

Risky Banks and Macro-Prudential Policy for Emerging Economies

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XV Meeting of Monetary Policy Managers CEMLA

República Dominicana - September 27, 2019

¹The views expressed herein are those of the authors and do not necessarily reflect those of Bank of Mexico, the Inter-American Development Bank, or their board members or managers.

Motivation

Degree of interconnectedness among financial institutions $\uparrow \Rightarrow$ exposure of EMEs to AE financial shocks \uparrow , **global banks** played a key role

Portfolio capital flows and **cross-border banking flows** (non-core liabilities) create challenges for **EMEs financial stability**

- volatile, short-term, and pro-cyclical
- important channel of international transmission of foreign shocks

What can **EMEs** do to mitigate the effects of volatile portfolio capital flows and cross-border banking flows, i.e. non-core liabilities? **Implement macro-prudential measures**

This paper

Do cross-border banking flows play a role on propagating AE financial shocks to EMEs?

What are the financial stability consequences in EMEs?

What can EMEs do to mitigate these effects?

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Do cross-border banking flows play a role on propagating AE financial shocks to EMEs?

What are the financial stability consequences in EMEs?

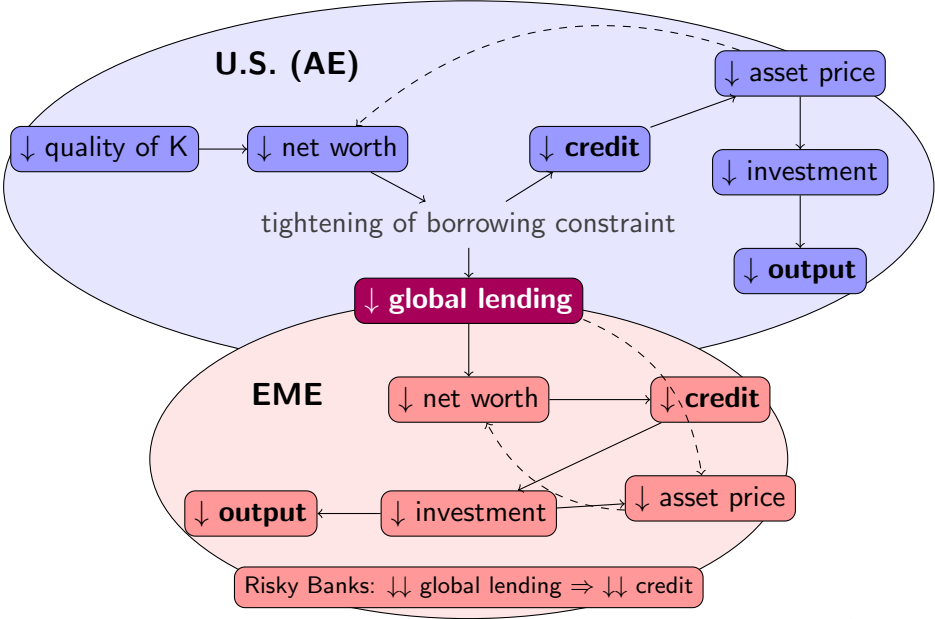
What can EMEs do to mitigate these effects?

- 1 Empirical Evidence + new VAR evidence on the transmission of financial shocks from the U.S. to Mexico and Turkey (**risky banks**)
- 2 Two-country DSGE model
 - ▶ financial intermediaries face an endogenous credit constraint *à la* Gertler and Kiyotaki, 2010
 - ▶ banks in the AE lend to banks in the EME, **cross-border banking flows** or **non-core liabilities**
 - ▶ banks in the EME might be constrained on how much they borrow from the AE, **risky EME banks**
- 3 Macro-prudential policy in the EME to mitigate the effects of the volatility of banks' non-core liabilities, *à la* Korea

Results

- ① VAR, a negative quality of capital shock in the U.S. prompts a negative impact in the EME
 - ▶ loans from U.S. banks to EME ↓
 - ▶ financial instability in the EME, credit ↓, GDP ↓
 - ▶ asset price co-movement across countries
 - ▶ when EME banks are risky for U.S. banks, macro variables fall more in the EME
- ② Model replicates the facts from the VAR and matches the impulse response functions
- ③ Macro-prudential policy in the EME by ↓ the volatility of cross-border banking flows
 - ▶ ↓ sources of financial instability
 - ▶ EME consumers are better off

Mechanism



Empirical Evidence

In the last few years, cross-border banking flows have been very volatile

- financial crisis \Rightarrow \downarrow of how much the U.S. lent to EMEs
- UMP, ZLB interest rate \Rightarrow \uparrow of capital flows to EMEs
- normalization of MP \Rightarrow a new reverse of the capital flows?

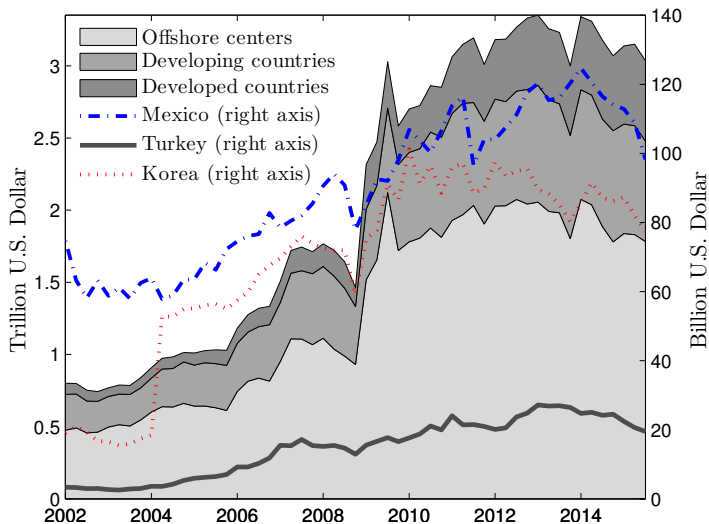
Non-core liabilities have been financing the increase in credit with respect to deposits in EMEs (Lane and McQuade, 2014)

How much are cross-border banking flows with respect to households' deposits for Turkish and for Mexican commercial banks?

- Turkey: 6.5%
- Mexico: 1.9%

\Rightarrow not big numbers but they can create lots of noise in the EME

Empirical Evidence Foreign Claims of U.S. Reporting Banks



Source: BIS Consolidated Bank Statistics, Immediate Borrower Basis

Empirical Evidence

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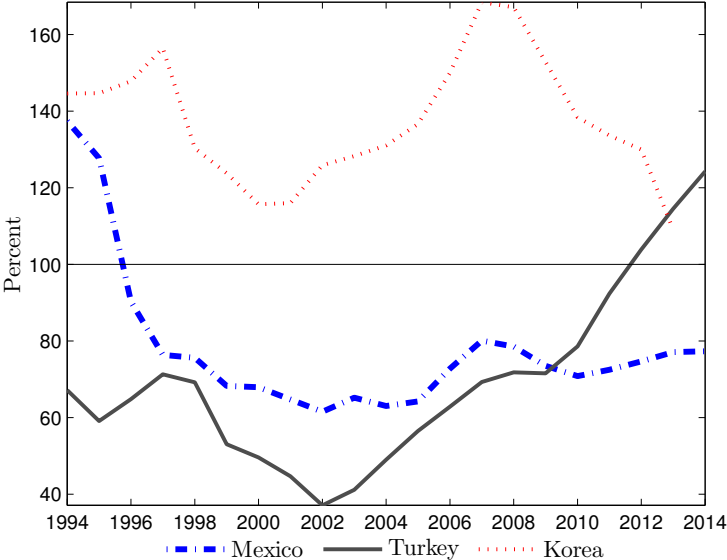
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Empirical Evidence Credit to Deposits Ratio



Source: Fred, Federal Reserve Bank of St. Louis

Empirical Evidence

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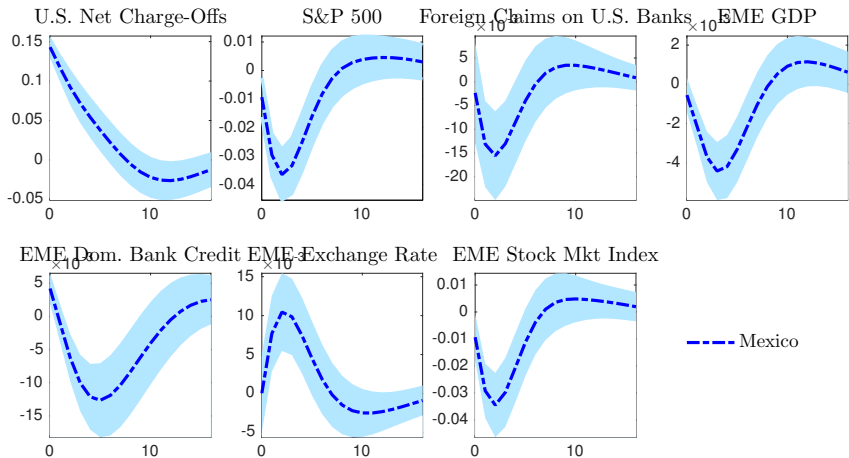
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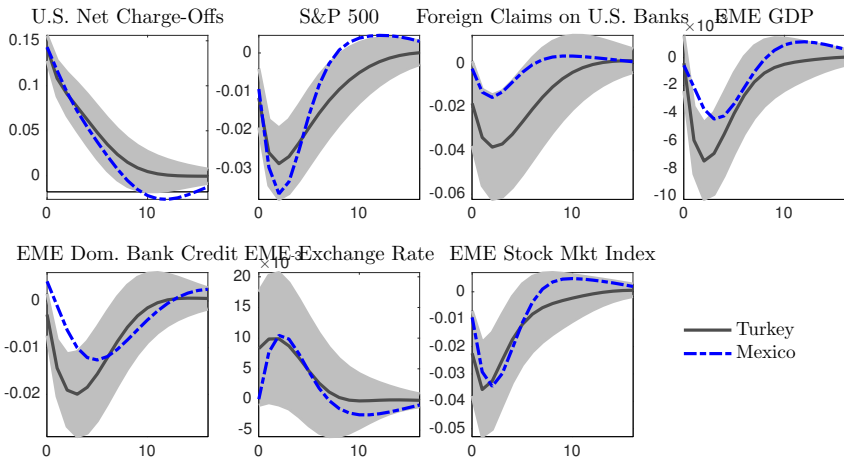
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Empirical Evidence VAR for Mexico



Note: Impulse Responses to Cholesky One-Std-Dev. Innovation to NCO on U.S. Commercial Banks. Mexican VAR estimated from 2002Q1 to 2015Q1. The shaded areas represent one standard confidence intervals and the lines represent the mean. The vertical axis shows the percent deviation from the trend, while the horizontal axis corresponds to quarters.

Empirical Evidence VAR for Mexico and Turkey



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Empirical Evidence VAR Results

The VAR evidence shows

- ① An \uparrow in the U.S. net charge-offs
 - ▶ \downarrow in the price of capital
 - ▶ \downarrow in bank lending to the EME
 - ▶ **financial instability** in the EME, \downarrow in credit and in GDP
- ② Asset price co-movement across countries
- ③ When EME banks are risky for U.S. banks, credit and GDP fall more in the EME

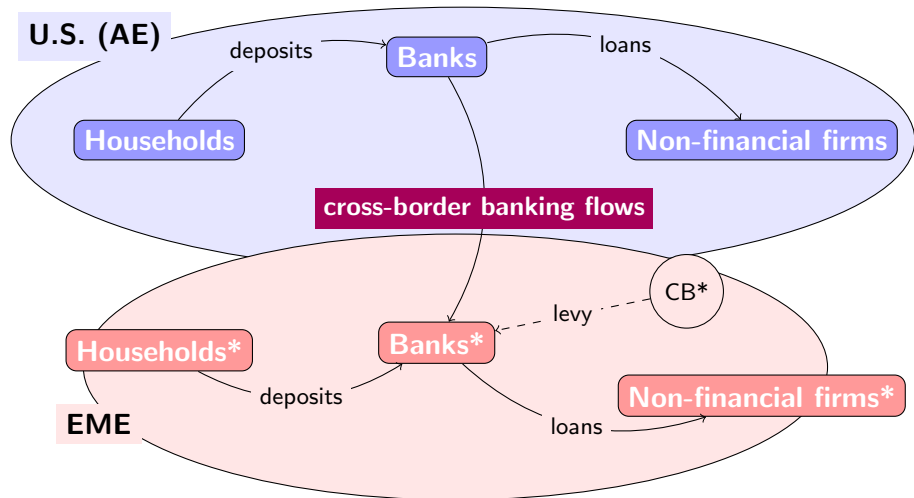
⇒ **We build a model that replicates these facts from the VAR**

- A model without global banks cannot replicate the VAR evidence
- Global banks explain the mechanism
 - ▶ safe vs. risky banks

The Model

- ① Two-country DSGE model
 - ▶ builds on Gertler and Kiyotaki (2010)
 - ▶ banking sector
 - ▶ endogenous credit constraint faced by financial intermediaries
 - ▶ U.S. (AE) banks invest (via EME banks) abroad - **non-core liabilities**
 - ★ U.S. is a relatively big economy with a big financial sector
 - ★ EME is a relatively small open economy with a small financial sector
 - ▶ EME banks might run away with debt from AE banks - **risky EME banks**
- ② Study the transmission of a shock to the quality of capital in the U.S.
 - ▶ models with global banks replicate the VAR-based evidence
- ③ Analysis of macro-prudential policy in the EME
 - ▶ welfare evaluation for different intensity of macro-prudential policy

The Model 2-country DSGE



AE Banks Financial Frictions

Gertler and Kiyotaki (2010) with international flows

- raise deposits from AE households, d_t
- lend
 - ▶ to AE non-financial firms, s_t
 - ▶ to EME banks, b_t

Assets	Liabilities
$Q_t s_t$	d_t
$Q_{bt} b_t$	n_t

Incentive compatibility constraint

$$V_t(s_t, b_t, d_t) \geq \theta (Q_t s_t + Q_{bt} b_t)$$

Aggregate net worth of AE banks

$$N_t = (\xi + \sigma) \{R_{k,t} Q_{t-1} S_{t-1} \Psi_t + R_{b,t} Q_{b,t-1} B_{t-1}\} - \sigma R_t D_{t-1}$$

At the end of the period $t - 1$ the value of the banks satisfies

$$V(s_{t-1}, b_{t-1}, d_{t-1}) = E_{t-1} \Lambda_{t-1,t} \left\{ (1 - \sigma) n_t + \sigma \left[\max_{s_t, b_t, d_t} V(s_t, b_t, d_t) \right] \right\}$$

▶ Problem of AE banks

EME Banks Financial Frictions

- raise funds from
 - ▶ EME households, d_t^*
 - ▶ AE banks, b_t^*
- make loans to EME non-financial firms, s_t^*

Assets	Liabilities
$Q_t^* s_t^*$	d_t^*
	$Q_{bt}^* b_t^*$
	n_t^*

Incentive compatibility constraint

- $\omega = 1$, **safe** EME banks

$$V_t(s_t^*, b_t^*, d_t^*) \geq \theta^*(Q_t^* s_t^* - Q_{bt}^* b_t^*)$$

- $0 < \omega < 1$, **risky** EME banks

$$V_t(s_t^*, b_t^*, d_t^*) \geq \theta^*(Q_t^* s_t^* - \omega Q_{bt}^* b_t^*)$$

Aggregate net worth of EME banks

$$N_t^* = (\sigma^* + \xi^*)[Z_t^* + (1 - \delta)Q_t^*]S_{t-1}^* - \sigma^*(R_t^* D_{t-1}^* + R_{bt}^* Q_{b,t-1}^* B_{t-1}^*)$$

Risky EME Banks

The parameter ω introduces a level of riskiness in the EME' cross-border banking flows. EME banks can run away with a fraction $\theta^*(1 - \omega)$ of international flows. \Rightarrow **risky** EME banks

For $\omega = 1$

$$E_t \Lambda_{t,t+1}^* \Omega_{t+1}^* R_{kt+1}^* = E_t \Lambda_{t,t+1}^* \Omega_{t+1}^* R_{bt+1}^* > E_t \Lambda_{t,t+1}^* \Omega_{t+1}^* R_{t+1}^*$$

For $0 < \omega < 1$

$$E_t \Lambda_{t,t+1}^* \Omega_{t+1}^* R_{kt+1}^* > E_t \Lambda_{t,t+1}^* \Omega_{t+1}^* R_{bt+1}^* > E_t \Lambda_{t,t+1}^* \Omega_{t+1}^* R_{t+1}^*$$

When EME banks can run away with a fraction of cross-border banking flows, EME banks are more exposed to events in the AE.

Experiments and Evaluation of the Model

- 1 Calibration and steady state comparison
- 2 Response of the model to a quality of capital shock in the AE
 - ▶ Model with safe global banks $\omega = 1$
 - ★ transmission across countries with asset price co-movement
 - ★ cross-border banking flows fall
 - ★ collapse of EME's credit, financial instability
 - ▶ Safe vs. risky EME banks $0 < \omega < 1$
 - ★ cross-border banking flows fall more
 - ★ deeper transmission of the financial instability
- 3 VAR-base evidence vs. model simulation
 - ▶ Relevance of modeling global banks
 - ▶ Difference between safe and risky banks
- 4 Macro-prudential policy carried out by the EME authority

Calibration

		AE	EME	
			$\omega = 1$	$\omega = 0.50$
β	discount factor	0.990	0.990	0.990
γ	inverse elasticity of labor supply	0.100	0.100	0.100
χ	relative utility weight of labor	2.000	2.000	2.000
α	effective capital share	0.330	0.330	0.330
δ	depreciation	0.018	0.023	0.023
κ	adjustment cost	3.000	3.000	3.000
\bar{g}	steady state gov expenditure	0.196	0.111	0.105
ν	home bias	0.775	0.975	0.975
η	elasticity of substitution	1.556	1.556	1.556
m	country size	0.900	0.100	0.100
ξ	start-up	0.002	0.002	0.002
θ	fraction of div assets	0.407	0.412	0.408
σ	survival rate	0.972	0.972	0.972
Φ	country-specific risk premium		0.010	
Ψ		-0.050		

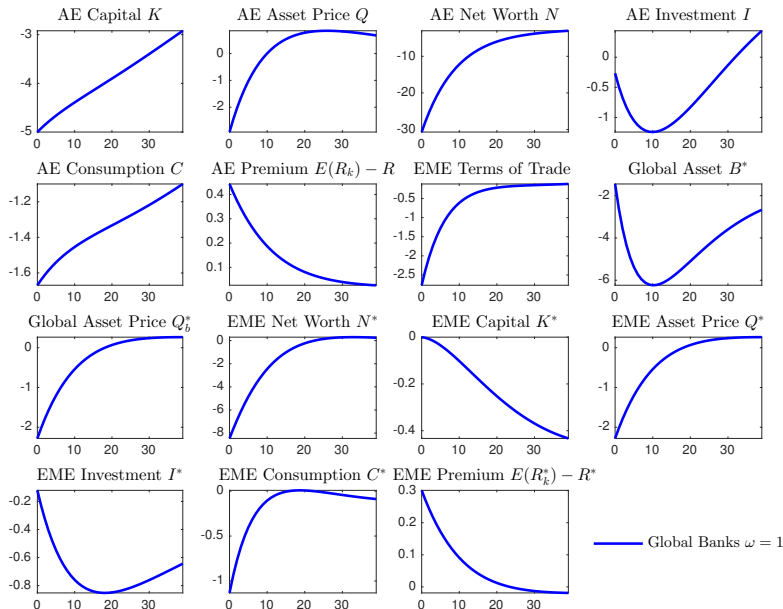
Calibration

Table: Deterministic Steady State, Model and Data

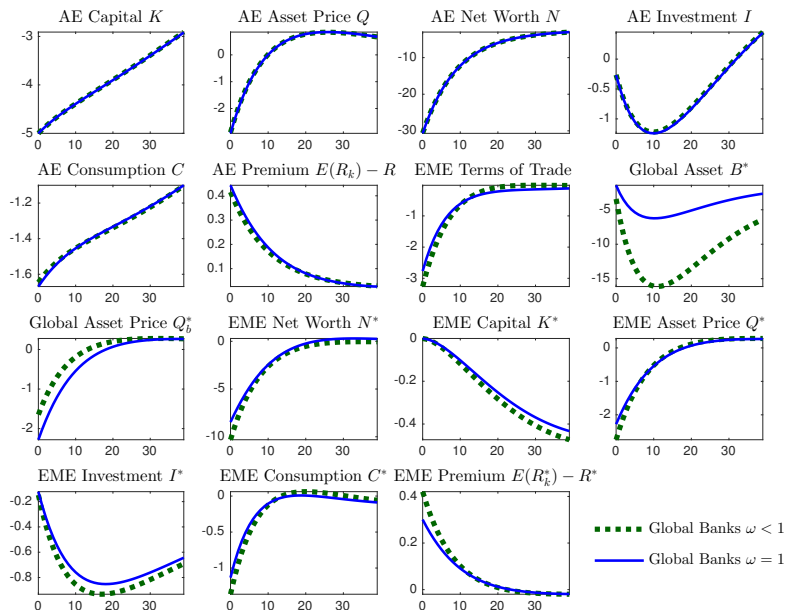
	Safe EME Banks $\omega = 1$			Risky EME Banks $\omega = 0.5$		
	Model	Data	CI 2sd	Model	Data	CI 2sd
<i>Advanced Economy: United States</i>						
Consumption/GDP	0.6115	0.6753	0.6820	0.6728	0.6753	0.6820
Investment/GDP	0.1924	0.1558	0.1774	0.1980	0.1558	0.1774
Government spending /GDP	0.1961	0.1909	0.2013	0.1961	0.1909	0.2013
<i>Emerging Market Economy:</i>						
	<i>Mexico</i>			<i>Turkey</i>		
Consumption/GDP	0.6771	0.6576	0.6682	0.6817	0.6782	0.6969
Investment/GDP	0.2120	0.2083	0.2193	0.2128	0.2158	0.2453
Government spending /GDP	0.1109	0.1094	0.1124	0.1055	0.1022	0.1087
Exports/GDP	0.2465	0.2749	0.3008	0.2479	0.2436	0.2570
Imports/GDP	0.2301	0.2722	0.3025	0.2339	0.2573	0.2852
Cross-border bnk fl/Deposits	0.0196	0.0105	0.0273	0.0670	0.0082	0.0793

Source: own calculations with data from FRED 2002Q1 - 2014Q4. For Mexico, the cross-border bank flows to deposits ratio is the ratio between deposits from financial institutions from abroad and deposits from households for the period 2004Q2-2015Q2, CF445, Bank of Mexico. For Turkey, it is the ratio between total deposits from financial foreign institutions and total deposits from households in TRY for the same time period, Central Bank of Turkey.

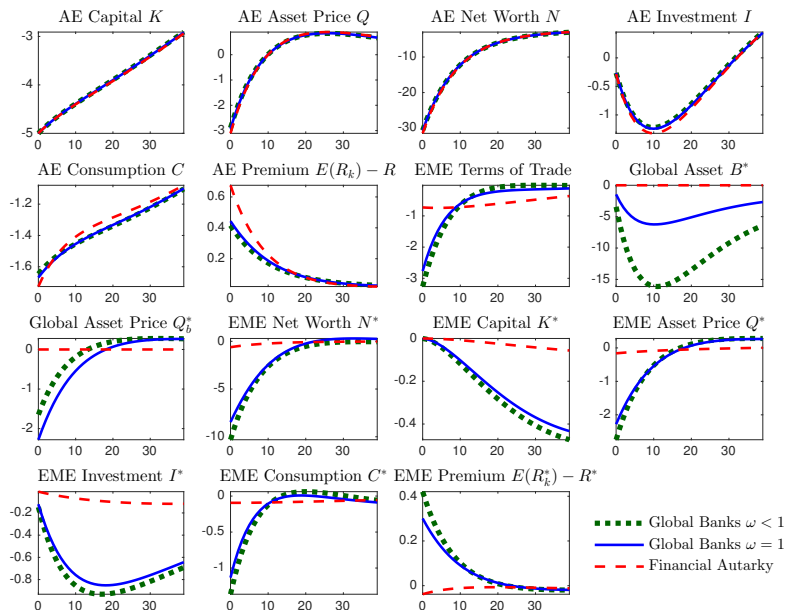
IRF to a Neg. Quality of K Shock - Global Banks



IRF to a Neg. Quality of K Shock - Risky Banks



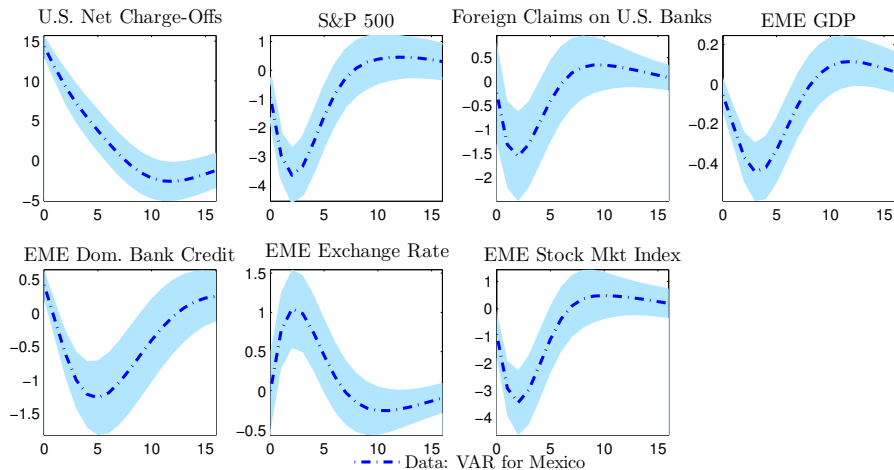
IRF to a Neg. Shock - No Global Banks



IRF to a Neg. Quality of K Shock in the AE

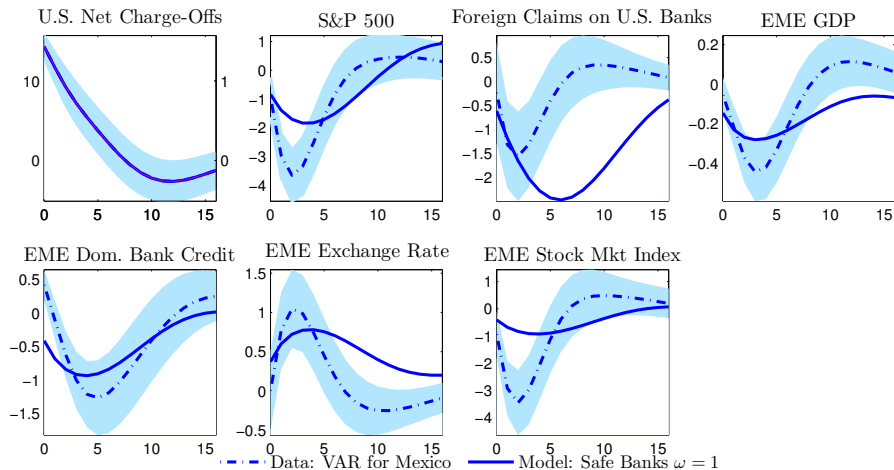
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 - ▶ collapse of EME's credit, financial instability
 - ▶ global financial crisis
- 2 Safe vs. risky EME banks $0 < \omega < 1$
 - ▶ cross-border banking flows fall more
 - ▶ deeper transmission of the financial crisis

VAR Evidence vs. the Model Mexico



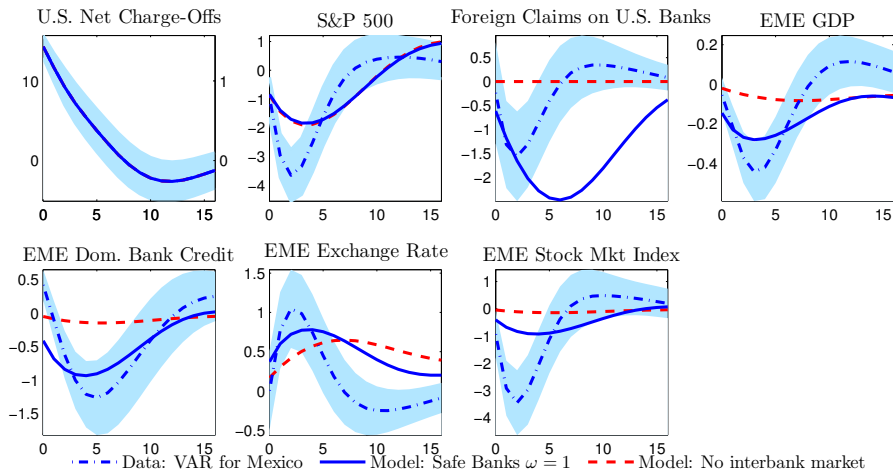
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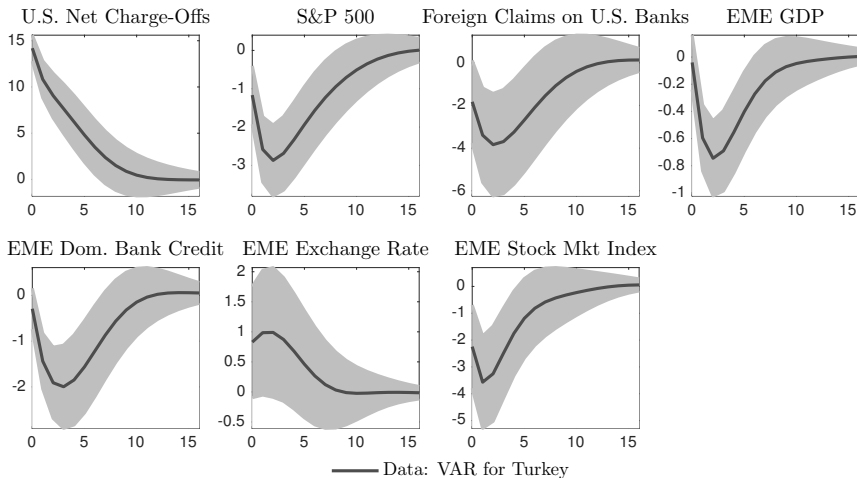
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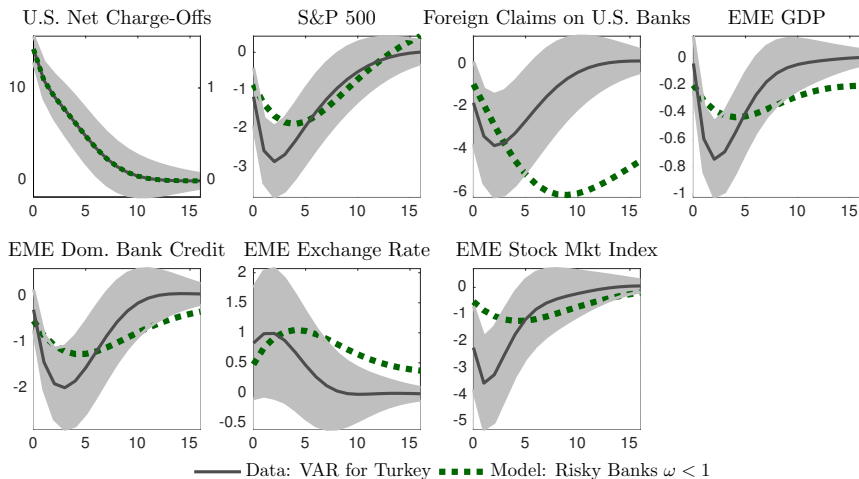
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VAR Evidence vs. the Model Turkey



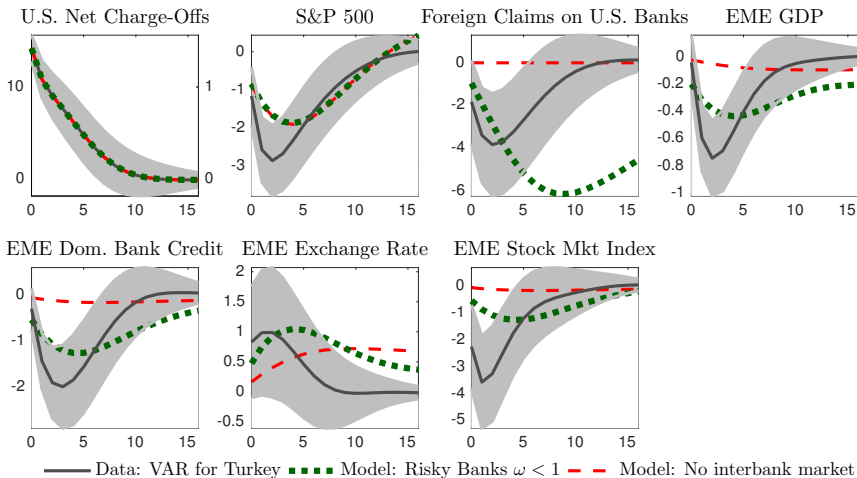
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VAR Evidence vs. the Model Turkey



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Macro-Prudential Policy in the EME

The Korean Experience

- August 2011, the Bank of Korea put a **levy on non-core liabilities**
- Purpose: non-core liabilities can generate systemic risk (procyclical and global interconnection of financial institutions)
- Result: share of short-term in total foreign borrowing by banks dropped from 64% as of end-June 2010 to 47% at end-December 2012

In the Model

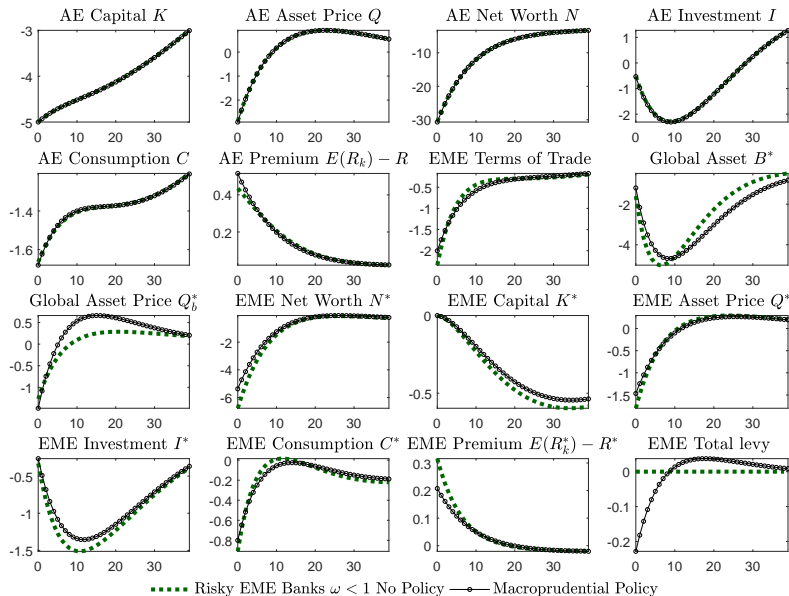
- There is a cost (tax) when assets grow faster than deposits

$$\varrho_{gt}^* = \left(\frac{\frac{S_{t+1}^* - S_t^*}{S_t^*}}{\frac{D_t^* - D_{t-1}^*}{D_{t-1}^*}} \right)^{\tau_g^*}$$

- Total net worth of EME banks

$$N_t^* = (\sigma^* + \xi^*) R_{kt}^* Q_{t-1}^* S_{t-1}^* - \sigma^* [R_t^* D_{t-1}^* + \varrho_{gt}^* R_{bt}^* Q_{b,t-1}^* B_{t-1}^*]$$

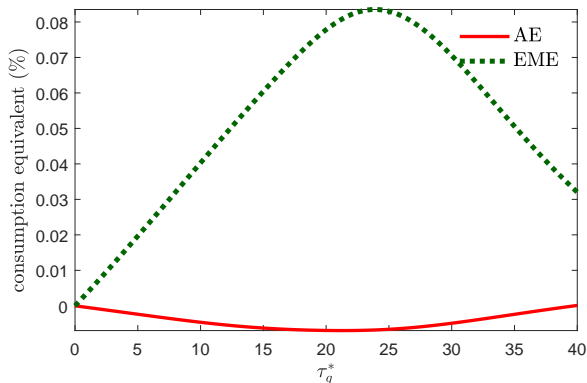
IRF to a Neg. Quality of K Shock - Macro-Prudential Pol.



Welfare analysis Consumption Equivalent

- Moments of the second order approximation of the model
- **Consumption Equivalent:** fraction of households' consumption that would be needed to equate the welfare under no policy to the welfare under policy;
 $\tau_g^* = 23.9$ and $\varrho_{gt}^* = 0.0284\%$

$$Welf_t = U(C_t, L_t) + \beta E_t Welf_{t+1} \quad \varrho_{gt}^* = \left(\frac{\text{asset growth}}{\text{deposits growth}} \right)^{\tau_g^*}$$



Conclusions

- 1 **Do cross-border banking flows (non-core liabilities) play a role on propagating AE financial shocks to EMEs? Yes!**
- 2 **What are the financial stability consequences in EMEs of these AE shocks?**
 - ▶ prompt instability for EMEs, credit and GDP fall
 - ▶ specially when EME banks are risky for the AE
 - ▶ models with global banks match qualitative evidence from the VAR
- 3 **What can EMEs do to mitigate these effects?**
 - ▶ Macro-prudential policy: levy on non-core liabilities, i.e. foreign debt, cross-border banking flows
 - ▶ EME shows a smoother reaction with the intervention
 - ▶ EME households are better off with the policy

Related Literature

Empirical Evidence

- Cross-border banking flows channel Cetorelli and Goldberg (2011) and Morais, Peydró, and Ruiz (2016)
- Large capital inflows increase the probability of credit booms Mendoza and Terrones (2008), Avdjiev, McCauley, and McGuire (2012), and Magud, Reinhart, and Vesperoni (2014)
- Credit growth linked to banks' net debt flows Lane and McQuade (2014)

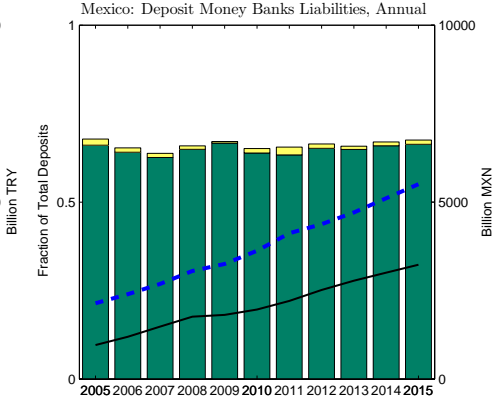
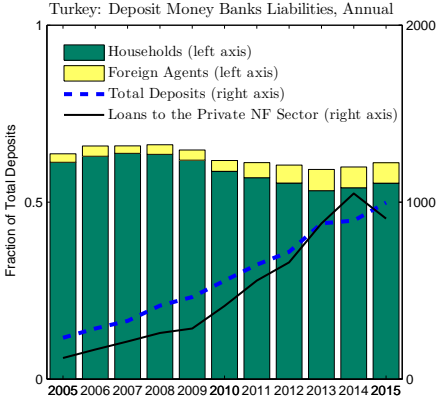
Theoretical Analysis

- Relevance of non-core liabilities Shin (2010), Shin and Shin (2010)
- 2-country model with global banks Dedola, Karadi, and Lombardo (2013) and Nuguer (2016)

This Paper's Contribution

- VAR evidence: EMEs response to a U.S. net charge-off shock, difference between safe and risky banks
- Theoretical model for EMEs with global banks and the cross-border banking flow channel and macro-prudential policy

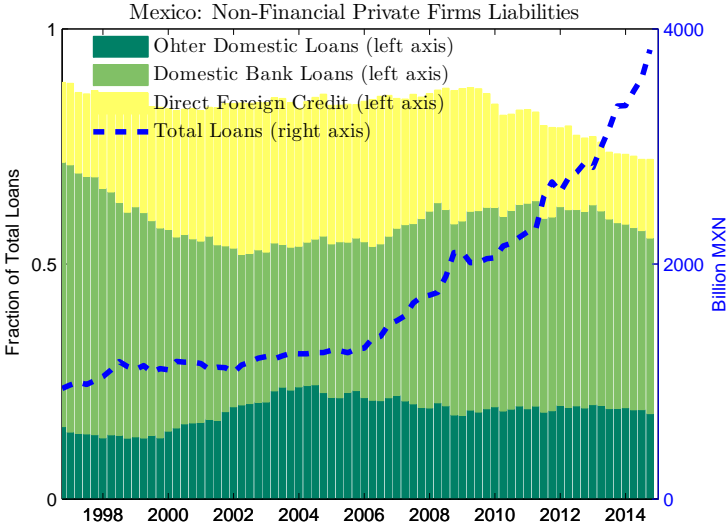
Empirical Evidence: Funding of Commercial Banks



Source: Turkish Central Bank and Bank of Mexico.

[Go back](#)

Empirical Evidence: Funding of Non-Financial Firms



Source: Bank of Mexico.

Households

Each household consists of a continuum of members

① Worker

- ▶ supplies labor

② Banker

- ▶ with prob. σ continues being a banker
- ▶ with prob. $1 - \sigma$ exits the banking business

Perfect consumption insurance within the household.

Problem

$$\max_{C_t, L_t, D_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \frac{\chi}{1+\gamma} L_t^{1+\gamma} \right]$$
$$\text{s.t. } C_t + D_t = W_t L_t + \Pi_t + R_t D_{t-1} + T_t$$

Non-Financial Firms

1 Good producers

$$X_t = A_t K_t^\alpha L_t^{1-\alpha} = X_t^H + X_t^{*H} \frac{1-m}{m}$$

In order to finance new investment, they sell state-contingent claims, S_t , to banks.

$$\begin{aligned} S_t &= I_t + (1 - \delta)K_t \\ K_{t+1} &= S_t \Psi_{t+1} \end{aligned}$$

► Firms

2 Capital good producers

They choose investment to maximize profit

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right) - E_t \Lambda_{t,t+1} \left[\frac{I_{t+1}}{I_t}\right]^2 f'\left(\frac{I_{t+1}}{I_t}\right)$$

► Adj Costs

Equilibrium

Resource constraint

$$Y_t = \left[\nu^{\frac{1}{\eta}} X_t^H \frac{\eta-1}{\eta} + (1-\nu)^{\frac{1}{\eta}} X_t^F \frac{\eta-1}{\eta} \right]^{\frac{\eta}{\eta-1}}$$

$$Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right) \right] I_t + G_t$$

Current Account

$$CA_t = Q_{b,t} B_t - R_{bt} Q_{b,t-1} B_{t-1} = X_t^{*H} \frac{1-m}{m} \frac{P_t^H}{P_t} - X_t^F \tau_t \frac{P_t^H}{P_t}$$

Benchmark: The RBC Model in Financial Autarky

Advanced Economy (AE)

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \frac{\chi}{1+\gamma} L_t^{1+\gamma} \right]$$

$$X_t = A_t K_t^\alpha L_t^{1-\alpha} = X_t^H + X_t^{*H} \frac{1-m}{m}$$

$$Y_t = \left[\nu^{\frac{1}{\eta}} X_t^H \frac{\eta-1}{\eta} + (1-\nu)^{\frac{1}{\eta}} X_t^F \frac{\eta-1}{\eta} \right]^{\frac{\eta}{\eta-1}}$$

$$Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right) \right] I_t + G_t$$

$$S_t = I_t + (1-\delta)K_t$$

$$K_{t+1} = S_t \underbrace{\Psi_{t+1}}_{\text{quality of capital shock}}$$

$$\text{Financial autarky case: } CA_t = \frac{1-m}{m} X_t^{H*} - X_t^F \tau_t = 0$$

EME is similar with variables with *.

Ψ_t and Ψ_t^* are i.i.d. and mutually independent. We study a shock in Ψ_t .

Non-financial firms

No-cost technology for the final good production, problem:

$$\begin{aligned} \max_{X_t^H, X_t^F} Y_t &= \left[\nu^{\frac{1}{\eta}} X_t^H^{\frac{\eta-1}{\eta}} + (1-\nu)^{\frac{1}{\eta}} X_t^F^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \\ \text{s.t. } P_t Y_t &\equiv Z_t = P_t^H X_t^H + P_t^F X_t^F \end{aligned}$$

The optimization problem yields

$$P_t = \left[\nu (P_t^H)^{1-\eta} + (1-\nu) (P_t^F)^{1-\eta} \right]^{\frac{1}{1-\eta}}.$$

We can define everything in terms of TOT ($\tau = \frac{P_t^F}{P_t^H}$),

$$\frac{P_t}{P_t^H} = \left[\nu + (1-\nu) \tau_t^{1-\eta} \right]^{\frac{1}{1-\eta}}.$$

The demands are defined by

$$X_t^H = \nu Y_t \left[\frac{P_t^H}{P_t} \right]^{-\eta} \quad \text{and} \quad X_t^F = (1-\nu) Y_t \left[\frac{P_t^F}{P_t} \right]^{-\eta}$$

Law of one price + home bias, the real exchange rate is

$$\varepsilon_t = \frac{S_t P_t^*}{P_t} = \left[\frac{\nu^* + (1-\nu^*) \tau_t^{1-\eta}}{\nu + (1-\nu) \tau_t^{1-\eta}} \right]^{\frac{1}{1-\eta}}$$

Non-financial firms - Adjustment Costs

CEE (2005)

$$F(i_t, i_{t-1}) = \left[1 - S \left(\frac{i_t}{i_{t-1}} \right) \right] i_t,$$

with $S(1) = S'(1) = 0$, $\varphi \equiv S''(1) > 0$.

GK (2010) problem

$$\max_{l_t} E_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ Q_{\tau} l_{\tau} - \left[1 + f \left(\frac{l_{\tau}}{l_{\tau-1}} \right) \right] l_{\tau} \right\}$$

$$\text{with } f \left(\frac{l_{\tau}}{l_{\tau-1}} \right) = \left[\varrho \frac{l_{\tau}}{l_{\tau-1}} - \varrho \right]^2$$

$$f(1) = 0, \quad f' \left(\frac{l_t}{l_{t-1}} \right) = 2\varrho \left[\varrho \frac{l_t}{l_{t-1}} - \varrho \right], \quad f''(1) = 0, \quad f'' \left(\frac{l_t}{l_{t-1}} \right) = 2\varrho^2 \equiv \varphi > 0.$$

The optimization problem yields

$$\begin{aligned} Q_t &= 1 + f \left(\frac{l_t}{l_{t-1}} \right) + \frac{l_t}{l_{t-1}} f' \left(\frac{l_t}{l_{t-1}} \right) - E_t \Lambda_{t,t+1} \left(\frac{l_{t+1}}{l_t} \right)^2 f' \left(\frac{l_{t+1}}{l_t} \right) \\ &= 1 + \left[\varrho \frac{l_t}{l_{t-1}} - \varrho \right]^2 + \frac{l_t}{l_{t-1}} 2\varrho \left[\varrho \frac{l_t}{l_{t-1}} - \varrho \right] - E_t \Lambda_{t,t+1} \left(\frac{l_{t+1}}{l_t} \right)^2 \left[\varrho \frac{l_{t+1}}{l_t} - \varrho \right] \end{aligned}$$

AE Banks Optimization

Bellman equation

$$\begin{aligned} V(s_t, b_t, d_t) &= \nu_{st}s_t + \nu_{bt}b_t - \nu_t d_t \\ &= E_t \Lambda_{t,t+1} \left\{ (1 - \sigma) n_{t+1} + \sigma \left[\max_{d_{t+1}, s_{t+1}, b_{t+1}} V(s_{t+1}, b_{t+1}, d_{t+1}) \right] \right\} \end{aligned}$$

The optimization implies

$$\begin{aligned} \nu_t &= E_t [\Lambda_{t,t+1} \Omega_{t+1} R_{t+1}] \\ \mu_t &= E_t [\Lambda_{t,t+1} \Omega_{t+1} (R_{kt+1} - R_{t+1})] \\ \phi_t &= \frac{\nu_t}{\theta - \mu_t} \\ \mu_t &= \frac{\nu_{st}}{Q_t} - \nu_t \\ \frac{\nu_{st}}{Q_t} &= \frac{\nu_{bt}}{Q_{bt}} \Rightarrow E_t \Lambda_{t,t+1} \Omega_{t+1} R_{kt+1} = E_t \Lambda_{t,t+1} \Omega_{t+1} R_{bt+1} \end{aligned}$$

where

$$\begin{aligned} \Omega_{t+1} &= 1 - \sigma + \sigma(\nu_{t+1} + \mu_{t+1}\phi_{t+1}) \\ R_{kt+1} &= \Psi_{t+1} \frac{Z_{t+1} + (1 - \delta)Q_{t+1}}{Q_t} \end{aligned}$$

EME Banks Optimization

Bellman equation

$$\begin{aligned}V(s_t^*, b_t^*, d_t^*) &= \nu_{st}^* s_t^* - \nu_{bt}^* b_t^* - \nu_t^* d_t^* \\ &= E_t \Lambda_{t,t+1}^* \left\{ (1 - \sigma^*) n_{t+1}^* + \sigma^* \left[\max_{d_{t+1}^*, s_{t+1}^*, b_{t+1}^*} V(s_{t+1}^*, b_{t+1}^*, d_{t+1}^*) \right] \right\}\end{aligned}$$

The optimization implies

$$\begin{aligned}\nu_t^* &= E_t [\Lambda_{t,t+1}^* \Omega_{t+1}^* R_{t+1}^*] \\ \mu_t^* &= E_t [\Lambda_{t,t+1}^* \Omega_{t+1}^* (R_{kt+1}^* - R_{t+1}^*)] = \frac{\nu_{st}^*}{Q_t^*} - \nu_t^* \\ \phi_t^* &= \frac{\nu_t^*}{\theta^* - \mu_t^*} \\ \mu_{bt}^* &= E_t [\Lambda_{t,t+1}^* \Omega_{t+1}^* (R_{bt+1}^* - R_{t+1}^*)] = \frac{\nu_{bt}^*}{Q_t^*} - \nu_t^* \\ \phi_{bt}^* &= \frac{\nu_{bt}^*}{\theta^* \omega - \mu_{bt}^*} \\ \omega = 1 \quad \frac{\nu_{st}^*}{Q_t^*} &= \frac{\nu_{bt}^*}{Q_{bt}^*} \Rightarrow E_t \Lambda_{t,t+1}^* \Omega_{t+1}^* R_{kt+1}^* = E_t \Lambda_{t,t+1}^* \Omega_{t+1}^* R_{bt+1}^* \\ \omega < 1 \quad \frac{\nu_{st}^*}{Q_t^*} &= \left[\frac{\nu_{bt}^*}{Q_{bt}^*} - (1 - \omega) \nu_t^* \right] \frac{1}{\omega} \Rightarrow \mu_{bt}^* = \omega \mu_t^*\end{aligned}$$

where

$$\begin{aligned}\Omega_{t+1}^* &= 1 - \sigma^* + \sigma^* (\nu_{t+1}^* + \mu_{t+1}^* \phi_{t+1}^*) \\ R_{kt+1}^* &= \Psi_{t+1}^* \frac{Z_{t+1}^* + (1 - \delta^*) Q_{t+1}^*}{Q_t^*}\end{aligned}$$