



# Network analysis and financial stability: Quantification of systemic risk from overlapping portfolios in the financial system

IX Meeting on Financial Stability

5 - 6 September 2019 – Montevideo, Uruguay

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# Introduction

# Systemic risk and indirect exposures

How the systemic risk (SR) arise from overlapping portfolios?

- Systemic risk is defined as the risk that a significant fraction of the financial system can no longer perform its function as credit provider and collapses.
- SR arises from the probability of default propagating through many different mechanisms and channels of contagion to the financial system and potentially to the real economy.
- SR arises from asset price shocks and funding liquidity shocks. Losses from asset price shocks can result in contagious failures.
- Financial contagion could arise from indirect links between financial institutions mediated by financial markets. When financial institutions invest in the same assets, their portfolios are said to overlap (overlapping portfolios).
- Contagion can occur because of shocks that cause common assets to be devaluated. Devaluations can cause further sales and devaluations leading to fire sales.

# The financial system as a (multilayer) network

- There has been a lot of recent research on financial networks for the purposes of studying systemic risk, performing stress testing or determining the relevance of financial institutions.
- A commonly shared view is that the financial system is highly interconnected.
- Financial institutions interact in different markets, which can be thought of as different networks within a meta-structure which can be interpreted as a multilayered network or a multiplex network. This gives rise to consider multiple channels of contagion.\*
- This is the first quantification of systemic risk on a national scale that includes overlapping portfolios.

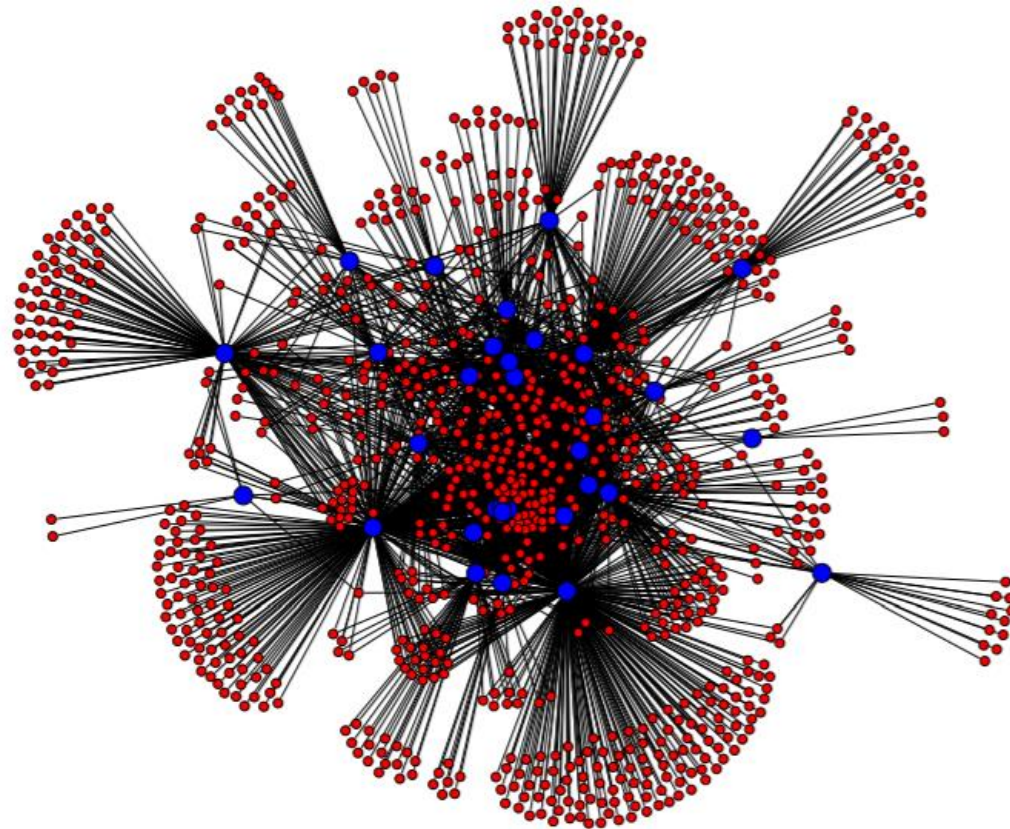
# Quantification of systemic risk from overlapping portfolios

# Methodology

- DebtRank is a recursive method\* that quantifies the systemic importance of financial institutions in terms of losses that they would contribute to the total loss in a crisis.
- We use a novel method to quantify the expected loss due to SR from overlapping portfolios (indirect exposures), where the loss for bank  $i$  is because the default of bank  $j$  causes the liquidation of  $j$ 's portfolios causing the devaluation of  $i$ 's common assets with  $j$ 
  - Bipartite networks of financial institutions and securities.
  - Compare SR from direct interbank exposures (default contagion) and indirect external exposures (overlapping portfolios).
  - Compare marginal contributions of individual direct and indirect exposures to the overall SR.

# Banks-assets bipartite network

- Nodes in the network represent banks (blue) and assets (red). Links the holding of an the asset by a bank.
- There are some banks that have independent portfolios or are even isolated. Also, there is an important degree of overlapping, the red nodes at the center of the plot; many banks are exposed to the same securities.





# Assumptions

- Linear market impact associated with the bank liquidating its position. Financial institutions liquidate their portfolios proportional to the relative loss of equity.
- Banks do not change the composition of their portfolios as they liquidate.
- Each bank knows the value of the capital of its counterparties at each step of the dynamic (mark-to-market valuation).
- The multilayer network consists of two layers: direct exposures and indirect exposures.
  - Direct exposures: deposits & loans, derivatives, cross holdings of securities, foreign exchange.
  - Indirect exposures result from overlapping portfolios.

# DebtRank I

- DebtRank is a recursive method suggested in Battiston et al. (2012) to determine the systemic importance of nodes in financial networks.
- **It is a number measuring the fraction of the total economic value in the network that is potentially affected by the distress of a node or a set of nodes.**
- The generalized version is made in Bardoscia et al. (2015)
- Adapted to the context of systemic risk
- Quantifies systemic relevance of node in financial network with economically meaningful number
- Takes capitalization/leverage of banks into account

# DebtRank II

- The nodes in the exposures network are banks.  $A_{ij}$  denotes links in the network (bank's  $i$  exposure to bank  $j$ ), and  $C_i$  is bank's  $i$  capital.
- We denote the total outstanding interbank exposures of bank  $i$  by  $A_i = \sum_j A_{ij}$ . Non interbank assets are denoted by  $A_i^E$  and liabilities by  $L_i^E$ . A bank is defaulted if  $C_i \leq 0$ .
- The set of active banks at time  $t$  is denoted by  $\mathcal{A}(t) = \{i: C_i(t) > 0\}$
- Interbank assets are mark-to-market while liabilities keep their face value
- When a bank defaults, the recovery rate on interbank loans is 0

# DebtRank III

- The shock propagation mechanism from borrowers to lenders is as follows
- Relative changes in the capital of the borrowers are reflected by relative changes on the interbank assets of the lenders:

$$A_{ij}(t + 1) = \begin{cases} A_{ij}(t) \frac{C_j(t)}{C_j(t - 1)} & \text{if } j \in \mathcal{A}(t - 1) \\ A_{ij}(t) = 0 & \text{if } j \notin \mathcal{A}(t - 1) \end{cases}$$

- The case  $j \notin \mathcal{A}(t - 1)$  ensures that, once bank  $j$  defaults, the corresponding interbank assets  $A_{ij}$  of its creditors will remain zero for the rest of the evolution
- We denote by  $h_i(t) = (C_i(0) - C_i(t))/C_i(0)$  the relative loss of capital between iterations 0 and  $t$ . By iterating in the balance sheet identity, the contagion dynamics can be written as:

$$h_i(t + 1) = \min \left[ 1, h_i(t) + \sum_{j=1}^N \Lambda_{ij}(t) [h_j(t) - h_j(t - 1)] \right]$$
$$\Lambda_{ij}(t) = \begin{cases} \frac{A_{ij}(0)}{C_j(0)} & \text{if } j \in \mathcal{A}(t - 1) \\ 0 & \text{if } j \notin \mathcal{A}(t - 1) \end{cases}$$

# Methodology

- The marginal SR of an individual exposure on Expected Systemic Loss is expressed as the difference of total expected systemic loss:

$$\Delta EL^{\text{sys}} \Big|_{\Delta X_{kl}} = \sum_{i=1}^b p_i [V(X_{ij} + \Delta X_{kl}) R_i(X_{ij} + \Delta X_{kl}, C_i) - V(X_{ij}) R_i(X_{ij}, C_i)]$$

$R_i(X_{ij} + \Delta X_{kl}, C_i)$  is the DebtRank

$V(X_{ij} + \Delta X_{kl})$  is the total economic value of the exposure network

$\Delta X_{kl}$  is the matrix with precisely one nonzero element for the exposure between k and l

A positive  $\Delta EL^{\text{sys}}$  means that the change in exposure  $\Delta X_{kl}$  increases total SR.

# Price Impact Function: Assumptions

- To compute this potential loss, we need to compute the impact of bank  $j$  on the value of each asset  $a$ , and then the importance of asset  $a$  for bank  $i$ :

Let us consider a network of  $b$  banks and  $m$  assets, and let us denote its equity by  $C_i$ , the number of shares of asset  $a$  owned by bank  $i$  by  $S_{ia}$ , the total number of outstanding shares of asset  $a$  by  $N_a$ , and the price of asset  $a$  by  $p_a$  respectively.

We assume the impact of bank  $j$  on asset  $a$  is proportional to the fraction of shares owned by the bank.

As a measure of the **direct impact of banks on assets** we define the matrix:

$$W'_{ja} = \frac{p_a S_{ja}}{N_a}$$

# Price Impact Function: Assumptions

- The underlying assumption here is that of a linear market impact associated with the bank liquidating its position on the asset: Should the bank liquidate its entire position; the price would shift from  $p_a$  to  $p_a(1 - \left(\frac{S_{ja}}{N_a}\right))$ .
- The importance of asset  $a$  for bank  $i$  is simply given by the number of shares  $i$  owns of asset  $a$ . Therefore, we define the indirect exposure of bank  $i$  to bank  $j$  from overlapping portfolios as (Guo et al., 2016; Schaanning, 2017).

$$X_{ij}^{\text{OP}} = \sum_a W'_{ja} S_{ia} = \sum_a \frac{p_a S_{ia} S_{ja}}{N_a}$$

# Price Impact Function: Assumptions

- $X_{ij}^{OP}$  is the appropriately weighted bank projection of the weighted bipartite network of banks and assets  $S_{ia}$ , so that the dynamic above is equivalent to the standard DebtRank on the projected network of overlapping portfolios.
- The matrix  $X_{ij}^{OP}$  is symmetrical, and its diagonal elements are non-zero even though the bipartite network itself has, by definition, no self-loops.
- Diagonal elements represent the self-inflicted loss of a bank from (rapidly) liquidating its portfolio (market impact). This loss will be high if bank  $i$  holds a large fraction of asset  $a$  in its portfolio, and is negligible if  $i$  holds only a small fraction of asset  $a$ .
- We assume that a bank liquidates a fraction of its portfolio proportional to its relative loss of equity. Our choice of proportional liquidation is a simplifying assumption that provides the smallest departure from the DebtRank algorithm, and allows us to use the DebtRank algorithm on the projected network of overlapping portfolios.



# Price Impact Function: Assumptions

- ❑ We assume an implicit 0% recovery rate. This implies that our measure of SR is more conservative with respect to one that would be obtained by considering a non-zero recovery rate.
- ❑ A second assumption is that banks do not change the composition of their portfolios as they liquidate. This is a common assumption in the literature on fire-sales (Huang et al., 2013; Greenwood et al., 2015; Cont and Schaanning, 2017), and it has recently been shown (Schaanning, 2017) to be a good approximation of the behavior of large banks.
- ❑ A further assumption we make is that each bank knows the value of the equity of its counterparties at each step of the dynamics. This is required because DebtRank assumes banks to compute the value of their interbank assets using an ex-ante mark-to-market valuation, according to which the value of an interbank asset depends on the value of the capital of the borrower (Battiston et al., 2012; Bardoscia et al., 2015; Barucca et al., 2016; Roncoroni et al., 2019).

# Price Impact Function: Assumptions

To consider contagion from asset liquidation we calculate the DebtRank of the indirect exposure network  $X_{ij}^{OP}$ ,

$$R_i^{OP} := R_i^{OP}(X_{ij}^{OP}, C_i, v_i^{OP})$$

where  $C_i$  is  $i$ 's capital and  $v_i^{OP}$   $i$ 's economic value. Given the current value of assets  $a$  in  $i$ 's investment portfolio, we define its economic value as:

$$v_i^{OP} = \frac{\sum_a p_a S_{ia}}{\sum_j \sum_a p_a S_{ja}}$$

i.e. the fraction of  $i$ 's investment portfolio from the total investment portfolios of all banks.

$R_i^{OP}$  measures the fraction of the total economic value ( $V^{OP} = \sum_i \sum_a p_a S_i^a$ ) that is affected by the distress of a bank  $i$  from indirect exposures, i.e. from overlapping portfolios.

Data

# The financial system as a (multilayer) network

- Data were collected and are owned by Banco de México, contains detailed information about various types of daily exposures between the major Mexican financial intermediaries (banks) over the period 2004-2013:
- **Securities holdings** of Mexican financial intermediaries by containing the International Securities Identification Number (ISIN) that uniquely identifies every security.
- **Capitalization** of banks at every month and the market data (prices) for the various securities.
- **Complete information about securities holdings** of major financial intermediaries and the ability to uniquely identify securities in the portfolios allows us to represent the Mexican financial system as a bipartite network of securities and financial institutions.

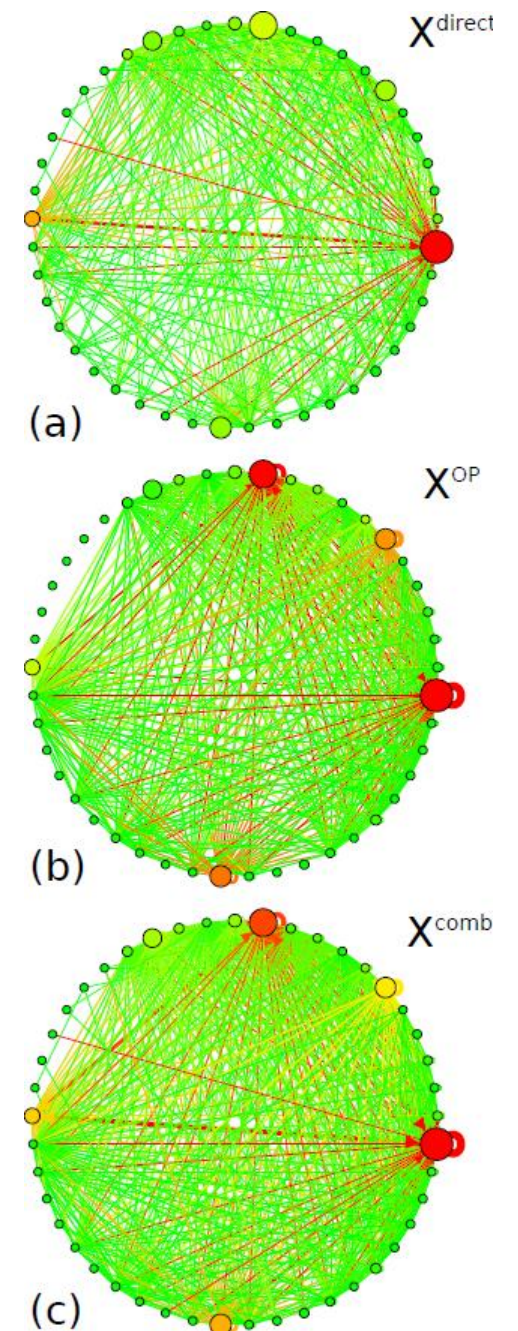
Results

# Mexican multi-layer banking network

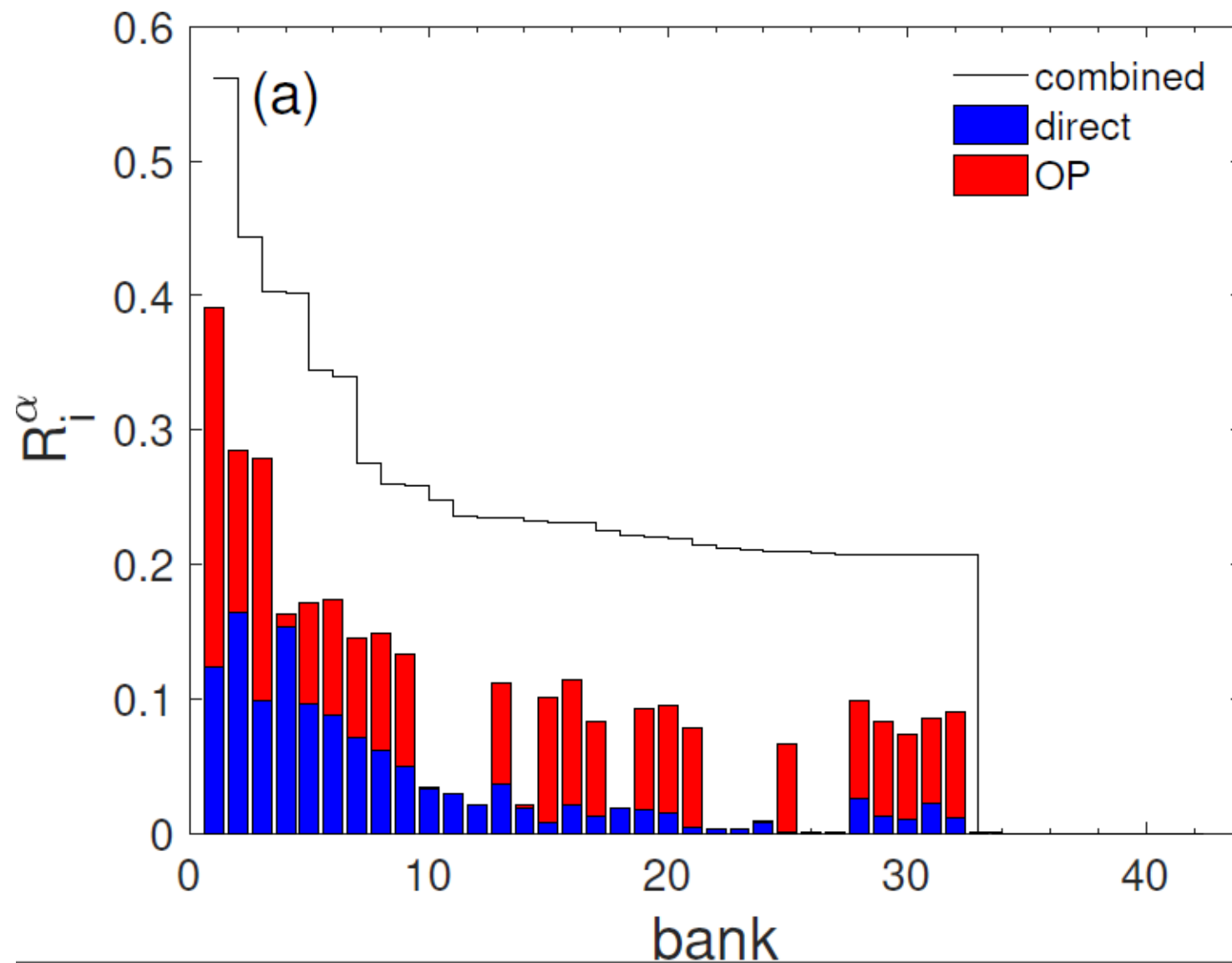
Node size represents the size of banks in terms of total assets. The important banks are red; unimportant ones are green, the width of links represents the size of the exposures in the layer, link color is the same as the counterparty's node color (DebtRank).

Diagonal elements represent the loss for a bank itself from liquidating its portfolio and are typically larger than the indirect exposure to other banks with similar portfolios. The different layers of exposure of the Mexican financial system are rather dense.

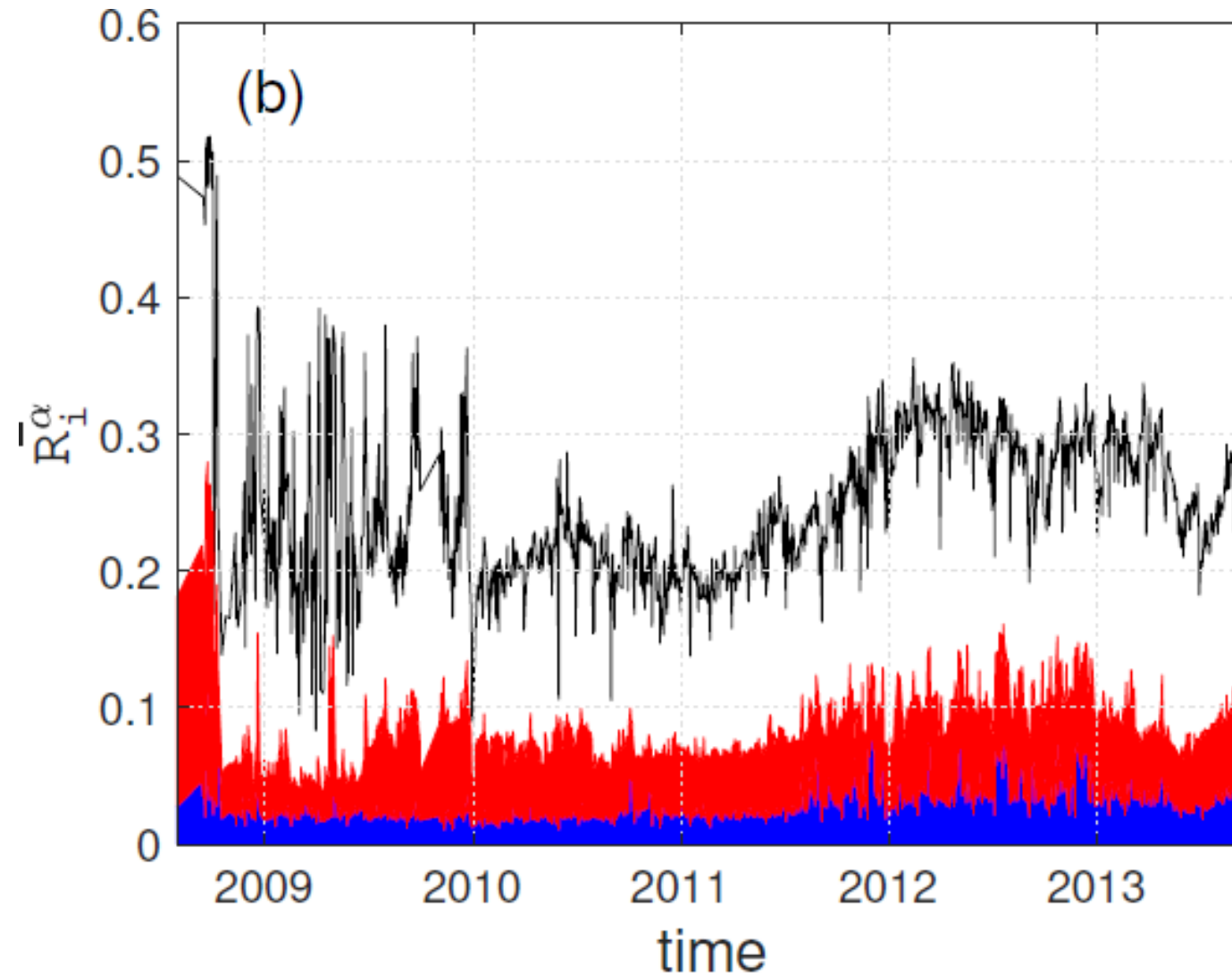
- (a) Network of direct interbank exposures. The density of this layer is 0.23.
- (b) Network of indirect external exposures from overlapping portfolios. The density of this layer is 0.43.
- (c) Combined banking network. The density of this layer is 0.49.



# SR profile for the different layers



# Time series for the average DebtRank from 31 July 2008 to 30 September 2013





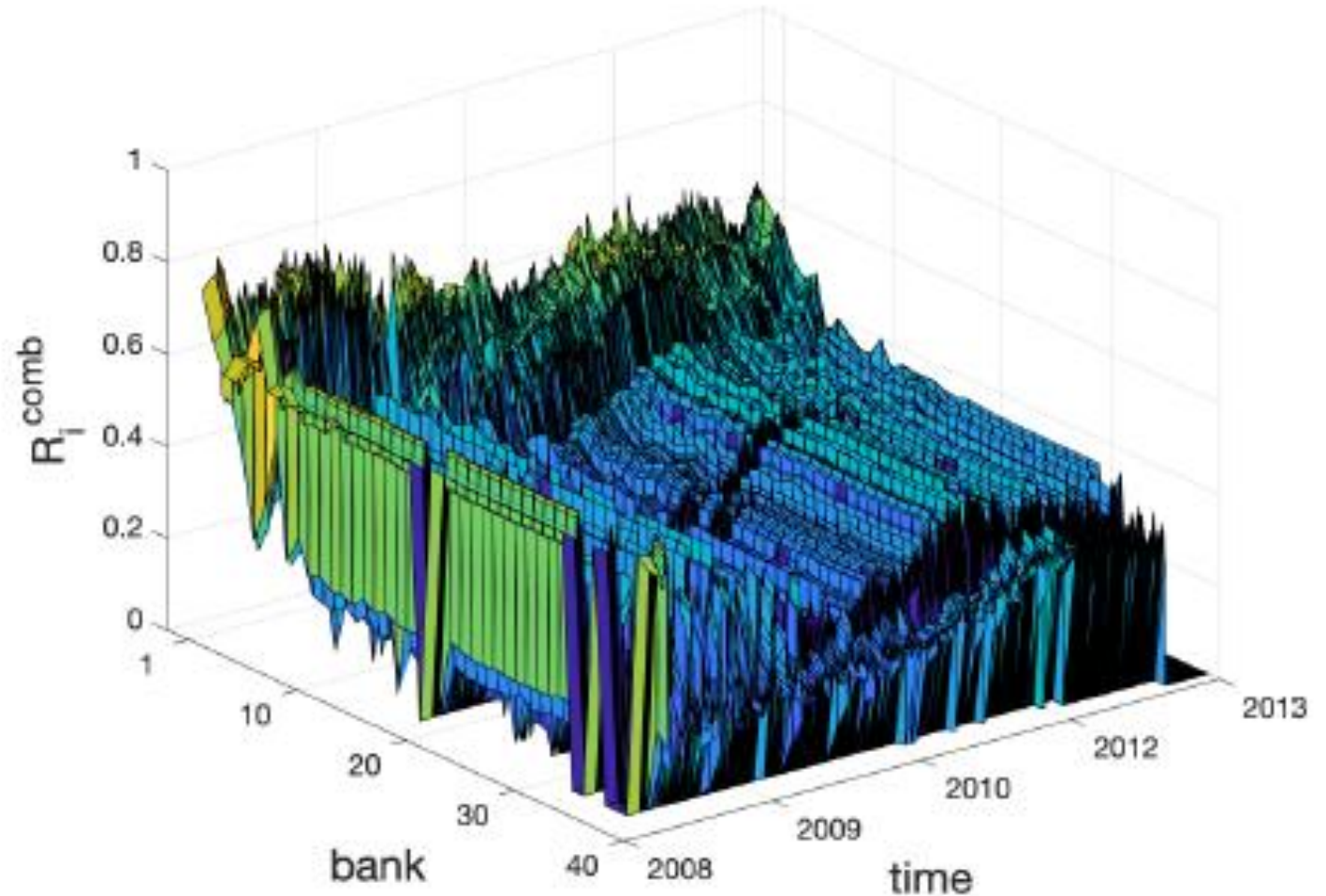
# Systemic Risk surface for the combined network from all layers, from 31 July 2008 to 30 September 2013.

In this figure, we show the daily DebtRanks in the combined network from all layers for each bank from 2008 to 2013.

The most systemically important banks do not change too much over time.

Systemic Risk was higher for almost all banks at the beginning of the measurement period (2008 financial crisis).

After the height of the financial crisis, there is a group of banks that are basically flat in terms of SR and over time.

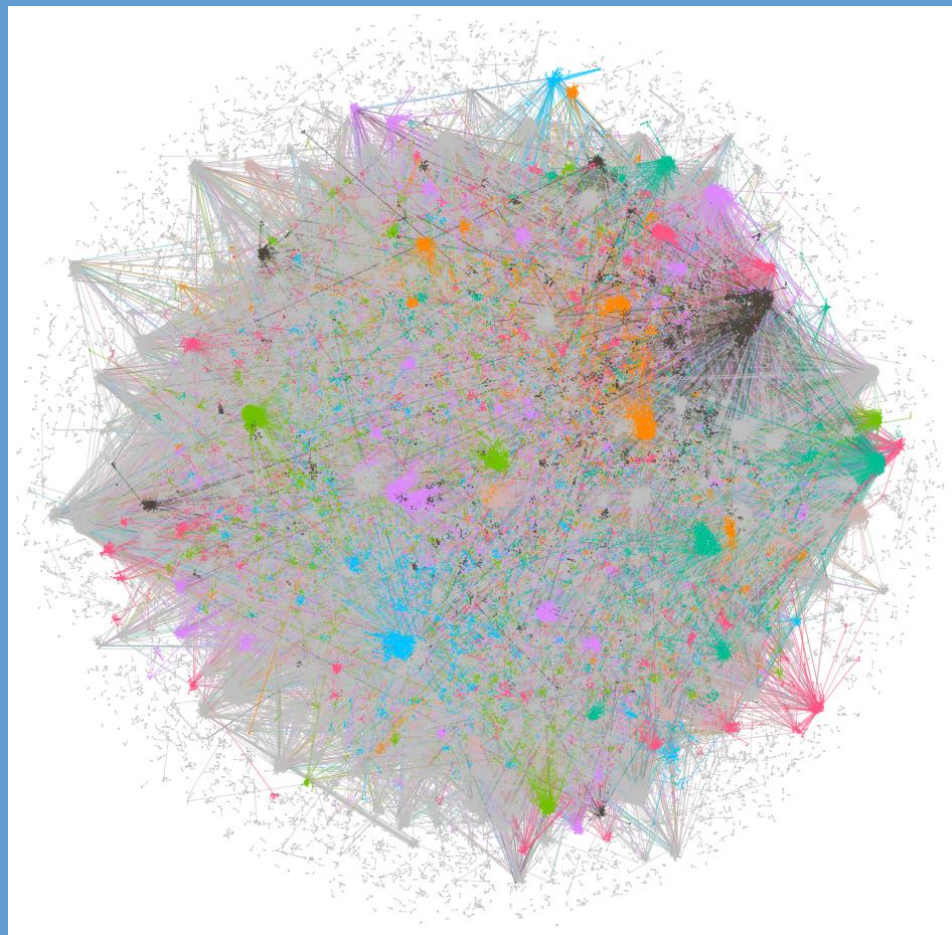


# Conclusions

- Systemic risk (SR) arises from indirect interconnections that occur when financial institutions invest in common assets (overlapping portfolios).
- Mutual influence of different channels of contagion were represented by a financial system as a multi-layer network of direct interbank exposures (default contagion) and indirect external exposures (overlapping portfolios).
- Indirect exposures represents an important form of financial contagion.
- Direct interbank exposures underestimates total systemic risk levels by up to 50 percent.
- There are many more aspects of the modeling of financial stability and systemic risk which can be tackled by using network theory and models.

# Future work

- Incorporate into this framework funding liquidity risk
- Include more financial intermediaries like investment funds and pension funds
- Possibly consider more asset types (equity)



Thanks a lot for your attention.