ruling out cointegrating relationships. See Appendix 4 for details.

3. RESULTS

During 1984 to 1985, monthly inflation almost reached 200 percent in Bolivia (Figure 1). Unlike other hyperinflations, the Bolivian case did not arise after a foreign war, a civil war or a political revolution, but rather in a period of political uncertainty that followed years of prosperity based on auspicious terms of trade and heavy foreign borrowing in the late 70s. The situation in Bolivia worsened as a consequence of the unfavorable international environment of high interest rates and falling commodities prices in the early 80s. By the year 1984, the inflation rate began to accelerate, the economy declined in real terms and the government was unable to borrow from international markets. The end of the crisis came along with an orthodox stabilization program proposed by the newly elected president Victor Paz Estensoro in august 1985 (Sachs, 1987).[12]

Cagan (1956) defined a hyperinflation as beginning in the month the rise in prices exceeds 50 percent and as ending in the month before the monthly rise in prices drops below that amount and stays below for at least a year. By this definition, the Bolivian hyperinflation lasted from April 1984 to September 1985, a total sample of 18 observations.

Figure 1
Bolivian Inflation and Hyperinflation (Shaded)



[12] This program prompted the devaluation and subsequent managed float of the exchange rate, a commitment to full currency convertibility on the current and capital accounts, a reduction of fiscal deficit through a sharp increase in public sector prices (e.g., the price of domestic oil), a public sector wage freeze, a tax overhaul proposal to broaden the tax base and raise tax revenue and the signing of an IMP stand-by arrangement, besides other policies of trade liberalization, internal price decontrol and decentralization or privatization of public enterprisers. See inter alia Sachs (1987), Morales and Sachs (1989) and Pastor (1991) for details of the Bolivian hyperinflation.

Table 1
Bayesian Estimates of the CSW Model[1]

	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$
α	−11.4615 [−17.7434, −4.1398]	−14.0747 [−19.4283,−9.1682]	−8.3639 [−9.8646, −6.8366]
λ	0.6691 [0.5096, 0.865]	0.7164 [0.6428, 0.7928]	0.6752 [0.6058, 0.7531]
σ_η^2	0.7020 [0.4176, 0.9902]	0.7768 [0.5500, 0.9998]	0.5442 [0.3233, 0.8070]
σ_ε^2	0.1795 [0.1376, 0.2329]	0.1750 [0.1261, 0.2229]	0.1753 [0.1246, 0.2234]

[1] $\mathcal{M}_1$, $\mathcal{M}_2$, $\mathcal{M}_3$: Bayesian estimates for models 1, 2 and 3, respectively. Bayesian credible intervals between brackets.

3.1 CSW Estimation And Comparison

Three CSW models $\mathcal{M}_1$, $\mathcal{M}_2$, $\mathcal{M}_3$ with different prior distributions were estimated with the sample of the Bolivian hyperinflation. The Bayes factor analysis supported $\mathcal{M}_2$, in which the estimation of the parameter of interest –the semi-elasticity of demand for real balances with respect to expected inflation– was equal to $\hat{\alpha}_{\mathcal{M}_2} = -14.0747$ (Table 1, Figure 2).

In order to mitigate possible frequentist criticisms of subjectivity, model $\mathcal{M}_1$ was design as an agnostic model that avoids assigning informative values for the unknown parameters $\boldsymbol{\theta}_{\mathcal{M}_1} = \left(\alpha, \lambda, \sigma_\varepsilon^2, \sigma_\eta^2\right)$:

In $\mathcal{M}_1$ uninformative but proper uniform priors with bounded support were chosen for $\boldsymbol{\theta}_{\mathcal{M}_1}$:

$$\mathbb{P}(\alpha \mid \mathcal{M}_1) \overset{d}{=} \mathcal{U}(-20,0), \mathbb{P}(\lambda \mid \mathcal{M}_1) \overset{d}{=} \mathcal{U}(0,1),$$
$$\mathbb{P}\left(\sigma_\eta^2 \mid \mathcal{M}_1\right) \overset{d}{=} \mathcal{U}(0,1), \mathbb{P}\left(\sigma_\varepsilon^2 \mid \mathcal{M}_1\right) \overset{d}{=} \mathcal{U}(0,1)$$

In $\mathcal{M}_2$ and $\mathcal{M}_3$ the prior distribution of λ was $\mathbb{P}(\lambda \mid \mathcal{M}_{2,3}) \overset{d}{=} \mathcal{N}(0.7093, 0.003)$, with $\hat{\lambda}_{\mathcal{L}} = 0.7093$ the maximum likelihood estimate of the CSW model. For these models, the prior elicitation of α followed two approaches:

In model $\mathcal{M}_2$ the prior of α was based on the maximum likelihood estimate $\hat{\alpha}_{\mathcal{L}} = -15.5$, then $\mathbb{P}(\alpha \mid \mathcal{M}_2) \overset{d}{=} \mathcal{N}(-15.59, 4.32)$.

In model $\mathcal{M}_3$ the location parameter of the prior distribution was elicitated according to scientific knowledge on the field, a common practice in Bayesian analysis. A value of 8.052 was chosen for the prior mean of α, $\mathbb{P}(\alpha \mid \mathcal{M}_3) \overset{d}{=} \mathcal{N}(-8.052, 1)$; this value is based on the Phylaktis and Taylor (1993) non-Bayesian estimation of the CSW model for the Bolivian hyperinflation.

Bayes factors were used both to compare and choose between models (Table 2), using $\mathcal{M}_1$ as a benchmark model. Substantial evidence against model 1 was found because $10^{-1} < \mathcal{B}_{\mathcal{M}_1,\mathcal{M}_2}, \mathcal{B}_{\mathcal{M}_1,\mathcal{M}_3} < 10^{-1/2}$.

The Bayes factor of $\mathcal{M}_2$ against $\mathcal{M}_3$ was equal to

$$\mathcal{B}_{\mathcal{M}_2,\mathcal{M}_3} = \frac{\exp(0.490932)}{\exp(0.350166)} = 1.1511,$$

favoring model 2. Since $\mathcal{M}_2$ was supported by the data, the Bayesian point estimation $\tilde{\alpha}_{\mathcal{M}_2}$ of model 2 was used in the validation analysis.

Table 2

Bayes Factors

	$\ln \hat{\mathbb{P}}(\mathrm{Y}_T \mid \mathcal{M}_i)$	$\mathcal{B}_{\mathcal{M}_i,\mathcal{M}_j}$
$\mathcal{M}_1$	−1.101327	1
$\mathcal{M}_2$	0.490932	0.2035
$\mathcal{M}_3$	0.350166	0.2342

Figure 2

Bayesian Estimation of α in Models $\mathcal{M}_1$ (top), $\mathcal{M}_2$ (middle) and $\mathcal{M}_3$ (bottom)

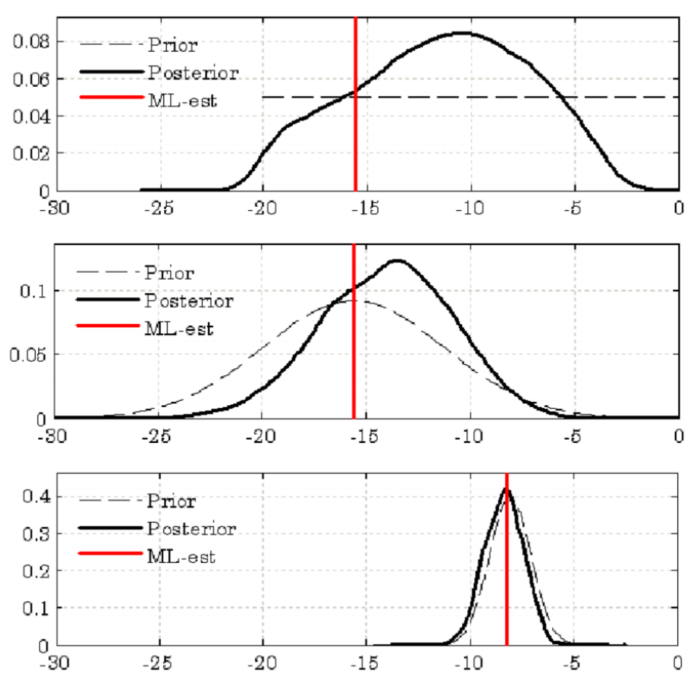
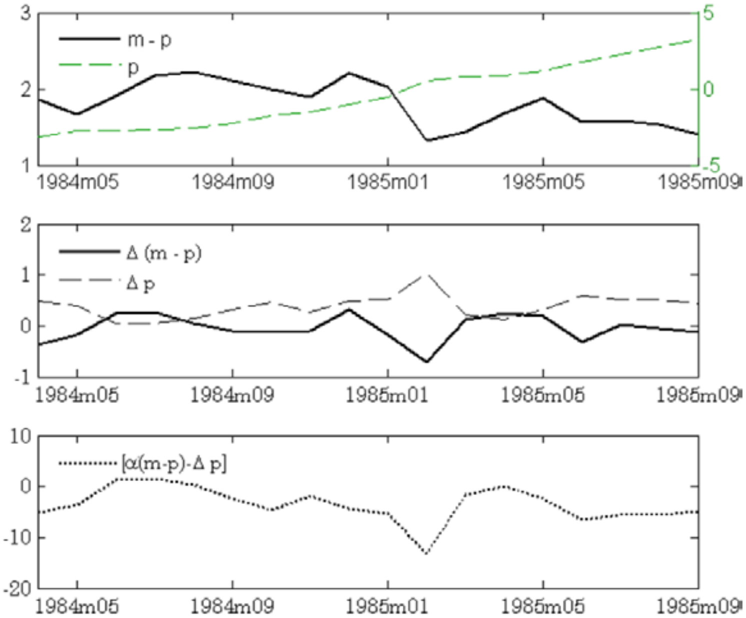


Figure 3

Dynamics of m, p in levels (top), differences (middle) and the transformed expression $[\alpha^{-1}_{\mathcal{M}_2}(m_t - p_t) - \Delta p_t]$ (bottom)



3.2 Bayesian Validation of the CSW Model

Testing the CSW Model (I): Unit Roots

The results of the Bayesian unit root test suggested that real money balances and prices were cointegrated during the Bolivian hyperinflation. Using a probability of 0.1 on the stationary part of the prior, flat on the interval [0.5,1], the linear combination $\left[\tilde{\alpha}^{-1}_{\mathcal{M}_2}(m_t - p_t) - \Delta p_t\right]$ was found to be stationary with a zero order of integration. On the contrary it was found that $(m_t - p_t) \sim I(1)$ and $p_t \sim I(1)$. Figure 3 shows that p_t evolves with an upward trend and $(m_t - p_t)$ seems to follow a random walk process. Then, as expected, the null of unit root of these series could not be rejected with the Sims' statistic, based on the small-sample limit of the Bayesian test. In contrast, the null of unit root is rejected for $\Delta(m_t - p_t), \Delta p_t$ and $\left[\tilde{\alpha}^{-1}_{\mathcal{M}_2}(m_t - p_t) - \Delta p_t\right]$, both with the Schwarz asymptotic limit and the small-sample limit (Table 3). The low value of the marginal probability of the test shows that the data provides strong evidence against the unit root hypothesis of $\left[\tilde{\alpha}^{-1}_{\mathcal{M}_2}(m_t - p_t) - \Delta p_t\right]$, supporting the evidence in favor of the CSW model $\mathcal{M}_2$.

Table 3

Results of the Bayesian Unit Root Test

	Sims statistic	*Asymptotic limit (Schwarz)*	*Small-sample limit*	*Marginal probability*
$(m_t - p_t)$	2.5923	3.0557	4.2260	0.2009
p_t	3.5037	3.7773	4.9476	0.1861
$\Delta(m_t - p_t)$	14.7348	2.8039	3.9742	0.0005
Δp_t	8.2053	2.8086	3.9789	0.0132
$\left[\tilde{\alpha}_{\mathcal{M}_2}^{-1}(m_t - p_t) - \Delta p_t\right]$	7.8217	2.8148	3.9851	0.0161

Table 4

Bayesian Testing of Rational Expectations Restrictions

Prior assumptions[1]		*Regressor matrix*[2]	
		H(0)	***H(1)***
$\mathbb{P}(\mathbb{H}_0) = P(\mathbb{H}_1)$	$\mathcal{B}_r$	0.00442	0.00522
	$\mathbb{P}(\mathbb{H}_0 \mid data)$	0.00440	0.00519
$\mathbb{P}(\mathbb{H}_0) > P(\mathbb{H}_1)$	$\mathcal{B}_r$	0.03978	0.04699
	$\mathbb{P}(\mathbb{H}_0 \mid data)$	0.26362	0.29720
Log of marginal likelihood		–51.5401	–51.7067

[1] In the first case $\mathbb{P}(\mathbb{H}_0) = P(\mathbb{H}_1) = 0.5$, and in the second case $\mathbb{P}(\mathbb{H}_0) = 0.9$ and $\mathbb{P}(\mathbb{H}_1) = 0.1$.

[2] $H(0) = \{1, \Delta^2 p_t, e_t\}$ and $H(0) = \{1, \Delta^2 p_t, \Delta^2 p_{t-1}, e_t, e_{t-1}\}$.

Testing the CSW Model (II): Rational Expectations

The rational expectations hypothesis was rejected, even when a high prior probability of rational expectations was assumed a priori. The log-marginal likelihood of the Bayesian model of $y := (\alpha \Delta^2 p_{t+1} - e_t)$ regressed on a constant term, was equal to –56.9618. Including $\Delta^2 p_{t+1}$ and e_t as regressors, the log-marginal likelihood increased to –51.5401. Based on the comparison of these marginal likelihoods, the probability of rational expectations is equal to 0.0044, with a Bayes factor of 0.00442 (Table 4). These values provide decisive evidence against the rational expectations hypothesis. The statistical significance of the parameters β_1 and β_2, related to $\Delta^2 p_{t+1}$ and e_t respectively, can be appreciated in the form of the marginal posteriors and the joint posterior density of these coefficients, which is concentrated away from zero (Figure 4).

Even when a high prior probability of rational expectations was assumed in advance $\mathbb{P}(\mathbb{H}_0) = 0.9$, the posterior probability of rational expectations is still low, equal to

$\mathbb{P}(\mathbb{H}_0 \mid data) = 0.29362$, providing again substantial evidence against the rational expectations restrictions. Similar results were obtained when lags of $\Delta^2 p_{t+1}$ and e_t were included in the Bayesian auxiliary model (Table 4).

Testing the CSW Model (III): Explosive Roots

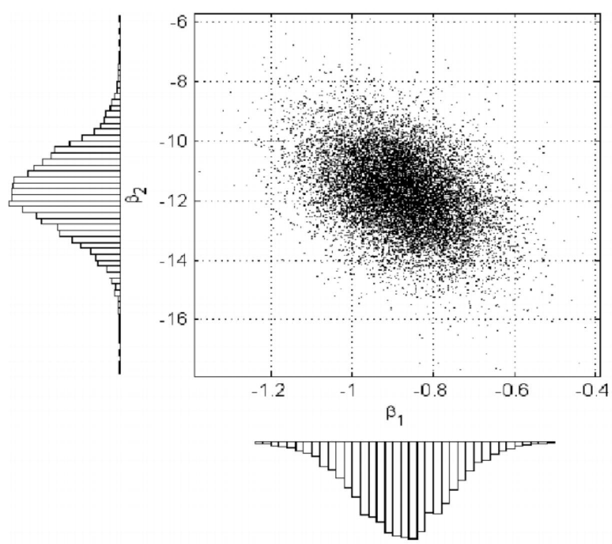
No evidence of co-explosiveness was found after estimating Bayesian vector auto-regressions with the vector of prices and money. Three BVAR(k) models (k = 1,2,3) where estimated, with a constant and a trend term as exogenous regressors (as in Nielsen, 2008), using a Sims-Zha prior equal to [$\varkappa_1$, $\varkappa_2$, $\varkappa_3$, $\varkappa_4$, $\varkappa_5$, $\varkappa_6$] = [0.7, 1, 0.9, 1, 0.4, 0.1].[13] In all the cases the roots of the companion matrix were inside the unit circle (Table 5, Figure 5), suggesting that prices and money were not co-explosive during the Bolivian hyperinflation.

4. DISCUSSION

The hyperinflation episode of Bolivia was modeled with Bayesian methods. A low probability that the agents' expectations have been rational was found, and there is no evidence that the behavior of prices and real balances was explosive during this hyperinflation. The results suggest that the excess of money was the cause of this hyperinflationary phenomenon, as the authorities expanded the money stock beyond the optimal rate that maximizes the sustainable real revenues.

Figure 4

Joint Posterior Density of β_1 and β_2



[13] The values of $\varkappa_5, \varkappa_6$ were chosen to reflect the observed non-stationary behavior of prices and money, and the evidence of cointegration between these variables. The values of $\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4$ were chosen minimizing the in-sample root mean squared error between the fitted values and the observed vector of prices and money. A similar approach can be found in Brandt and Freeman (2002).

Figure 5

Roots of D^B for three Bayesian VAR Models

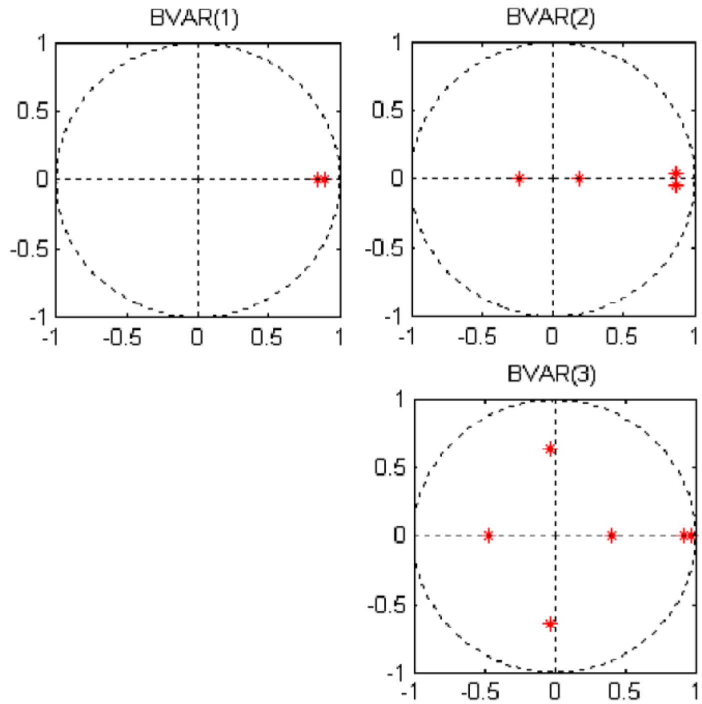


Table 5

Bayesian Stability Evaluation

BVAR order	*Eigenvalues of the companion matrix*		
	Real	***Imaginary***	***Modulus***
1	0.8970	-	-
	0.8450	-	-
2	0.8790	–0.0343	0.8716
	0.8709	–0.0343	0.8716
	–0.2351	–	–0.2351
	0.1850	–	0.1850
3	0.9688	–	0.9688
	0.9111	–	0.9111
	–0.0332	0.6449	0.6458
	–0.0332	–0.6449	0.6458
	–0.4752	-	0.4752
	0.4126	-	0.4126

The Bayesian estimation of $\tilde{\alpha}_{\mathcal{M}_2} = -14.07$ indicates a theory-consistent negative semi-elasticity in the money demand equation. This estimate implies an optimal rate of inflation equal to $7.10 \ \ (-1/\tilde{\alpha}_{\mathcal{M}_2})$ with a Bayesian credible interval of 5.15% to 10.91%. Average inflation during the Bolivian hyperinflation was 50.8%; a value outside the credible intervals that exceeds seven times the optimal-inflation-rate point estimation. This result indicates an excessively fast money supply expansion in the Bolivian hyperinflation, far above the optimal rate that maximizes the real revenues that the central bank could have obtained (i.e., the Cagan's paradox). This finding is in line with the debt-crisis' explanation of the origins of the Bolivian hyperinflation proposed by Morales and Sachs (1989): as net resource transfers to Bolivia from the rest of the world turned negative in 1982, seigniorage financing substituted the decline in resource flows and the government focused their resources on foreign debt service, resorting to printing money to finance the domestic spending that no longer could be sustained with taxes, due to the collapse of the main export commodities. This monetary emission promote a repeated and accelerating depreciation of the exchange rate, further prompting inflation.

Since no evidence of co-explosiveness between prices and money supply was found for the Bolivian hyperinflation, this event would have been different from the hyperinflation episode of Yugoslavia, for which Nielsen (2008, 2010) found evidence of co-explosiveness, requiring the use of vector auto-regressive models with explosive roots of the type described in Juselius and Mladenovic (2002) and Nielsen (2010).

The orthogonality-constraint rejection of the rational expectations hypothesis (REH) implies that there were demand shocks (not explicitly detailed in the CSW model) that explain money demand in the Bolivian hyperinflation, besides expectations. However, the rejection of the REH could be negligible to the findings about the presence of the Cagan's paradox, since, as suggested by Engsted (1998), the estimation of α does not require the precise specification of the mechanism behind the formation of expectations or the exact nature of the disturbance μ_t, if money balance and inflation are cointegrated. Evidence of this inherent cointegration was found with a Bayesian unit root test applied to $\left[\tilde{\alpha}_{\mathcal{M}_2}^{-1}\left(m_t - p_t\right) - \Delta p_t\right]$.

Methodologically, it was shown that the Bayesian approach allows to properly model an economic crisis with a short sample of observations, since these methods increase the precision of statistical inference by means of incorporating information about the parameters of interest in the form of prior probability assumptions. Furthermore, the validity of the prior assumptions can be evaluated empirically with Bayes factors, and the implications of the model can be tested with modern Bayesian techniques, thus improving the quality of the specification process in small samples.

APPENDIX 1

Intertemporal Utility Maximizing Framework of the Cagan's Model

Cagan's model can be derived from an intertemporal utility maximizing framework under rational expectations, as in Gray (1984). Consider an economy composed of identical, infinitely lived households, each of which maximizes the utility function,

$$U = \int_0^{\infty} e^{\rho t}\left[v\left(c_t\right)+w\left(n_t\right)\right]dt,$$

subject to the budget constraint,

$$P_t y = P_t c_t + \dot{M}_t,$$

being ρ the internal rate of discount, c_t and m_t real household consumption and money balances at time t, P_t the price level, and $\dot{\boldsymbol{M}}_t$ the instantaneous time rate of change of nominal money balances. The solution to the household's problem is found by maximizing the Hamiltonian function H with respect to the control variable c_t,

$$\mathcal{H} = \left[v\left(c_t\right)+w\left(n_t\right)\right]e^{-\rho t} + \chi_t\left[P_t y - P_t c_t\right].$$

Any consumption plan that maximizes $\mathcal{H}$ must satisfy,

$$\frac{\partial \mathcal{H}}{\partial c_t} = v_c\left(c_t\right)e^{-\rho t} - \chi_t P_t = 0,$$

$$\frac{\partial \mathcal{H}}{\partial M_t} = -\dot{\chi}_t = \chi_t w_n\left(n_t\right)\left(1/P_t\right)e^{-\rho t},$$

$$\frac{\partial \mathcal{H}}{\partial \lambda_t} = \dot{M}_t = P_t y - P_t c_t.$$

Equilibrium in the markets of goods and money requires that a fixed stock of nominal money be demanded by households at each point in time. This implies zero planned net increments to the nominal money balances of the representative household, $\dot{M}_t = 0$, thus allowing to solve the equilibrium price path,

$$\frac{\dot{P}_t}{P_t} = \frac{w_n\left(n_t\right)}{v_c\left(y\right)} - \rho.$$

The money demand function can be obtained from the Euler equation if $\rho = 0$ and the function $w\left(n\right) = n - n\ln n$. See Gray (1984).

APPENDIX 2

State-space Representation of the CSW Model

The system of equations in section 2.2 can be obtained after rewritten the bivariate process,

$$(1-L)x_t = \left(\lambda+\alpha\left(1-\lambda\right)\right)^{-1}\left(1-\lambda L\right)\left(\varepsilon_t-\eta_t\right),$$

$$\left(1-L\right)\mu_t = \left[\left(\lambda+\alpha\left(1-\lambda\right)\right)^{-1}\right]\left(1-\lambda\right)\left(\varepsilon_t-\eta_t\right)-\varepsilon_{t-1}+\varepsilon_t.$$

So,

$$\varepsilon_t - \eta_t = \frac{\left(\lambda + \alpha\left(1-\lambda\right)\right)\left(1-\lambda L\right)}{1-\lambda L} x_t,$$

substituting in $\varepsilon_t = \mu_t - \dfrac{1-\lambda}{1-\lambda L} x_t$ and rearranging gives,

$$\eta_t = \mu_t - \frac{\left(1-\lambda + (\lambda + \alpha\left(1-\lambda\right)\right)\left(1-\lambda L\right)}{1-\lambda L} x_t,$$

in vector notation,

$$\varepsilon_t \eta_t = \begin{bmatrix} -\dfrac{1-\lambda}{1-\lambda L} & 1 \\ -\dfrac{\left(1-\lambda + (\lambda + \alpha\left(1-\lambda\right)\right)\left(1-\lambda L\right)}{1-\lambda L} & 1 \end{bmatrix} x_t \mu_t .$$

Multiplying both sides of the equation by $\left(1-\lambda L\right)\mathbf{I}$ where $\mathbf{I}$ is a 2×2 identity matrix,

$$\varepsilon_t \eta_t - \lambda \mathbf{I} \varepsilon_{t-1} \eta_{t-1} = \begin{bmatrix} -\left(1-\lambda\right) & 1 \\ -\left(1+\alpha\left(1-\lambda\right)\right) & 1 \end{bmatrix} x_t \mu_t + \begin{bmatrix} 0 & -\lambda \\ \lambda + \alpha\left(1-\lambda\right) & -\lambda \end{bmatrix} x_{t-1} \mu_{t-1}.$$

Let $\mathrm{G}_0 = \begin{bmatrix} -\left(1-\lambda\right) & 1 \\ -\left(1+\alpha\left(1-\lambda\right)\right) & 1 \end{bmatrix}$. Premultiplying the preceding equation by,

$$G_0^{-1} = \begin{bmatrix} 1 & -1 \\ 1+\alpha\left(1-\lambda\right) & -\left(1-\lambda\right) \end{bmatrix} \Big/ \left(\lambda + \alpha\left(1-\lambda\right)\right).$$

Gives,

$$a_{1t} a_{2t} - \lambda \mathbf{I} a_{1t-1} a_{2t-1} = x_t \mu_t + G_0^{-1} \begin{bmatrix} 0 & -\lambda \\ \lambda + \alpha\left(1-\lambda\right) & -\lambda \end{bmatrix} x_{t-1} \mu_{t-1},$$

where $a_{1t} a_{2t} \equiv G_0^{-1} \varepsilon_t \eta_t$. Computing $G_0^{-1} \begin{bmatrix} 0 & -\lambda \\ \lambda + \alpha\left(1-\lambda\right) & -\lambda \end{bmatrix}$ explicitly and rearranging gives,

$$x_t \mu_t = \begin{pmatrix} 1 & 0 \\ \left(1-\lambda\right) & \lambda \end{pmatrix} \begin{pmatrix} x_{t-1} \\ \mu_{t-1} \end{pmatrix} + a_{1t} a_{2t} - -\lambda \mathbf{I} a_{1t-1} a_{2t-1}$$

And the innovations $a_{1t} a_{2t} = G_0^{-1} \varepsilon_t \eta_t$ are equal to,

$$a_{1t} a_{2t} = \begin{bmatrix} 1 & -1 \\ 1+\alpha\left(1-\lambda\right) & -\left(1-\lambda\right) \end{bmatrix} \Big/ \left(\lambda + \alpha\left(1-\lambda\right)\right) \varepsilon_t \eta_t,$$

$$= \frac{1}{\lambda + \alpha\left(1-\lambda\right)} \left(\varepsilon_t - \eta_t\right) \frac{1}{\lambda + \alpha\left(1-\lambda\right)} \left(\varepsilon_t - \eta_t\right).$$

See Sargent (1977).

APPENDIX 3

Natural-conjugate Bayesian Estimators for Testing Rational Expectations Restrictions

Let $y := \alpha\Delta^2 p_{t+1} - e_t$ and remove the time and lag-length subscripts of $\mathbf{H}_t^n$ for convenience, $\mathbf{H} := [\mathbf{1} \quad \mathbf{H}_t^n]$. The likelihood function for two linear regression models, one with $\mathbf{H}_t^n$ as a matrix of regressors and the other one only with a constant as a regressor, would be,

$$\mathcal{L}\left(\boldsymbol{\beta}_c, \sigma_c^2 \mid \mathbf{1}, y\right) = \left(2\pi\sigma_c\right)^{-\frac{T}{2}} \exp\left(-\frac{1}{2\sigma_c^2}\left(y - \mathbf{1}\boldsymbol{\beta}_c\right)'\left(y - \mathbf{1}\boldsymbol{\beta}_c\right)\right),$$

$$\mathcal{L}\left(\boldsymbol{\beta}_{\mathbf{H}}, \sigma^2 \mid \mathbf{H}, y\right) = \left(2\pi\sigma_{\mathbf{H}}\right)^{-\frac{T}{2}} \exp\left(-\frac{1}{2\sigma_{\mathbf{H}}^2}\left(y - \mathbf{H}\boldsymbol{\beta}_{\mathbf{H}}\right)'\left(y - \mathbf{H}\boldsymbol{\beta}_{\mathbf{H}}\right)\right),$$

where $\beta\,.$ is a $n \times 1$ vector of coefficients and $\mathbf{1}$ is a $T \times 1$ unitary vector (the constant term). The values for which $\mathcal{L}\left(\boldsymbol{\beta}_c, \sigma_c^2 \mid \mathbf{1}, y\right)$ and $\mathcal{L}\left(\boldsymbol{\beta}_{\mathbf{H}}, \sigma^2 \mid \mathbf{H}, y\right)$ are at its maximum are well known from standard likelihood theory and ordinary least square principles,

$$\hat{b}_c = \left(\mathbf{H}'\mathbf{H}\right)^{-1}\mathbf{H}'y,\ \hat{b}_{\mathbf{H}} = \left(\mathbf{1}'\mathbf{1}\right)^{-1}\mathbf{1}'y,$$

$$\hat{\sigma}_c^2 = \frac{\left(y - \mathbf{1}\hat{b}_c\right)'\left(y - \mathbf{1}\hat{b}_c\right)}{T - n},\ \hat{\sigma}_{\mathbf{H}}^2 = \frac{\left(\text{y-}\mathbf{H}\hat{b}_{\mathbf{H}}\right)'\left(\text{y-}\mathbf{H}'\hat{b}_{\mathbf{H}}\right)}{\text{T-n}}.$$

If these values were plugged into $\mathcal{L}\left(\boldsymbol{\beta}_c, \sigma_c^2 \mid \mathbf{1}, y\right)$ and $\mathcal{L}\left(\boldsymbol{\beta}_{\mathbf{H}}, \sigma^2 \mid \mathbf{H}, y\right)$, then,

$$\mathcal{L}\left(\boldsymbol{\beta}_c, \sigma_c^2 \mid \mathbf{1}, y\right) = \sigma_c^{-T} \exp\left[-\frac{1}{2\sigma_c^2}\left(\hat{\sigma}_c^2\left(T - n\right) + \left(\boldsymbol{\beta}_c - \hat{b}_c\right)\mathbf{1}'\mathbf{1}\left(\boldsymbol{\beta}_c - \hat{b}_c\right)\right)\right],$$

$$\mathcal{L}\left(\boldsymbol{\beta}_{\mathbf{H}}, \sigma^2 \mid \mathbf{H}, y\right) = \sigma_{\mathbf{H}}^{-T} \exp\left[-\frac{1}{2\sigma_{\mathbf{H}}^2}\left(\hat{\sigma}_{\mathbf{H}}^2\left(T - n\right) + \left(\boldsymbol{\beta}_{\mathbf{H}} - \hat{b}_{\mathbf{H}\%}\right)\mathbf{H}'\mathbf{H}\left(\boldsymbol{\beta}_{\mathbf{H}} - \hat{b}_{\mathbf{H}}\right)\right)\right].$$

Let $\mathbb{P}\left(\boldsymbol{\beta}_c, \sigma_c^2\right) = \mathbb{P}\left(\boldsymbol{\beta}_c \mid \sigma_c^2\right)\mathbb{P}\left(\sigma_c^2\right)$ and $\mathbb{P}\left(\boldsymbol{\beta}_{\mathbf{H}}, \sigma_{\mathbf{H}}^2\right) = \mathbb{P}\left(\boldsymbol{\beta}_{\mathbf{H}} \mid \sigma_{\mathbf{H}}^2\right)\mathbb{P}\left(\sigma_{\mathbf{H}}^2\right)$ be two compounded priors were the components are specified by the conjugates, $\boldsymbol{\beta}_c \mid \sigma_c^2 \sim \mathcal{N}\left(\mathbb{B}_c, \sigma_c^2\right), \sigma_c^2 \sim \mathcal{IG}\left(a_c, b_c\right)$ and $\boldsymbol{\beta}_{\mathbf{H}} \mid \sigma_{\mathbf{H}}^2 \sim \mathcal{N}\left(\mathbb{B}_{\mathbf{H}}, \sigma_{\mathbf{H}}^2\right), \sigma_{\mathbf{H}}^2 \sim \mathcal{IG}\left(a_{\mathbf{H}}, b_{\mathbf{H}}\right)$:

$$\mathbb{P}\left(\boldsymbol{\beta}_c \mid \sigma_c^2\right) = \left(2\pi\right)^{-\frac{n}{2}} \exp\left[-\frac{1}{2}\left(\boldsymbol{\beta}_c - \mathbb{B}_c\right)'\boldsymbol{\Sigma}_c^{-1}\left(\boldsymbol{\beta}_c - \mathbb{B}_c\right)'\right],$$

$$\mathbb{P}\left(\boldsymbol{\beta}_{\mathbf{H}} \mid \sigma_{\mathbf{H}}^2\right) = \left(2\pi\right)^{-\frac{n}{2}} \exp\left[-\frac{1}{2}\left(\boldsymbol{\beta}_{\mathbf{H}} - \mathbb{B}_{\mathbf{H}}\right)'\boldsymbol{\Sigma}_{\mathbf{H}}^{-1}\left(\boldsymbol{\beta}_{\mathbf{H}} - \mathbb{B}_{\mathbf{H}}\right)'\right],$$

$$\mathbb{P}\left(\sigma_c^2\right) \propto \sigma_c^{-(a_c - n)} \exp\left[-\frac{b_c}{\sigma_c^2}\right],$$

$$\mathbb{P}\left(\sigma_{\mathbf{H}}^2\right) \propto \sigma_{\mathbf{H}}^{-(a_{\mathbf{H}} - n)} \exp\left[-\frac{b_{\mathbf{H}}}{\sigma_{\mathbf{H}}^2}\right].$$

Combining the data likelihoods with the prior specifications and applying Bayes' Law, the joint posteriors are,

$$\mathbb{P}\left(\boldsymbol{\beta}_c,\sigma_c^2 \mid \mathbf{1},y\right) \propto \left(\sigma_c^2\right)^{-\frac{n+a_c}{2}} \exp\left[-\frac{1}{2\sigma_c^2}\left(\tilde{s}_c+\left(\boldsymbol{\beta}_c-\tilde{\boldsymbol{\beta}}_c\right)'\left(\boldsymbol{\Sigma}_c^{-1}+\mathbf{1}'\mathbf{1}\right)\left(\boldsymbol{\beta}_c-\tilde{\boldsymbol{\beta}}_c\right)\right)\right]$$

$$\mathbb{P}\left(\boldsymbol{\beta}_{\mathbf{H}},\sigma_{\mathbf{H}}^2 \mid \mathbf{H},y\right) \propto \left(\sigma_{\mathbf{H}}^2\right)^{-\frac{n+a_{\mathbf{H}}}{2}} \exp\left[-\frac{1}{2\sigma_{\mathbf{H}}^2}\left(\tilde{s}_{\mathbf{H}}+(\boldsymbol{\beta}_{\mathbf{H}}-\tilde{\boldsymbol{\beta}}_{\mathbf{H}})'(\boldsymbol{\Sigma}_{\mathbf{H}}^{-1}+\mathbf{H}'\mathbf{H}(\boldsymbol{\beta}_{\mathbf{H}}-\tilde{\boldsymbol{\beta}}_{\mathbf{H}})\right)\right]$$

The marginalization of these joint posteriors gives,

$$\mathbb{P}\left(\boldsymbol{\beta}_c \mid \mathbf{1},y\right) \propto \left[\tilde{s}_c+\left(\boldsymbol{\beta}_c-\tilde{\boldsymbol{\beta}}_c\right)'\left(\boldsymbol{\Sigma}_c^{-1}+\mathbf{H}'\mathbf{H}\right)\left(\boldsymbol{\beta}_c-\tilde{\boldsymbol{\beta}}_c\right)\right]^{-\frac{n+a_c}{2}+1},$$

$$\mathbb{P}\left(\boldsymbol{\beta}_H \mid H,y\right) \propto \left[\tilde{s}_H+\left(\boldsymbol{\beta}_H-\tilde{\boldsymbol{\beta}}_H\right)'\left(\boldsymbol{\Sigma}_H^{-1}+\mathbf{H}'\mathbf{H}\right)\left(\boldsymbol{\beta}_H-\tilde{\boldsymbol{\beta}}_H\right)\right]^{-\frac{n+a_H}{2}+1},$$

which are kernels of a multivariate-t distribution with $n+a-k-2$ degrees of freedom. Therefore the mean of the parameter estimates (the Bayesian estimators for each equation) would be,

$$\begin{aligned}\mathbb{E}\left(\boldsymbol{\beta}_c \mid \mathbf{1},y\right) &= \tilde{\boldsymbol{\beta}}_c \\ &= \left(\boldsymbol{\Sigma}_c^{-1}+\mathbf{1}'\mathbf{1}\right)^{-1}\left(\boldsymbol{\Sigma}_c^{-1}\mathbb{B}_c+\mathbf{1}'\mathbf{1}\hat{b}_c\right), \\ \mathbb{E}\left(\boldsymbol{\beta}_{\mathbf{H}} \mid \mathbf{H},y\right) &= \tilde{\boldsymbol{\beta}}_{\mathbf{H}} \\ &= \left(\boldsymbol{\Sigma}_{\mathbf{H}}^{-1}+\mathbf{H}'\mathbf{H}\right)^{-1}\left(\boldsymbol{\Sigma}_{\mathbf{H}}^{-1}\mathbb{B}_{\mathbf{H}}+\mathbf{H}'\mathbf{H}\,\hat{b}_{\mathbf{H}}\right).\end{aligned}$$

APPENDIX 4

Sims-Zha Bayesian Method for Dynamic Multivariate Models

Sims and Zha (1998) considered linear multivariate models of the form $A(L)y(t) + C = \varepsilon(t)$, where $y(t)$ is the $m\times 1$ vector of observations, $A(L)$ is a $m\times m$ matrix polynomial, L is the lag operator and C is a constant vector that can be generalized to more complicated sets of exogenous regressors. Assuming $\varepsilon(t) \mid y(s), s<t \sim N(0,1)$, the likelihood function is,

$$\mathcal{L}\left(y(t),t=1,\ldots,T \mid A(L)\right) \propto |A(0)|^T \exp\left[-\frac{1}{2}\sum_t \left(A(L)y(t)+C\right)'\left(A(L)y(t)+C\right)\right]$$

Rewriting the model in matrix form, $\mathbf{YA}_0-\mathbf{XA}_+=\mathbf{E}$ and letting,

$$\mathbf{Z}=\begin{bmatrix}\mathbf{Y} & -\mathbf{X}\end{bmatrix}, \quad \text{and } \mathbf{A}=\begin{bmatrix}\mathbf{A}_0 \\ \mathbf{A}_+\end{bmatrix}.$$

The likelihood can now be expressed in compact form,

$$\mathcal{L}\left(\mathbf{Y} \mid \mathbf{A}\right) \propto |\mathbf{A}_0|^T \exp\left[-0.5\mathbf{a}'\left(\mathbf{I}\otimes\mathbf{Z}'\mathbf{Z}\right)\mathbf{a}\right]$$

A prior probability density function for a would be,

$$\pi(\mathbf{a})=\pi_0(\mathbf{a}_0)\varphi\left(\mathbf{a}_+-\mu(\mathbf{a}_0);H(\mathbf{a}_0)\right).$$

Where $\pi_0(\;)$ is a marginal distribution of $\mathbf{a}_0$ and $\varphi(\;)$ is the standard normal p.d.f. with covariance matrix $\boldsymbol{\Sigma}$. The posterior density function of $\mathbf{a}$ is:

$$q(\mathbf{a}) \propto \pi_0(\mathbf{a}_0) \,|\, \mathbf{A}(0)|^T \,|\, H(\mathbf{a}_0)|^{-1/2}$$
$$\times \exp[-0.5\left(\mathbf{a}_0'\left(\mathbf{I}\otimes\mathbf{Z}'\mathbf{Z}\right)\mathbf{a}_0 - 2\mathbf{a}_+'\left(\mathbf{I}\otimes\mathbf{X}'\mathbf{Y}\right)\mathbf{a}_0 + \mathbf{a}_+'\left(\mathbf{I}\otimes\mathbf{X}'\mathbf{X}\right)\mathbf{a}_+\right) + \left(\mathbf{a}_+ - \mu(\mathbf{a}_0)\right)' \mathrm{H}(\mathbf{a}_0)^{-1}\left(\mathbf{a}_+ - \mu(\mathbf{a}_0)\right)].$$

The a_+ coefficient can be read off directly as $\left(\mathbf{I}\otimes\mathbf{X}'\mathbf{X}\right) + H(\mathbf{a}_0)^{-1}$, and to preserve the Kronecker-product structure of the first term, $H(\mathbf{a}_0) = \mathbf{B}\otimes\mathbf{G}$, where $\mathbf{B}$ and $\mathbf{G}$ have the same order as $\mathbf{I}$ and $\mathbf{X}'\mathbf{X}$. In the Litterman (1986) prior the beliefs about the reduced form coefficient matrix $\mathbf{B} = \mathbf{A}_+\mathbf{A}_0^{-1}$ are centered on an identity matrix for the top m rows and zeros for the remaining rows. This notion can be made concrete by making the conditional distribution for $\mathbf{A}_+$ Gaussian with mean $\mathbf{A}_0$ in the first m rows and zero in the remaining rows, $\pi_t = \mathbb{E}_t x_{t+1}$.

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