



Introduction to Financial Networks

Course on RegTech and SupTech

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**The views expressed in this presentation are exclusively the responsibility of the author and do not necessarily reflect those of CEMLA or Banco de México.*

Introduction

Financial Networks. Introduction

- Financial networks are useful to model the complexity of interactions among banks and other users of the banking system. Networks are an effective visual method to model and identify all the connections in the financial system.
- Two approaches to financial networks:
 - A. Prices
 - B. Balance sheet data
- Balance sheet data approach consists on using balance sheet data to construct a network (not necessarily entirely known).
 - It could be used to analyze or study systemic risk and financial contagion.
- Price based networks normally resort to correlations.
 - Then a filtering technique can be applied.

Financial Networks. Introduction

- Financial networks are related to systemic risk and might have important implications in financial stability, Battiston and Martínez-Jaramillo (2018) pointed out the insights and the challenges related to systemic risk, stress testing and financial network models.
- Within the policy insights the authors identify that networks effects do matter and financial networks allow to understand externalities in presence of incomplete. On the other hand, they identified as challenges and research avenues, the multiplex financial network; big data; endogenous networks; climate change as a source of instability for the financial system; and network effects on real economy.
- Battiston et al. (2016): *“From the point of view of financial regulators, our findings show that the complexity of financial networks may decrease the ability to mitigate systemic risk, and thus it may increase the social cost of financial crises”*.
 - Battiston, S., Caldarelli, G., May, R., Roukny, T., and Stiglitz J., (2016) *“The price of complexity in financial networks”*, *Proceedings of the National Academy of Science*, Vol. 113, No. 36, pp. 10031–10036.

Network Analysis*

**Martinez-Jaramillo, S., Alexandrova-Kabadjova B., Solórzano-Margain JP. (2014) “An empirical study of the Mexican banking system’s network and its implications for systemic risk” Journal of Economic Dynamics & Control, Vol. 40, pp. 242–265.*

Network Analysis. Definitions

- **Non-directed graph:** is defined as a set of nodes connected to a set of edges.
- **Directed graph:** is a set of nodes connected to a set of edges with an specific order (i, j) .
- **Non-directed network:** it is assign a specific weight for each edge and is the connection among vertices (information).
- **Directed network:** it is assign a specific weight for each edge and the weight is the connection among vertices with an specific order (i, j) .
- **Adjacency Matrix:** is a matrix representation of a order list of arcs (i, j) . This could be divided by in-degree and out-degree
- **Neighbour:** is a neighbour if there exists an edge that connects the nodes.
- **Weight Matrix:** design from the adjacency matrix of a directed network. In the financial context, the weight of the arcs in the directed networks represent money flow.

Network Analysis. Topological Measures*

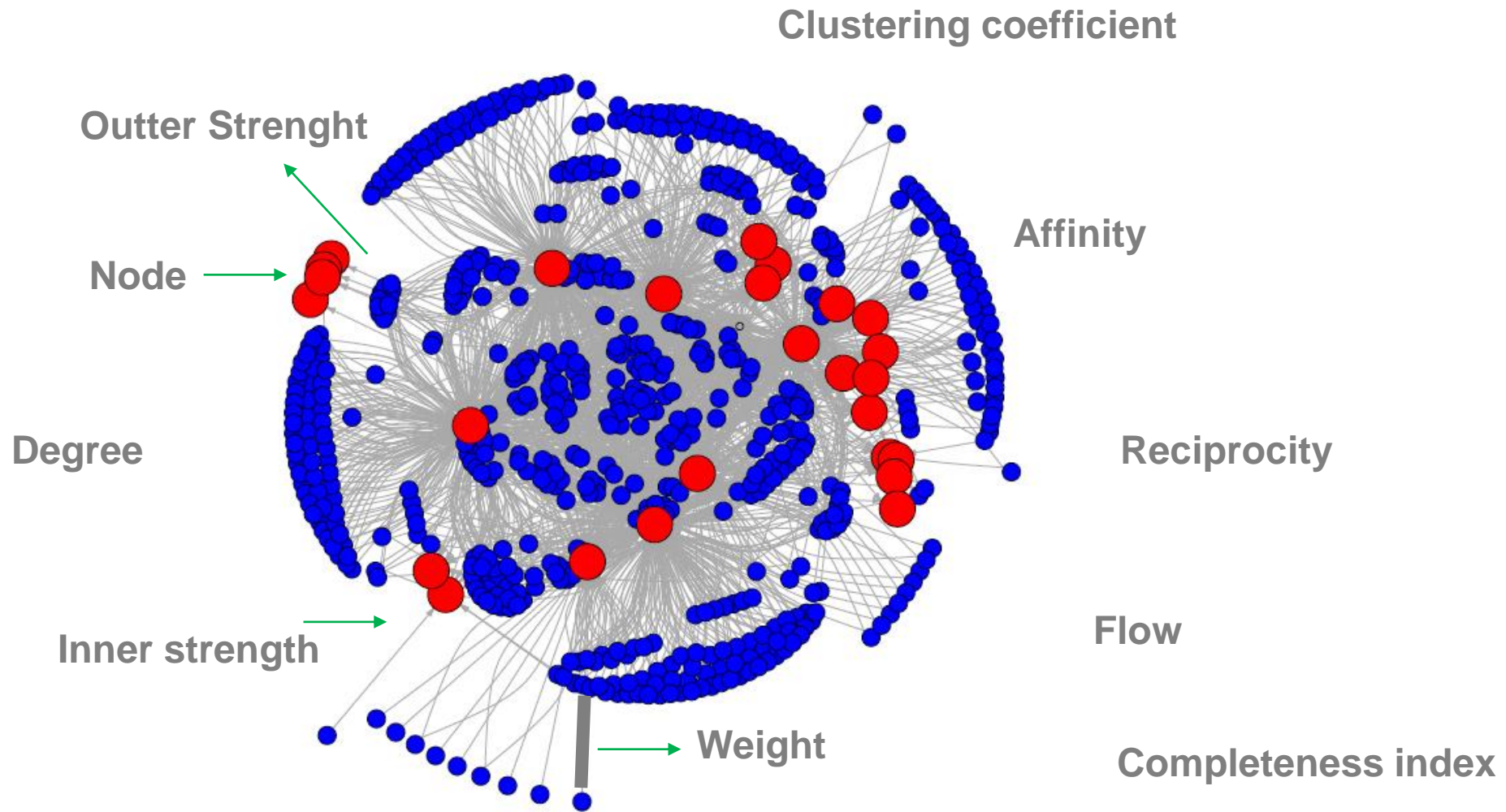
- **Degree:** the number of nodes that a node is connected to.
- **Clustering coefficient (CC):** is a measure of the density of the connections around a vertex i . The Clustering Coefficient indicates that if two vertices, which have a connection with a third vertex, have a connection between them; that is, it indicates if they form a triangle. the average CC measures the density of triangles in the graph.
- **Reciprocity:** is the fraction of arcs in any direction for which there exists an arc in the opposite direction.
- **Affinity:** describes the type of nodes to which such a node tends to have a link. If the nodes in a network tend to have relationships with nodes of similar degree or nodes with different degree. Conversely, nodes with low degree tend to have relationships with high degree nodes.
- **Completeness index:** is a measure of how close a graph is to the complete graph. The complete graph has an index of 1, whereas the graph with no edges has an index of 0. the closer the index is to 1, the closer the graph is to being fully connected.

**Martinez-Jaramillo, S., Alexandrova-Kabadjova B., Solórzano-Margain JP. (2014) "An empirical study of the Mexican banking system's network and its implications for systemic risk" Journal of Economic Dynamics & Control, Vol. 40, pp. 242–265.*

Network Analysis. Other Measures*

- **Strenght:** is a simple measure but an important one, and can be interpreted as a intensity-of-interaction measure. It is used as a criteria to determine centrality in a network.
- **Inner and outer strength** are relevant measures because they could be useful to determine if a bank plays a more important role as a lender or a borrower, in the case of the interbank exposures network.
- **Flow:** is a measure that can be used to characterize a node as a net lender or net borrower in the network. This characterization in turn can be used to take some actions in order to manage systemic risk depending on the importance of the node in the network.

Network Analysis. Example*



Network Analysis. Centrality

- Centrality is a useful tool to identify institutions that are more relevant to the financial stability and monitor systemic risk.
- Martinez-Jaramillo et al. (2014)* measure and monitor systemic risk through topological metrics for payment system and interbank networks. Additionally, the authors suggest non-topological measures to describe individual behavior of banks in both networks. They found that structures of payments and exposures networks are different regarding their connectivity.
- The lineal combination of all the centrality measures allow to rank the nodes according their relevance in the network. The larger the centrality measure, the greater importance such a node has in a network. Some of them are listed below:
 1. Degree centrality
 2. Strength centrality
 3. Betweenness centrality
 4. Closeness centrality
 5. Eigenvector centrality
 6. DebtRank, PageRank

**Martinez-Jaramillo, S., Alexandrova-Kabadjova B., Solórzano-Margain JP. (2014) "An empirical study of the Mexican banking system's network and its implications for systemic risk" Journal of Economic Dynamics & Control, Vol. 40, pp. 242–265.*

Network Structural Analysis. Centrality

- **Degree centrality** refers to those nodes which are important in a network as far it is connected to many other nodes. In presence of risk, it could spread to a higher amount of institutions because of its number of connections and the properties of the networks.
 - *Out-Degree*: The out-degree centrality for a node is the fraction of nodes its outgoing edges are connected to.
 - *In-Degree*: The in-degree centrality for a node is the fraction of nodes its incoming edges are connected to.
- **Strength centrality** refers to the sum of its interbank assets and liabilities. Inner strength is the sum of its interbank assets while the outer strength is the sum of its interbank liabilities. This is very important to determine which bank is lending (borrowing) the most in the network.
- **Betweenness centrality** is important in the payment systems network because a node with high betweenness centrality would have an important influence on other nodes as it can stop or distort the information that passes through it.

Network Structural Analysis. Centrality

- **Closeness centrality** can be associated with the capacity of a node to spread contagion, as such a node is close to the rest of the network.
- **Eigenvector centrality** takes into consideration the centrality of the neighbors to compute the centrality of a node. The eigenvector centrality take into account direct connections as well as indirect ones.
- **PageRank centrality** is a measure that considers the relevance of neighbors to determine the relevance of a node in the network.
- The main findings in Martinez-Jaramillo et al. (2014), are the wide range of empirical measures for two networks of the Mexican banking system: interbank exposures and the payments system flows. Additionally, the authors tested and provided good evidence of the robustness on financial networks of centrality measures.
- Also noteworthy is that contagion is not necessarily related to assets size. Those banks ranked very high in terms of interconnectedness are important to determined the systemic importance in financial networks.

Main channels of financial contagion:
Default cascades, funding contagion, fire sales
externality*

**van der Leij, M. (2019) "Financial networks and financial stability" CEMLA Course on Financial Stability, 20 September 2019.*

Financial contagion channels

- Financial contagion refers to the spread of a shock among banks through the financial network. Additionally, it is associated with higher connectivity, funding liquidity, common assets contagion. Financial contagion is one of the main components of the systemic risk.

- The different types of contagion are the following:
 - I. Default cascades (Furfine, 2003; Eisenberg & Noe, 2001; and DebtRank, 2012 and 2015)
 - II. Funding liquidity contagion (H. Lee, 2010)
 - III. Fire sales externality (Gai and Kapadia, 2010; Greenwood, 2015)

Default cascades

- The default cascade shock is transmitted through **asset side**. It could be amplified by bankruptcy cost, fire sales externality and by incorporating default risk in the asset values.
- **Eisenberg & Noe (2001)** methodology is based on optimization problem, in particular solving for payment that bank i makes to bank j (P_{ij}). They show that it is a unique payment vector (\vec{P}) that clears the system of payment equations if all banks default. Shock leads to default if $w_i < 0$ (equity).
- The assumptions are:
 1. External assets are always paid out.
 2. If a bank is solvent, then the bank pays what it owes.
 3. If a bank defaults, then the bank pays out all its assets. Assets are divided equally among all its creditors (equal seniority).
 4. Payments clear if:

$$P_i = \min_i \{ \bar{P}_i, c_i + \sum_k \alpha_{ki} P_k \} \forall i$$

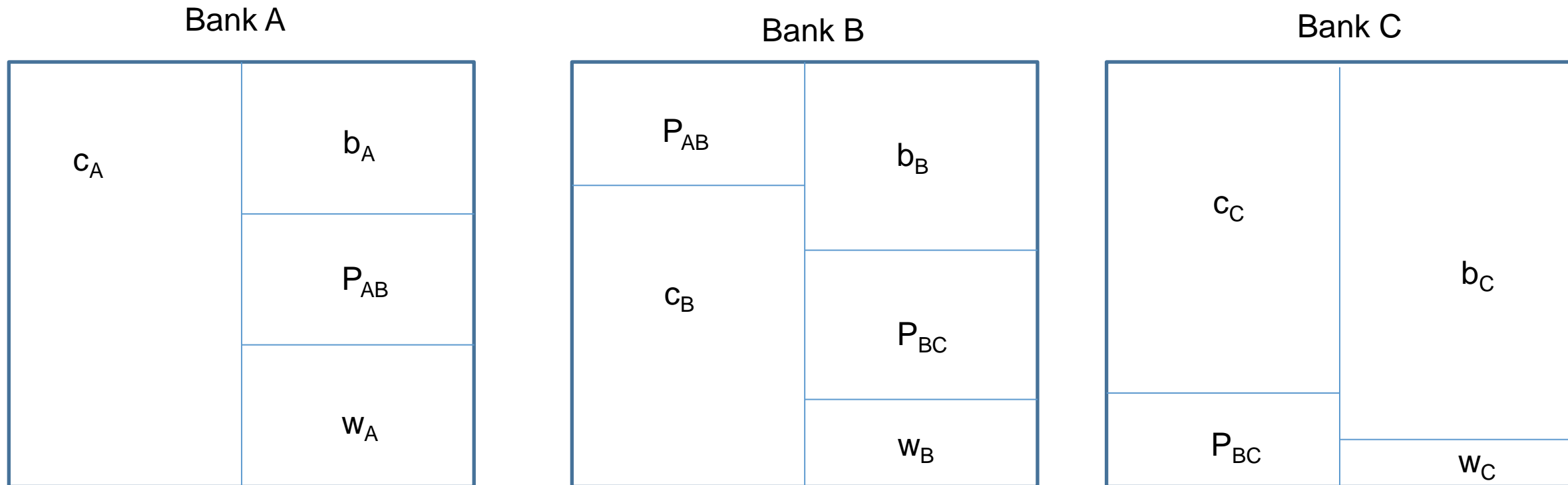
Bank balance sheet notation*

Bank i

Assets	Liabilities
Outside assets (c_i) <ul style="list-style-type: none">Liquid assets<ul style="list-style-type: none">Cash, government bonds	Outside liabilities (b_i) <ul style="list-style-type: none">Deposits
<ul style="list-style-type: none">Illiquid assets<ul style="list-style-type: none">Loans to firms and consumers	In-network Liabilities (P_{ij})
In-network Assets (P_{ji})	Equity (w_i) <ul style="list-style-type: none">Capital + reserves

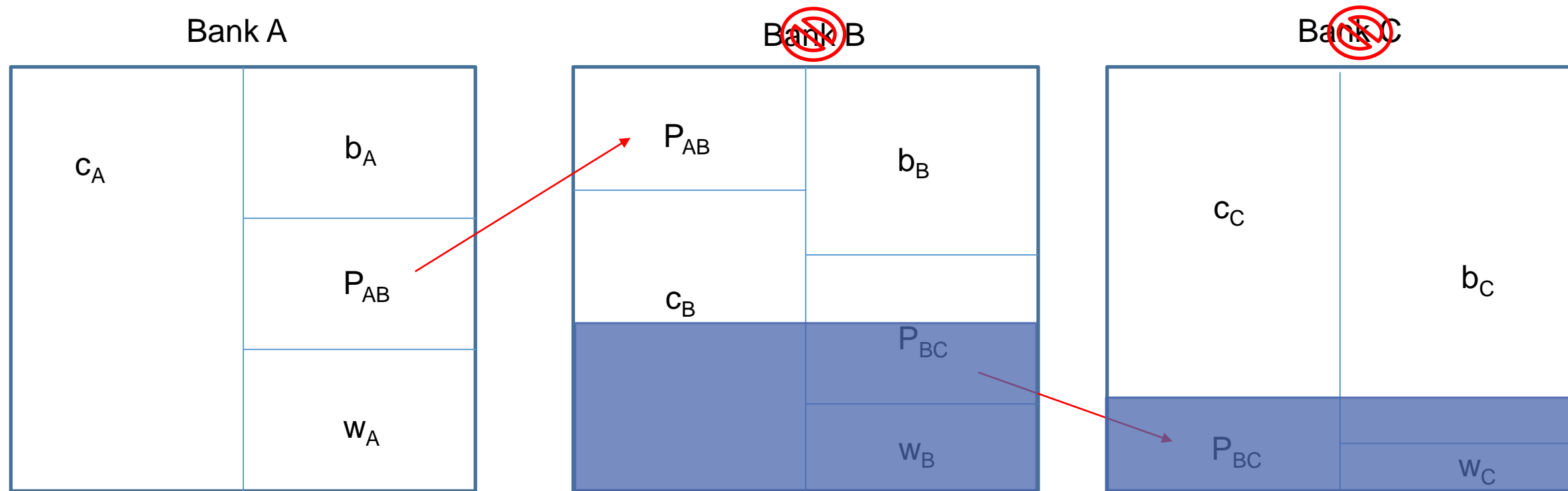
Eisenberg & Noe (2001)

- The following is an example of the Eisenberg-Noe methodology:



Eisenberg & Noe (2001)

- The following is an example of the Eisenberg-Noe methodology:



DebtRank algorithm

- DebtRank algorithm consisted on the market value of bank A's interbank debt may drop **before** bank A defaults. If assets are value at mark-to-market, a shock of A leads to a loss at other Banks that own debt issued by A.
- DebtRank algorithm outline is the following:
 - I. $t=0$, initiate balance sheets;
 - II. $t=1$, apply shocks to banks;
 - III. $t \geq 2$, revalue interbank assets proportional to drop in debt issuer's equity.
- $P_{ij}(t) = P_{ij}(0) \frac{w_i(t-1)}{w_i(0)}$; market value of debt issued by i and owned by j in round t .
- $w_i(t) = \max(0, w_i(0) - s_i - \sum_k (P_{ki}(0) - P_{ki}(t)))$; equity.

DebtRank algorithm

- Example of DebtRank:

Round 0: initial situation

Bank A

$c_A = 10$	$b_A = 4$
	$P_{AB} = 4$
	$w_A = 2$

Bank B

$P_{AB} = 4$	$b_B = 4$	
$c_B = 12$		$P_{BC} = 8$
		$w_B = 4$

Bank C

$c_C = 4$	$b_C = 10$
$P_{BC} = 8$	
$w_C = 2$	

Round 1: shock to bank A

Bank A

$c_A = 9$	$b_A = 4$
	$P_{AB} = 4$
	$w_A = 1$

Bank B

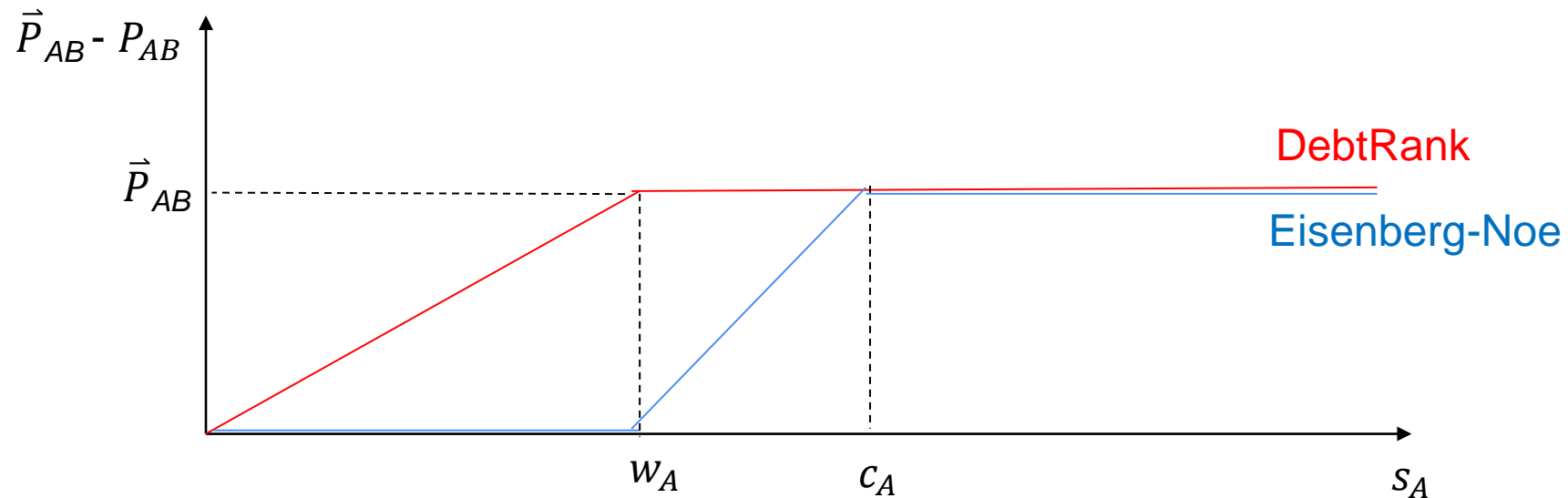
$P_{AB} = 2$	$b_B = 4$	
$c_B = 12$		$P_{BC} = 8$
		$w_B = 2$

~~Bank C~~

$c_C = 4$	$b_C = 10$
$P_{BC} = 4$	
$w_C = 0$	

DebtRank and Eisenberg-Noe

- The **differences** between DebtRank and Eisenberg-Noe are:
 - Eisenberg-Noe: is based on accounting identities, a fixed point clearing vector, it identifies a lowerbound for contagion, the **contagion only occurs after default**. And there is no contagion in quiet periods.
 - DebtRank: is a dynamic process, represents an upperbound on contagion, the **contagion occurs before default** and it is always volatile.



Funding liquidity contagion

- H. Lee (2010) proposed a new methodology to capture the systemic nature of funding liquidity risk in a foreign currency to analyze Korean banking system. Additionally, his framework included non-financial sectors and foreign financial institutions outside the domestic banking system and he limited the number of counterparties on the basis of the size of bank for estimating bank-to-bank exposures.
- This novel framework consisted on four measures of systemic funding risk:
 1. **Systemic funding liquidity indicator:** refers to the amount of assets directly or indirectly liquidated in the banking system when the system is unable to roll over external borrowing.
 2. **Systemic vulnerability indicator:** identifies which Banks are most exposed in the case of a systemic funding liquidity crisis.
 3. **Systemic importance indicator:** identifies systemically important financial institutions.
 4. **Systemic liquidity shortage indicator:** calculates the amount of the bank's liquidity needs.

Funding liquidity contagion

- In the funding contagion, the shock is transmitted through **liability side**, the net worth is not directly affected (no defaults). However, the mechanism gets amplified by sales of illiquid assets (fire sales) and by liquidity hoarding (bank B converts its remaining loan to A into cash).
- Some examples for the funding liquidity contagion are:

Funding liquidity contagion

- Case A: Bank can deal with their liquidity needs

Bank A

Interbank Assets	Interbank Liabilities
Liquid Assets	Other Liabilities
Illiquid Assets	Capital

The bank can cope with the withdrawal and selling interbank positions liquidating liquid assets (AFS).

- Case B: Liquidity shortage

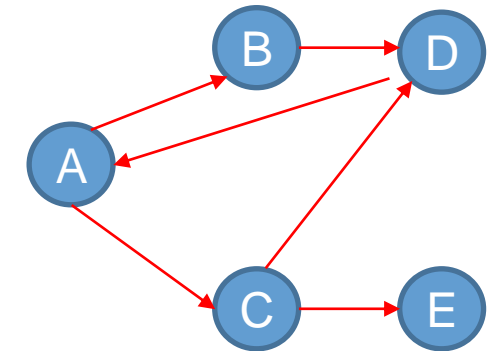
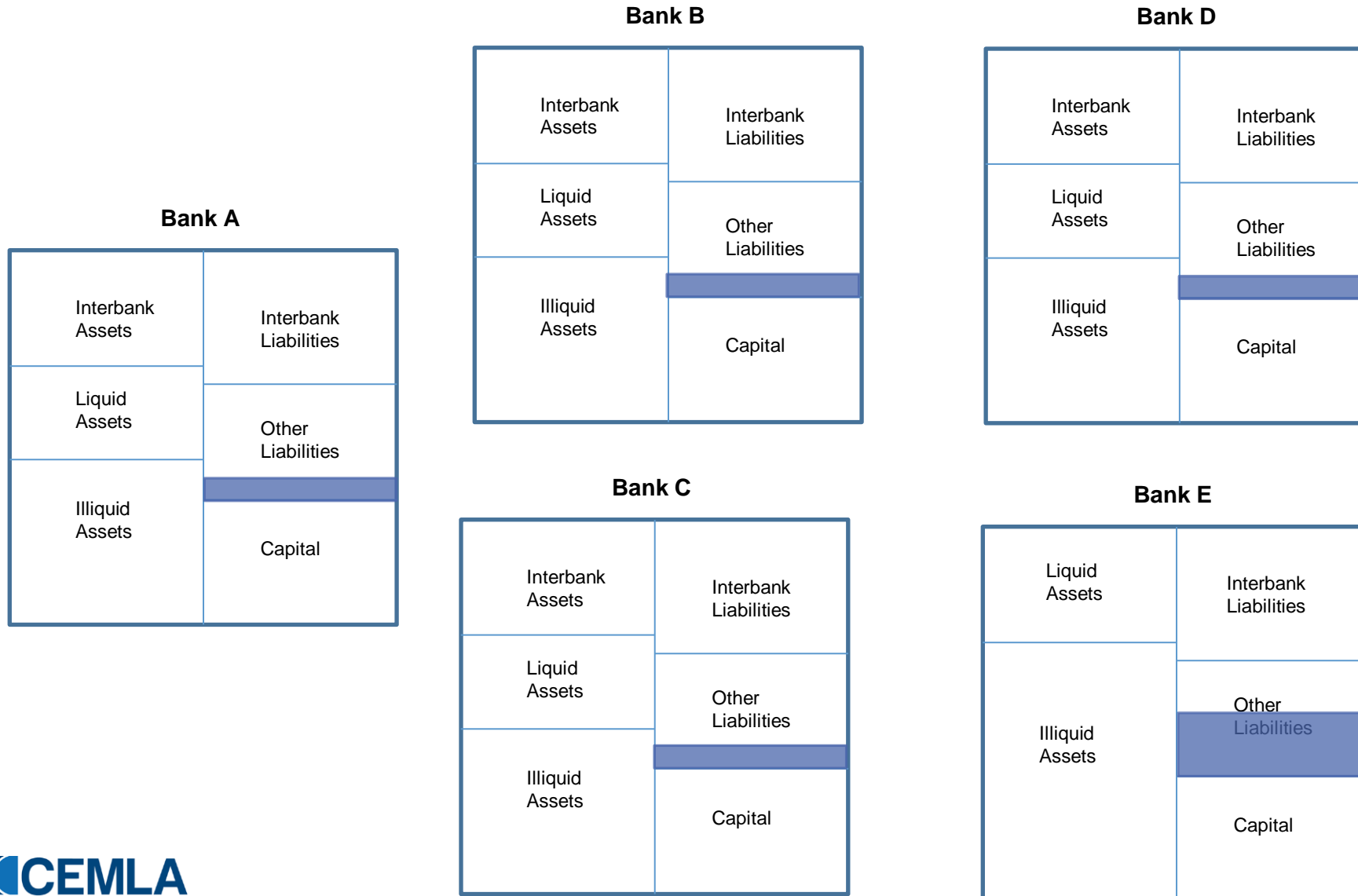
Bank A

Interbank Assets	Interbank Liabilities
Liquid Assets	Other Liabilities
Illiquid Assets	Capital

The bank can not cope with the withdrawal and selling interbank positions liquidating liquid assets having to resort to the sale of illiquid assets (AFS).

Funding liquidity contagion. Default Cascade

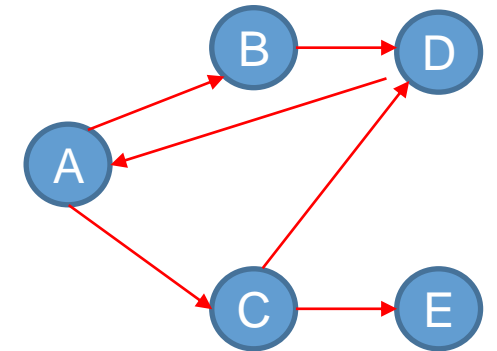
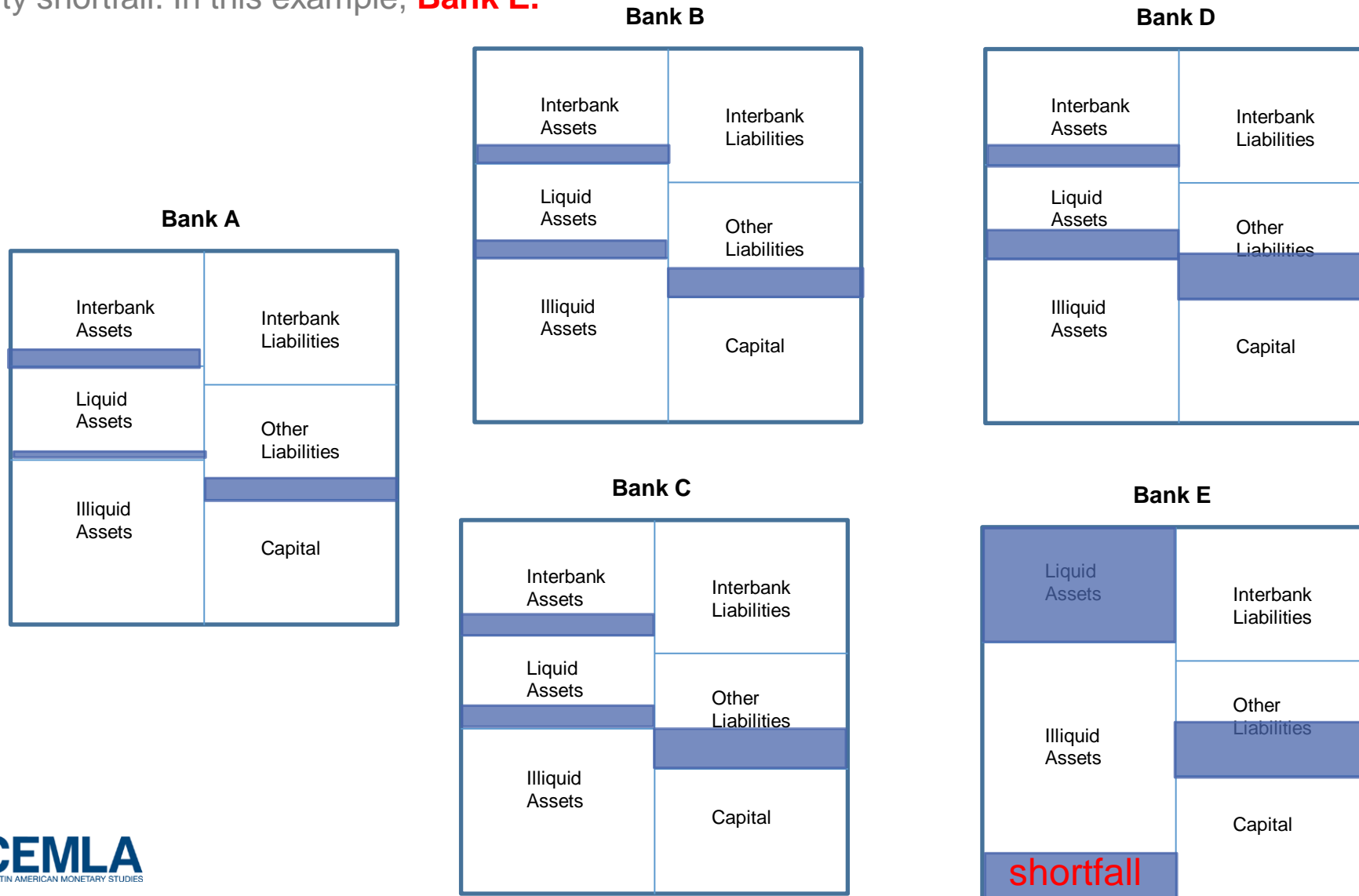
1. Bank i receives a shock to its funding denoted by $\Delta d_i (\forall i)$



Funding liquidity contagion. Default Cascade

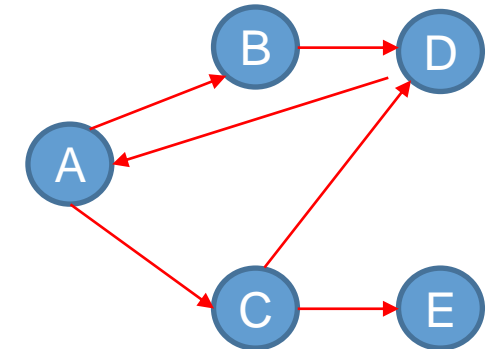
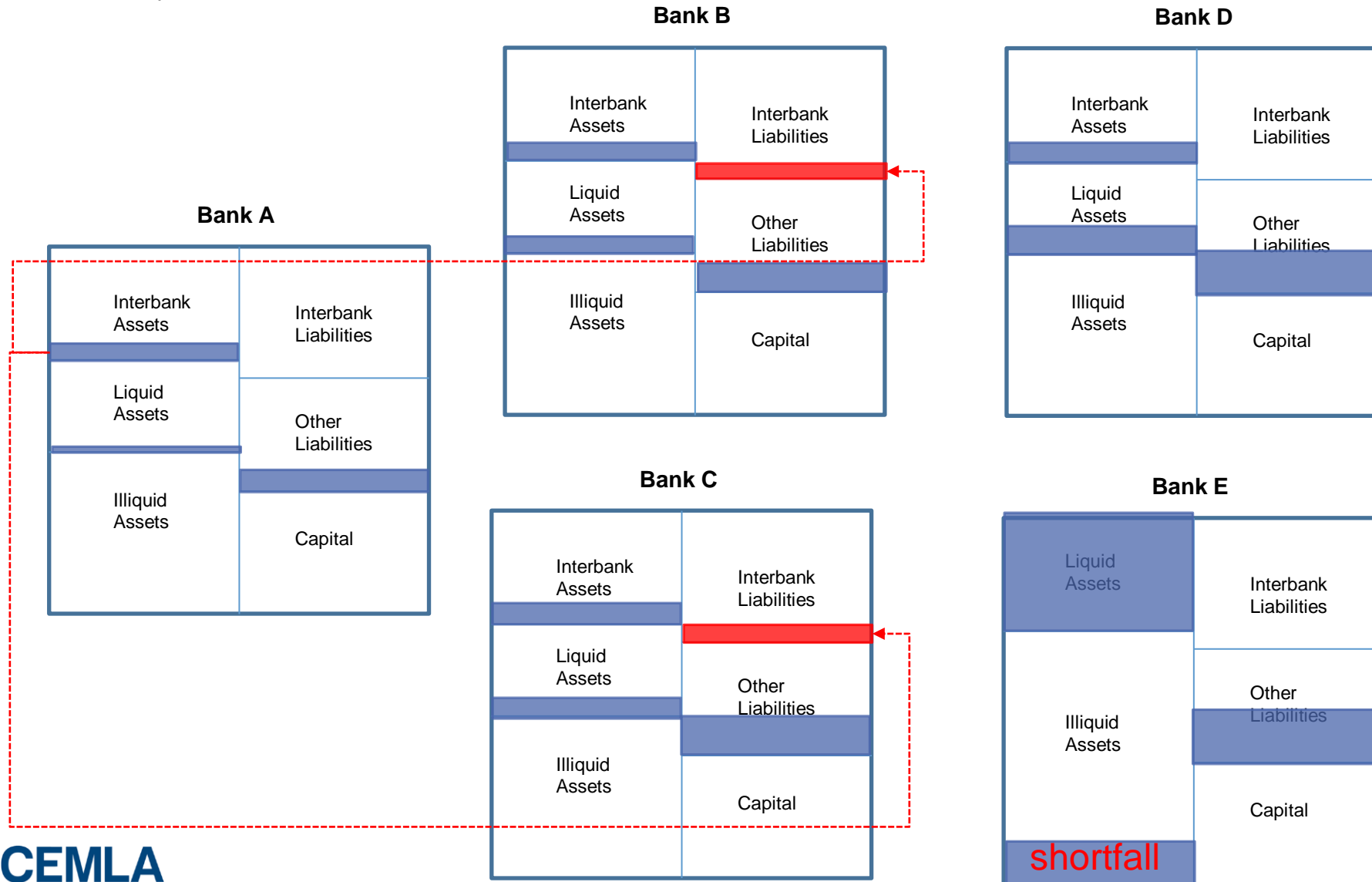
1.1 Banks sell liquid assets and cancel interbank loans to meet their liquidity needs.

1.2 If the bank does not have enough liquid and interbank assets to meet its liquidity needs, this gap represents the bank's liquidity shortfall. In this example, **Bank E**.



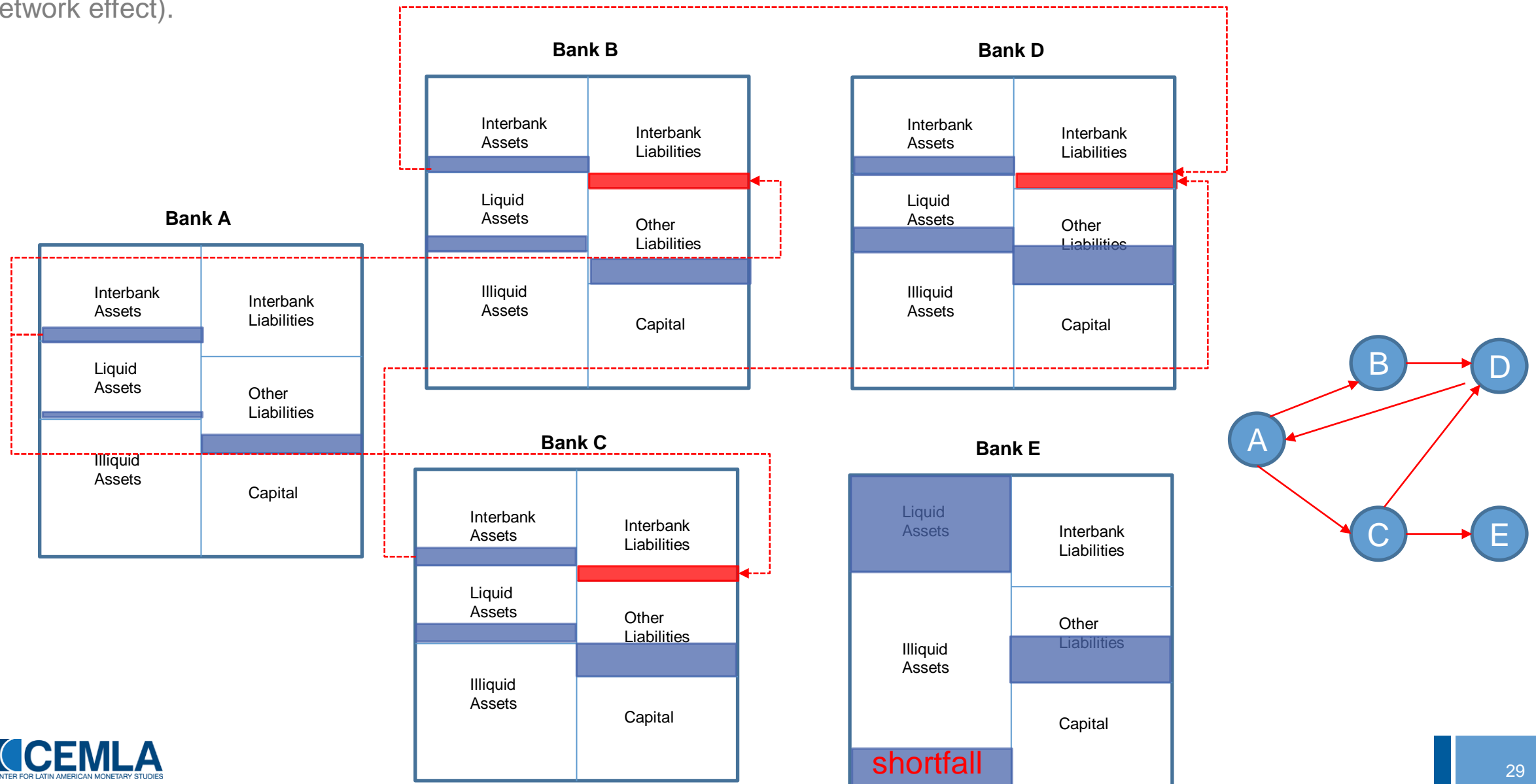
Funding liquidity contagion. Default Cascade

1. 3 The liquidity needs of each bank are updated, increasing by the interbank liabilities that were liquidated by other Banks (network effect).



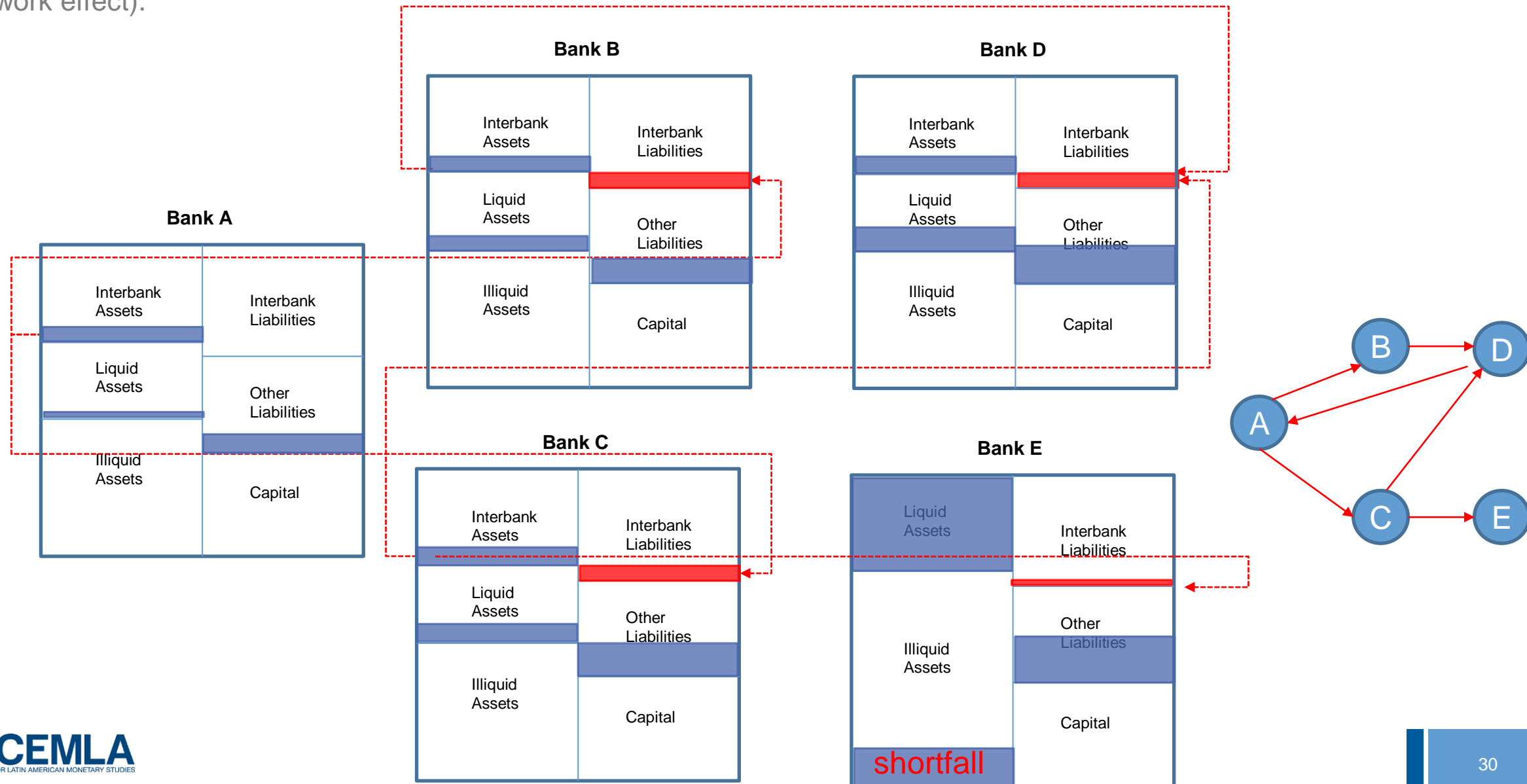
Funding liquidity contagion. Default Cascade

1. 3 The liquidity needs of each bank are updated, increasing by the interbank liabilities that were liquidated by other Banks (network effect).



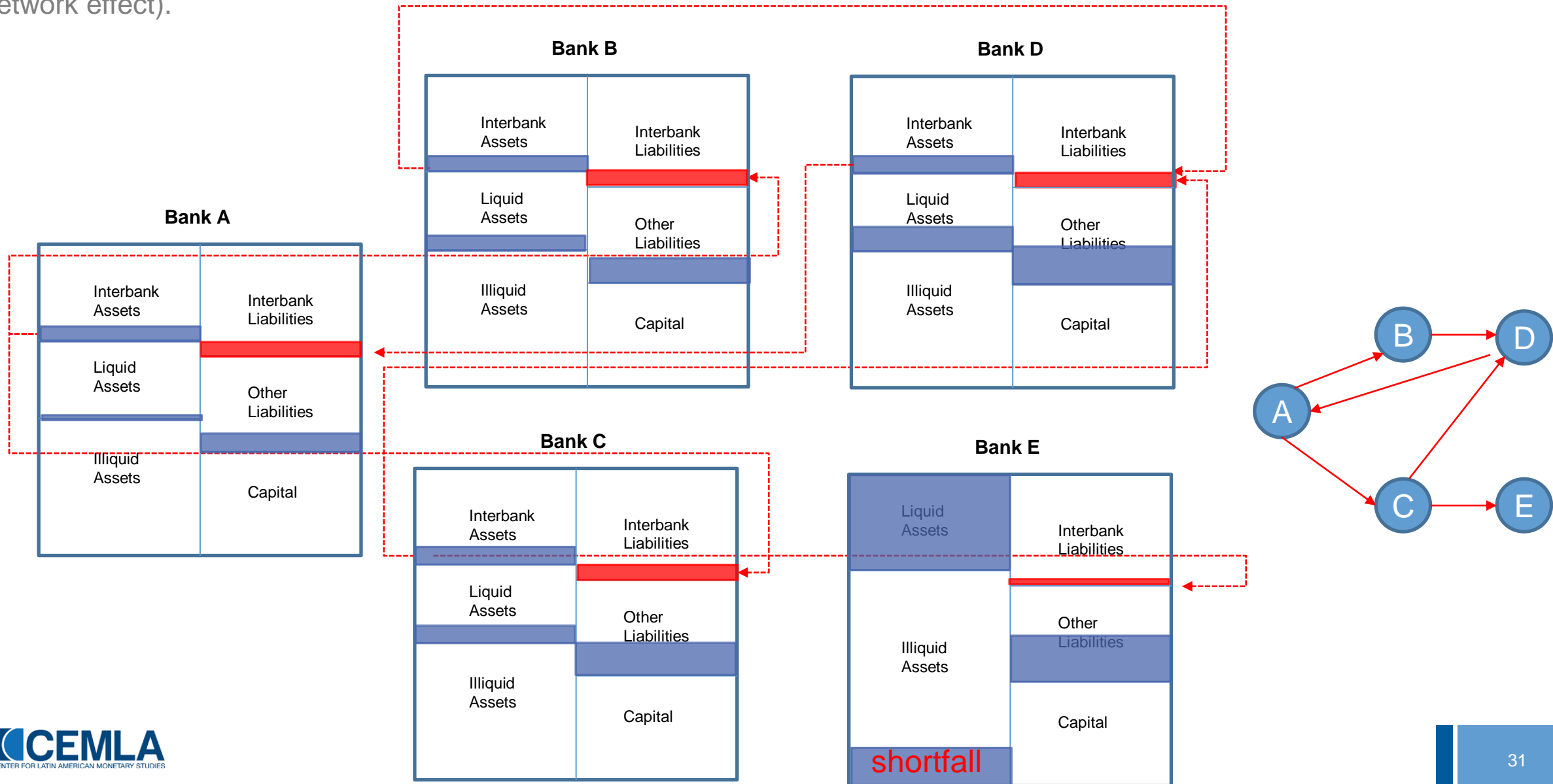
Funding liquidity contagion. Default Cascade

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Funding liquidity contagion. Default Cascade

1. 3 The liquidity needs of each bank are updated, increasing by the interbank liabilities that were liquidated by other Banks (network effect).



Funding liquidity contagion

- Funding liquidity algorithm outline is the following:

- i. $t=0$, Bank i receives a shock to its funding denoted by $\Delta d_i (\forall i)$.

Banks sell liquid assets and cancel interbank loans to meet their liquidity needs.

If the bank does not have enough liquid and interbank assets to meet its liquidity needs, this gap represents the bank's liquidity shortfall.

The liquidity needs of each bank are updated, increasing by the interbank liabilities that were liquidated by other Banks (network effect).

- ii. $t \geq 1$, Banks act in the same way to meet their current liquidity needs. The algorithm ends when all Banks have fulfilled their liquidity needs or, exhausted its liquid and interbank assets.

Funding liquidity contagion

- Possible Banks' reactions:

1. H. Lee (2013)

- Banks face his need for liquidity in proportion to the amount of liquid assets and interbank assets with.
- In interbank assets, funding will be withdrawn proportionally to each bank (they are more removed those representing more and less to those who represent less).
- Lee, Seung Hwan. "Systemic liquidity shortages and interbank network structures." *Journal of Financial Stability* 9, no. 1 (2013): 1-12.

2. H. Lee with preference index

- Banks face their need for liquidity in proportion to the amount of liquid assets and interbank assets with.
- In interbank assets, it takes into account the monthly index and the corresponding funding preference is first removed to interbank relations with a low rate of preference.

Funding liquidity contagion

- **Repo funding shocks outline is the following:**
 1. A run on the repo funding from brokerage firms, investment and pension funds.
 2. The level 1 securities returned by the repo counterparts are re-incorporated as liquid assets, the rest go to the non-liquid assets.
 3. Banks will suffer a loss in value of its liquid assets and a loss in value of the securities returned by their repo counterparties.
 4. However, banks' face a funding shock of the size of the repo funding withdrawn by these counterparties.

Funding liquidity contagion

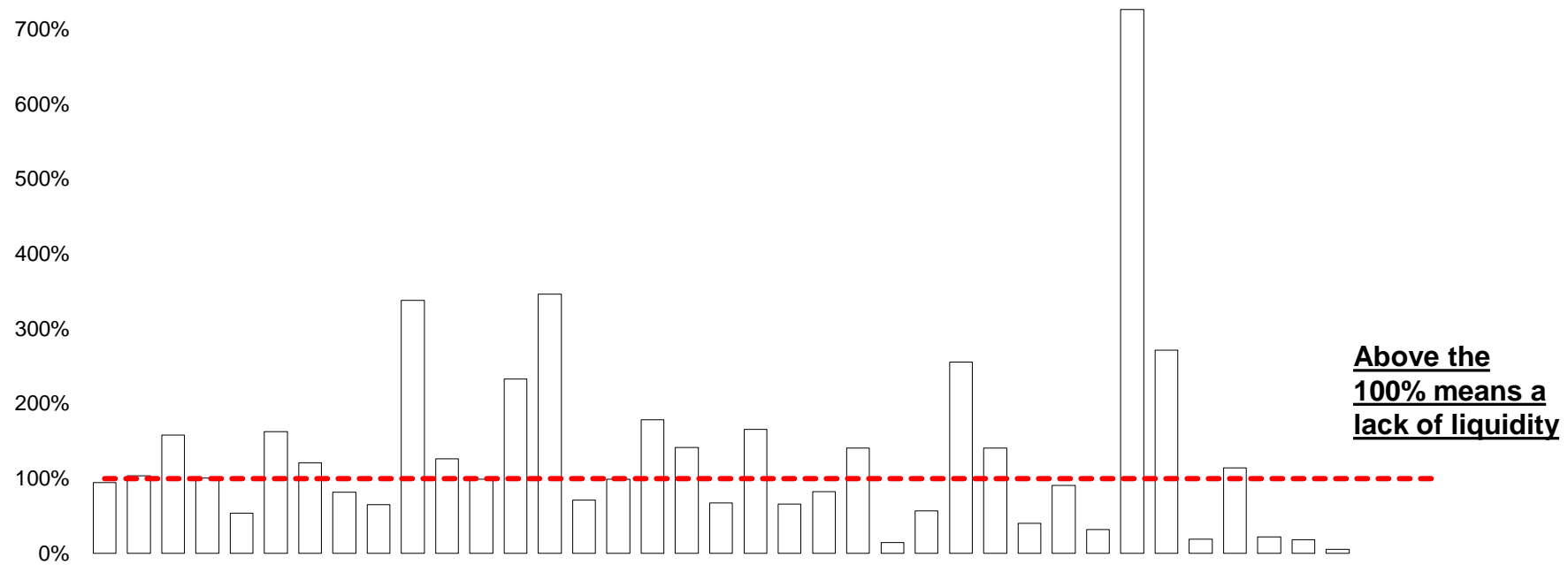
■ Repo funding shocks

The main results of the shocks to the repo funding are:

1. A 100% funding shock to all the repos in which the counterparts **are brokerage houses, investment funds and pensions funds** proved to be extremely harmful to the banking system, under any specification.
2. A 100% funding shock to all the repos in which the counterparts **are only investment funds and pension funds** is less harmful but three institutions could face extremely high liquidity shortages under the different specifications.
3. Network effects increase and the interbank network structure could be seriously affected. This enhances the importance of performing liquidity stress tests to different funding sources.

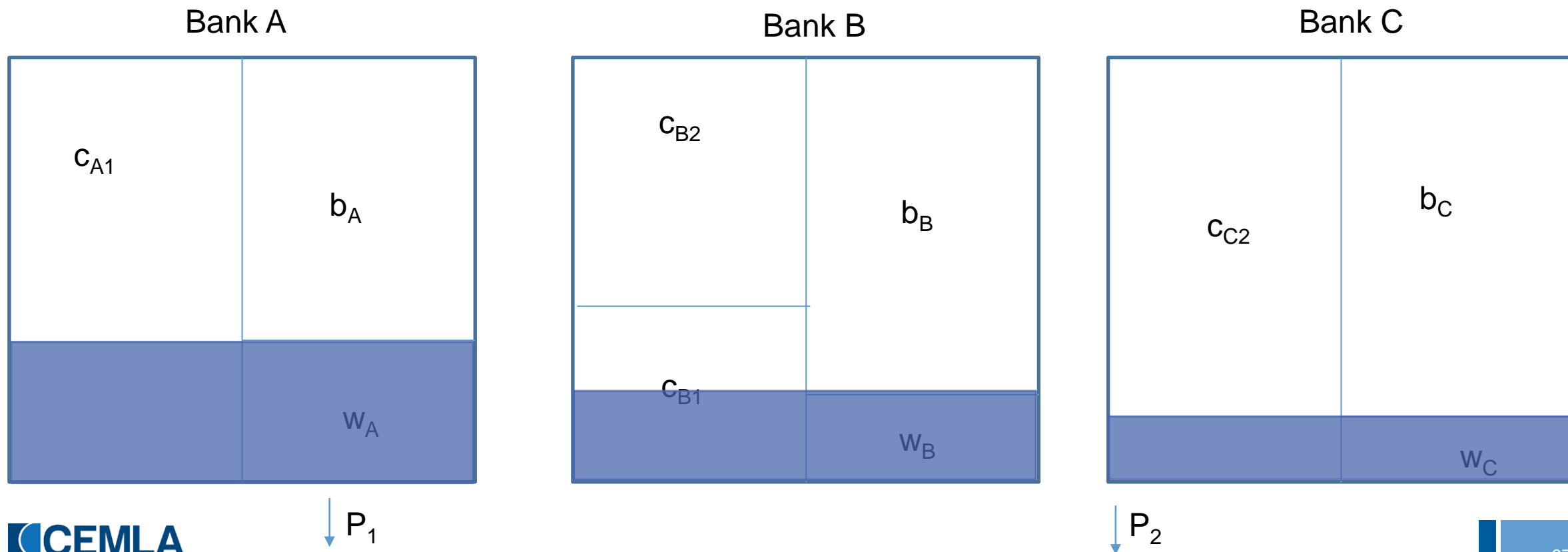
Funding liquidity contagion

% of illiquid assets and interbank assets



Fire sales externality

- In the fire sales externality, there is a shock on asset Price. The assumptions consisted on Banks that would like to keep their leverage ratio constant. The assets are illiquid and the balance sheet assets are valued at mark-to-market.
- Some examples for the fire sales externality are:



Fire sales externality

- For instance, in *Poledna et al (2019)** , the authors pointed out an important form of financial contagion by indirect links among financial institutions, in other words, financial institutions invest in the same assets. This means that their portfolios overlap and the contagion could be amplified because of those common assets to be devalued. Devaluations can cause further asset sales and devaluations leading to **fire sales**.

Measures of financial contagion*

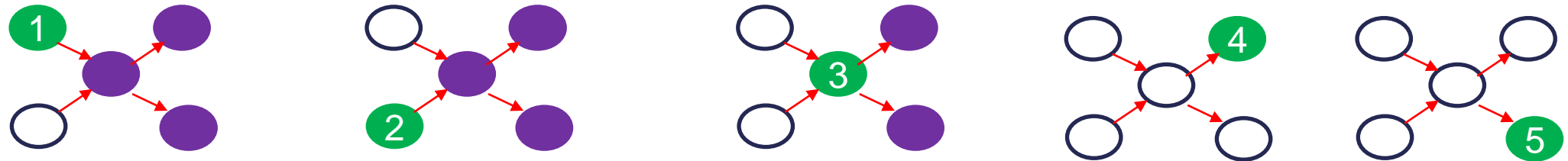
**van der Leij, M. (2019) "Financial networks and financial stability" CEMLA Course on Financial Stability, 20 September 2019.*

Measures of financial contagion

- A. System level: systemic risk, expected systemic loss.
 - B. Bank level: systemic importance and vulnerability.
-
- Poledna et al. (2015) defined systemic risk as *“the risk that a significant proportion of the financial market can no longer perform its function as a credit provider and collapses.”*
 - Systemic risk is a consequence of the interconnectedness among institutions in the financial system.
 - Measuring systemic risk is to allow better decision making and risk management for central Banks and regulators.

Measures of financial contagion

- Vulnerability of a bank refers to the fraction of equity lost averaged across all initial shocks.



- Most systemically important: 1 and 2
- Most vulnerable: 4 and 5
- $Systemic\ Importance = Systemic\ Risk(g) - Systemic\ Risk(g|\{i\ defaults\})$

Importance for assessing systemic risk:
The multi-layer network nature of systemic risk and its
implications for the cost of financial crises*

**Poledna, S., Martínez-Jaramillo, S., van der Leij, M. (2015) "The multi-layer network nature of systemic risk and its implications for the costs of financial crises" Journal of Financial Stability, Vol. 20, pp. 70–81.*

The financial system as a (multilayer) network

- There has been a lot of recent research on financial networks for the purposes of studying systemic risk, performing stress testing or determining the relevance of financial institutions.
- A commonly shared view is that the financial system is highly interconnected.
- Financial institutions interact in different markets, which can be thought of as different networks within a meta-structure which can be interpreted as a multilayered network or a multiplex network. This gives rise to consider multiple channels of contagion.*
- This is the first quantification of systemic risk on a national scale that includes overlapping portfolios.

Quantification of systemic risk in multilayer networks*

- Banks interact in different markets and generate different types of exposure. Banks issue securities that are later bought by other banks. By holding these securities, banks expose themselves to other banks. Foreign exchange transactions can lead to large exposures between banks. Their exposures are associated with settlement risk. Another market activity that can lead to considerable exposures is trading in financial derivatives.
- In Poledna et al. (2015), we analyze four different types of financial exposure:
 - i. derivatives,
 - ii. securities,
 - iii. foreign exchange,
 - iv. deposits & loans.

Quantification of systemic risk in multilayer networks

- Poledna et al (2015) is based on transaction data converted to bilateral exposures and various balance sheet data on the 43 Mexican banks, such as the capitalization measured at a monthly scale.
- The four exposure types are obtained in the following ways:
 - 1) **Deposits & loans:**
 - i. Daily exposures arise from interbank deposits & loans in local and foreign currency and from credit lines extended for settlement purposes.
 - ii. In the case of deposits & loans, the calculation of exposures is straightforward. We are only concerned with the quantification of the loss-given-default of a counterparty, so maturity and funding risk are not relevant.
 - iii. The exposures are calculated by adding up all deposits & loans between bank i and j. We calculate the gross exposure instead of net exposure.

Quantification of systemic risk in multilayer networks

2) Security cross-holdings:

- i. Daily exposures also arise from cross-holding of securities between banks, securities lending, securities used as collateral, and securities trading.
- ii. Cross-holding of securities between banks means that bank j holds securities issued by bank i .
- iii. We use the gross exposure because security contracts must be honored, even when the counterparty defaults.
- iv. The daily cross-holdings gross exposures are calculated by adding up all cross-holdings of securities that exist between bank i and j .

Quantification of systemic risk in multilayer networks

3) Derivatives:

- i. Daily exposures arise from the valuation of derivatives transactions, including swaps, forwards, options, and repo transactions.
- ii. For the derivatives layer, for each type of derivative contract (swaps, forwards or options) between any two given banks, the contract is valued and the resulting net exposure (at the contract level) is then calculated and assigned to the corresponding bank.
- iii. Options with the same underlying security are added up on each side and the exposures are then assigned to the counterparty with a positive net position. This process is replicated for each type of derivative with the same underlying security.
- iv. The resulting net exposures are then added up to calculate the final exposure arising from derivative contracts between bank i and bank j .

Quantification of systemic risk in multilayer networks

4) Foreign exchange:

- i. As far as foreign exchange (FX) transactions are concerned, exposures reflect settlement risk (or Herstatt risk; the risk that a counterparty will not pay as obligated at the time of settlement).
- ii. Mexican banks that are subsidiaries of internationally active banks are members of CLS (Continuous Linked Settlement) and are in a position to settle their FX transactions in a secured way.
- iii. However, not all active banks in Mexico are in this situation and large exposures related to FX transactions do arise. If banks settle FX transactions between themselves by using the clearance service provided by CLS – which eliminates time differences in settlement –there is no exposure. Otherwise the exposure includes both foreign currency receivable and foreign currency payable between bank i and bank j.

Quantification of systemic risk in multilayer networks

- Banking multi-layer network of Mexico on 30 September 2013.

(a) Network of exposures from derivatives,

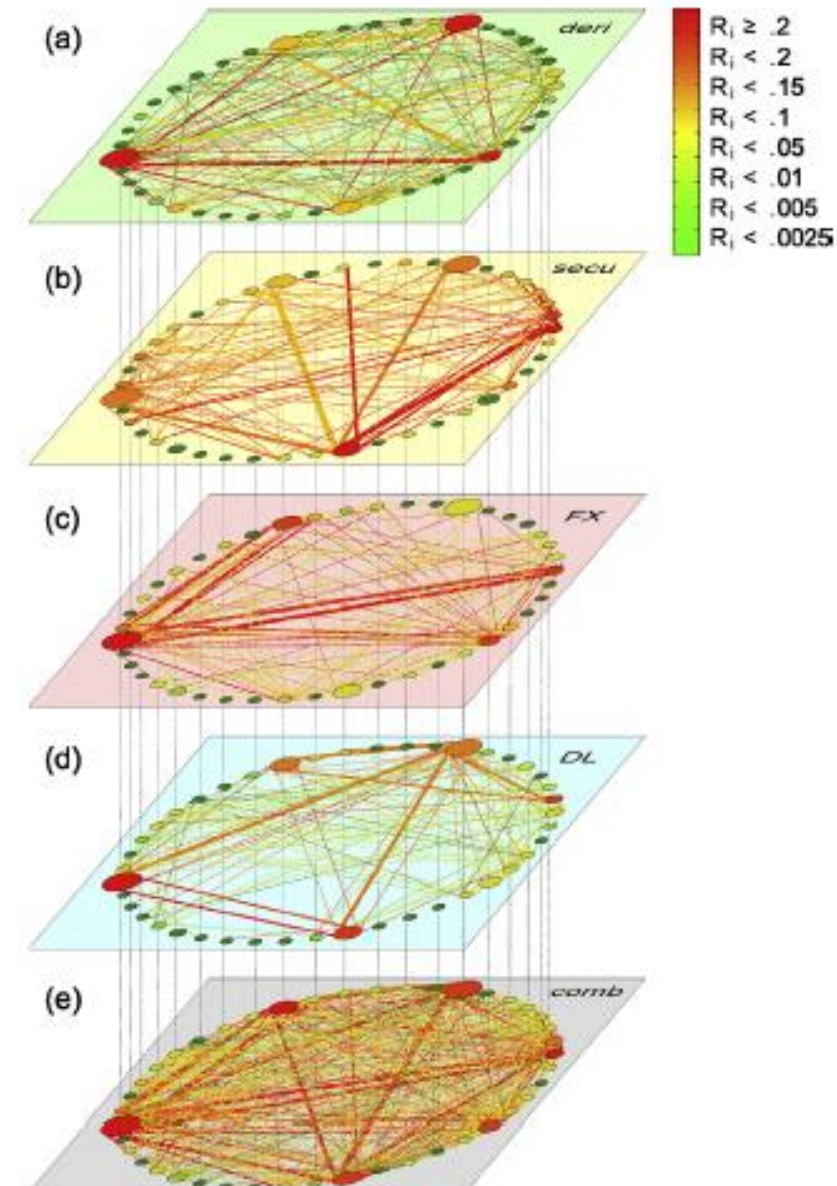
(b) security cross-holdings,

(c) foreign exchange exposures,

(d) deposits & loans and

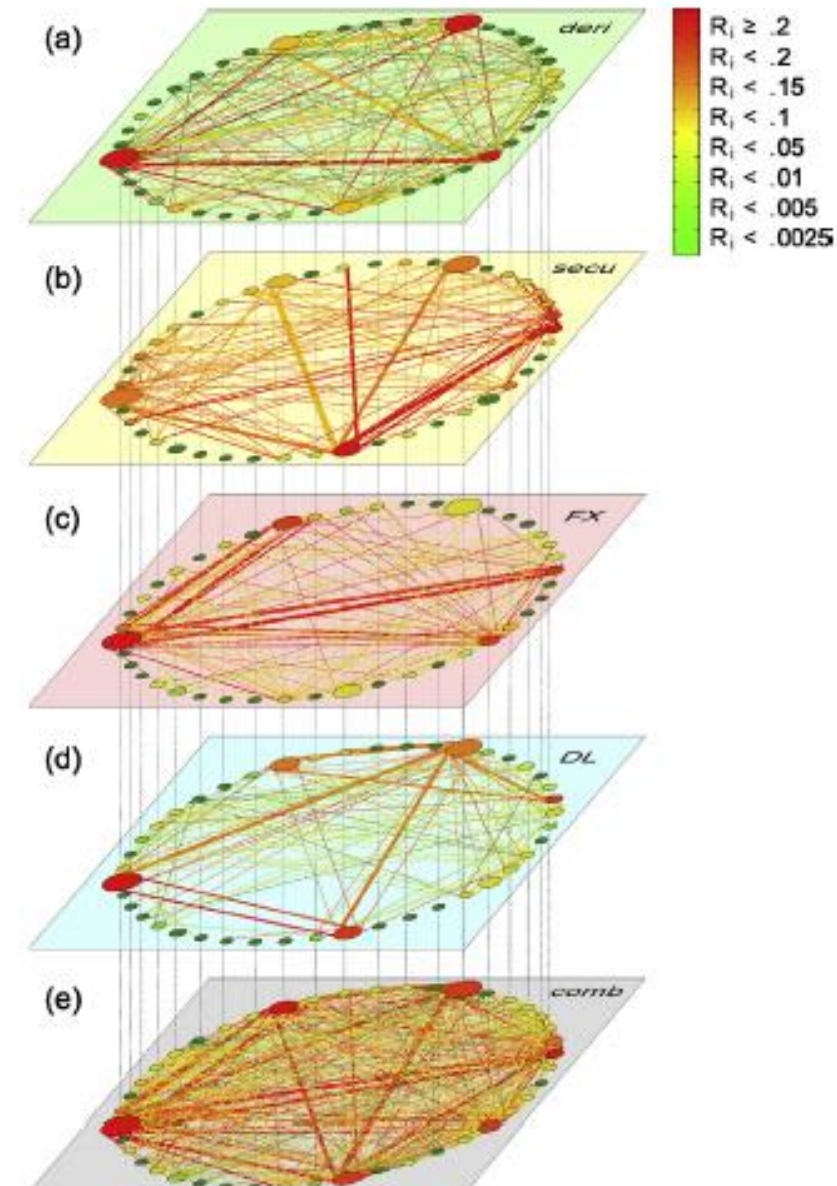
(e) combined banking network.

- Nodes (banks) are colored according to their systemic impact in the respective layer: from systemically important banks (red) to systemically safe (green).
- Node size represents banks' total assets.
- Link width is the exposure size between banks, link color is taken from the counterparty.



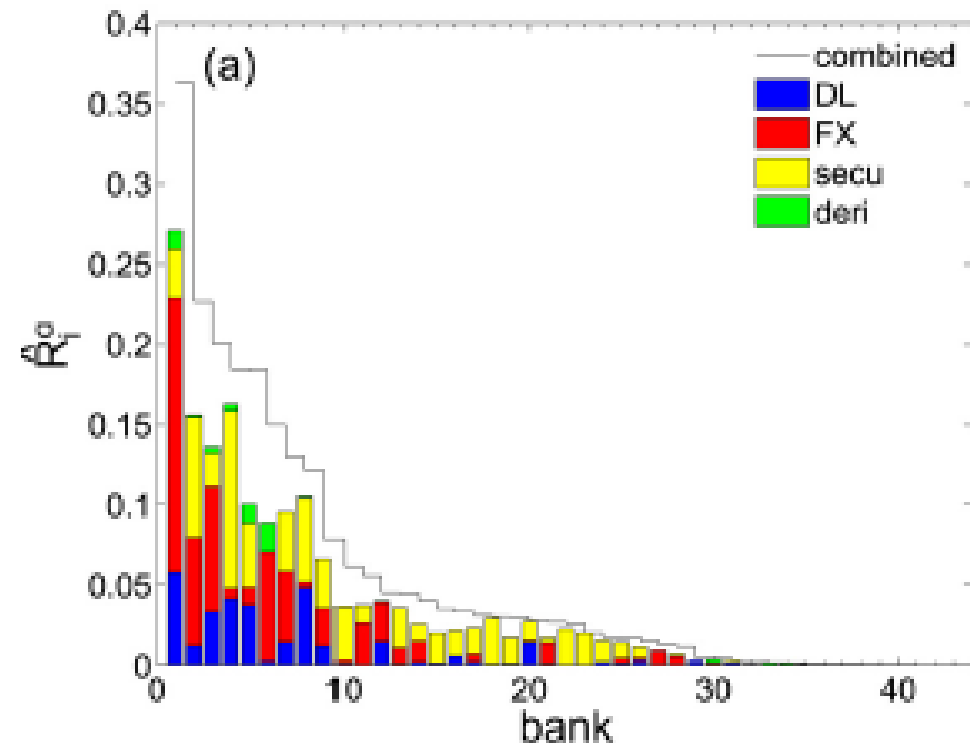
Quantification of systemic risk in multilayer networks

- Nodes i are colored according to their systemic impact, as measured by the DebtRank, in the respective layer.
- Systemically important banks are red and unimportant ones green.
- The width of links represents the size of the exposures in the layer; link color is the same as the counterparty's node color (DebtRank).
- Note that the data for derivative exposures also contains exposures from so-called repo transactions; the respective amounts are small (less than 2%) because the repo involves collateral.



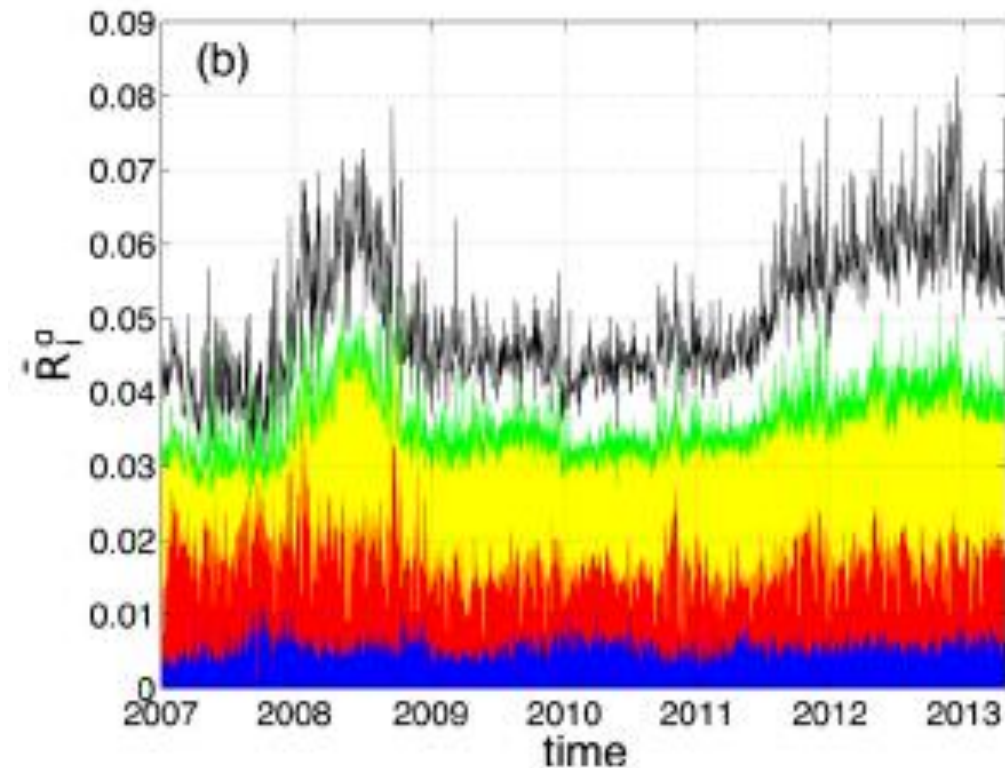
Quantification of systemic risk in multilayer networks

- This graph shows the SR-profile for the combined exposures (combined line) and stacked for different layers (colored bars) for 30 September 2013.
- Individual banks have different SR contributions from the different layers, reflecting their different trading strategies. A number of smaller banks have systemic impact in the securities market only.
- The SR contribution from the interbank (deposits & loans) and the derivative markets is clearly smaller than the contributions from the foreign exchange and securities markets. The systemic impact of the combined layers (line) is always larger than the sum of the layers separately.



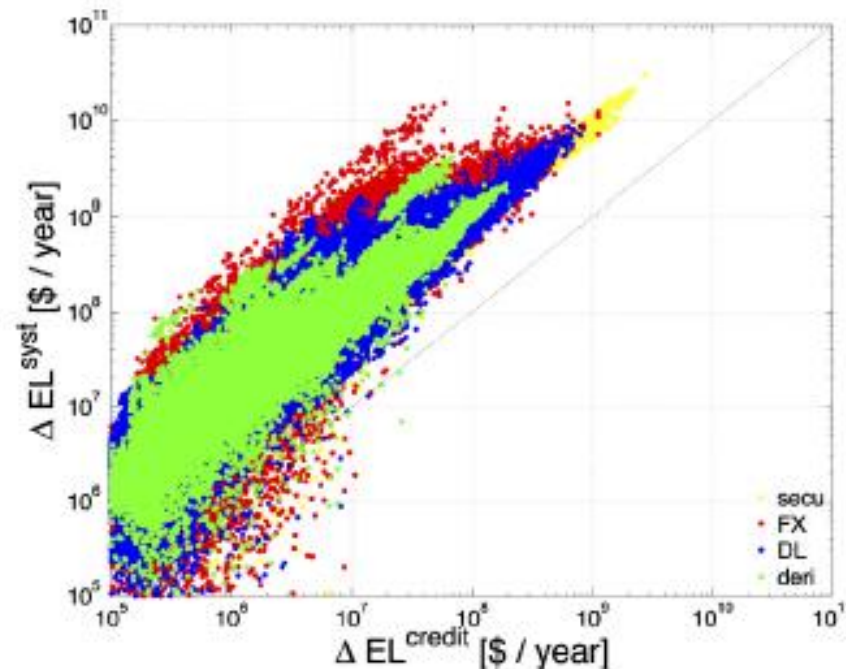
Quantification of systemic risk in multilayer networks

- The combined systemic impact is always larger than the combination of all layers separately. \bar{R} combined increases about 50% from roughly 1.7 before the financial crisis of 2007–2008 to about 2.6 in 2013. The contributions of the individual exposure types are more or less constant over time.
- The interbank (deposits& loans) and derivative markets have smaller average DebtRank contributions than foreign exchange or securities. The derivatives market gained importance in Mexico after 2009. Note the relative SR increase of securities at the beginning of the subprime crisis (Dec 2007) and the subsequent decrease shortly before the collapse of Lehman Brothers.



Quantification of systemic risk in multilayer networks

- Finally, we compare the marginal contribution of individual exposures on SR and credit risk. The different layers are distinguished by colors.
- We observe that marginal increase of expected systemic loss $>$ increase of credit risk for individual exposures between institutions. The marginal contributions from individual liabilities depend not only on the two parties involved, but also on the conditions of all nodes in the network.
- Deposits & loans and derivatives show the lowest variability, whereas for foreign exchange the variability is a bit higher. Derivatives show clusters of transactions with particularly high SR contributions for the corresponding liability size. Exposures from security cross-holdings have the highest contributions to SR.



Quantification of systemic risk from overlapping portfolios*

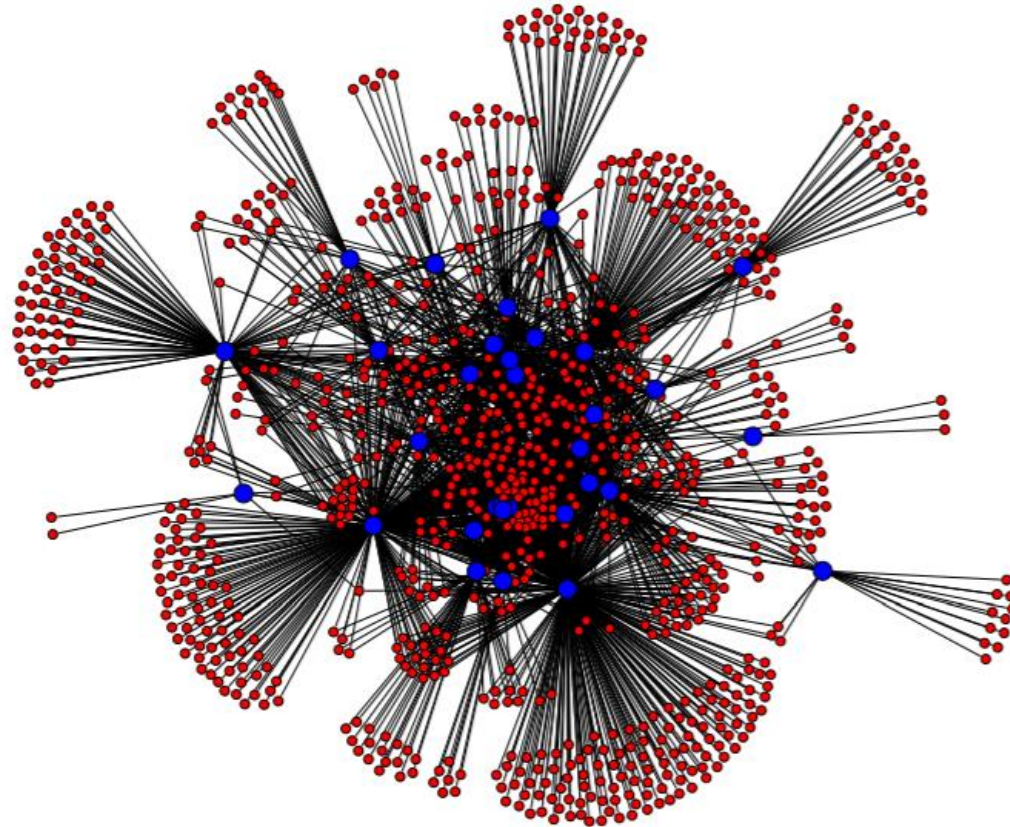
**Poledna, S., Martinez-Jaramillo, S., Caccioli, F., Thurner, S., 2019. "Quantification of systemic risk from overlapping portfolios in the financial system." To be published, Journal of Financial Stability*

Methodology

- DebtRank is a recursive method* that quantifies the systemic importance of financial institutions in terms of losses that they would contribute to the total loss in a crisis.
- We use a novel method to quantify the expected loss due to SR from overlapping portfolios (indirect exposures), where the loss for bank i is because the default of bank j causes the liquidation of j 's portfolios causing the devaluation of i 's common assets with j
 - Bipartite networks of financial institutions and securities.
 - Compare SR from direct interbank exposures (default contagion) and indirect external exposures (overlapping portfolios).
 - Compare marginal contributions of individual direct and indirect exposures to the overall SR.

Banks-assets bipartite network

- Nodes in the network represent banks (blue) and assets (red). Links the holding of an the asset by a bank.
- There are some banks that have independent portfolios or are even isolated. Also, there is an important degree of overlapping, the red nodes at the center of the plot; many banks are exposed to the same securities.



Assumptions

- Linear market impact associated with the bank liquidating its position. Financial institutions liquidate their portfolios proportional to the relative loss of equity.
- Banks do not change the composition of their portfolios as they liquidate.
- Each bank knows the value of the capital of its counterparties at each step of the dynamic (mark-to-market valuation).
- The multilayer network consists of two layers: direct exposures and indirect exposures.
 - Direct exposures: deposits & loans, derivatives, cross holdings of securities, foreign exchange.
 - Indirect exposures result from overlapping portfolios.

DebtRank I

- DebtRank is a recursive method suggested in Battiston et al. (2012) to determine the systemic importance of nodes in financial networks.
- It is a number measuring the fraction of the total economic value in the network that is potentially affected by the distress of a node or a set of nodes.
- The generalized version is made in Bardoscia et al. (2015)
- Adapted to the context of systemic risk
- Quantifies systemic relevance of node in financial network with economically meaningful number
- Takes capitalization/leverage of banks into account

DebtRank II

- The nodes in the exposures network are banks. A_{ij} denotes links in the network (bank's i exposure to bank j), and C_i is bank's i capital.
- We denote the total outstanding interbank exposures of bank i by $A_i = \sum_j A_{ij}$. Non interbank assets are denoted by A_i^E and liabilities by L_i^E . A bank is defaulted if $C_i \leq 0$.
- The set of active banks at time t is denoted by $\mathcal{A}(t) = \{i: C_i(t) > 0\}$
- Interbank assets are mark-to-market while liabilities keep their face value
- When a bank defaults, the recovery rate on interbank loans is 0

DebtRank III

- The shock propagation mechanism from borrowers to lenders is as follows
- Relative changes in the capital of the borrowers are reflected by relative changes on the interbank assets of the lenders:

$$A_{ij}(t+1) = \begin{cases} A_{ij}(t) \frac{C_j(t)}{C_j(t-1)} & \text{if } j \in \mathcal{A}(t-1) \\ A_{ij}(t) = 0 & \text{if } j \notin \mathcal{A}(t-1) \end{cases}$$

- The case $j \notin \mathcal{A}(t-1)$ ensures that, once bank j defaults, the corresponding interbank assets A_{ij} of its creditors will remain zero for the rest of the evolution
- We denote by $h_i(t) = (C_i(0) - C_i(t))/C_i(0)$ the relative loss of capital between iterations 0 and t . By iterating in the balance sheet identity, the contagion dynamics can be written as:

$$h_i(t+1) = \min \left[1, h_i(t) + \sum_{j=1}^N \Lambda_{ij}(t) [h_j(t) - h_j(t-1)] \right]$$
$$\Lambda_{ij}(t) = \begin{cases} \frac{A_{ij}(0)}{C_j(0)} & \text{if } j \in \mathcal{A}(t-1) \\ 0 & \text{if } j \notin \mathcal{A}(t-1) \end{cases}$$

Methodology

- The marginal SR of an individual exposure on Expected Systemic Loss is expressed as the difference of total expected systemic loss:

$$\Delta EL^{\text{sys}} \Big|_{\Delta X_{kl}} = \sum_{i=1}^b p_i [V(X_{ij} + \Delta X_{kl}) R_i(X_{ij} + \Delta X_{kl}, C_i) - V(X_{ij}) R_i(X_{ij}, C_i)]$$

$R_i(X_{ij} + \Delta X_{kl}, C_i)$ is the DebtRank

$V(X_{ij} + \Delta X_{kl})$ is the total economic value of the exposure network

ΔX_{kl} is the matrix with precisely one nonzero element for the exposure between k and l

A positive ΔEL^{sys} means that the change in exposure ΔX_{kl} increases total SR.

Price Impact Function: Assumptions

- To compute this potential loss, we need to compute the impact of bank j on the value of each asset a , and then the importance of asset a for bank i :

Let us consider a network of b banks and m assets, and let us denote its equity by C_i , the number of shares of asset a owned by bank i by S_{ia} , the total number of outstanding shares of asset a by N_a , and the price of asset a by p_a respectively.

We assume the impact of bank j on asset a is proportional to the fraction of shares owned by the bank.

As a measure of the direct impact of banks on assets we define the matrix:

$$W'_{ja} = \frac{p_a S_{ja}}{N_a}$$

Price Impact Function: Assumptions

- The underlying assumption here is that of a linear market impact associated with the bank liquidating its position on the asset: Should the bank liquidate its entire position; the price would shift from p_a to $p_a(1 - (\frac{S_{ja}}{N_a}))$.
- The importance of asset a for bank i is simply given by the number of shares i owns of asset a . Therefore, we define the indirect exposure of bank i to bank j from overlapping portfolios as (Guo et al., 2016; Schaanning, 2017).

$$X_{ij}^{OP} = \sum_a W'_{ja} S_{ia} = \sum_a \frac{p_a S_{ia} S_{ja}}{N_a}$$

Price Impact Function: Assumptions

- ❑ X_{ij}^{OP} is the appropriately weighted bank projection of the weighted bipartite network of banks and assets S_{ia} , so that the dynamic above is equivalent to the standard DebtRank on the projected network of overlapping portfolios.
- ❑ The matrix X_{ij}^{OP} is symmetrical, and its diagonal elements are non-zero even though the bipartite network itself has, by definition, no self-loops.
- ❑ Diagonal elements represent the self-inflicted loss of a bank from (rapidly) liquidating its portfolio (market impact). This loss will be high if bank i holds a large fraction of asset a in its portfolio, and is negligible if i holds only a small fraction of asset a .
- ❑ We assume that a bank liquidates a fraction of its portfolio proportional to its relative loss of equity. Our choice of proportional liquidation is a simplifying assumption that provides the smallest departure from the DebtRank algorithm, and allows us to use the DebtRank algorithm on the projected network of overlapping portfolios.

Price Impact Function: Assumptions

- ❑ We assume an implicit 0% recovery rate. This implies that our measure of SR is more conservative with respect to one that would be obtained by considering a non-zero recovery rate.
- ❑ A second assumption is that banks do not change the composition of their portfolios as they liquidate. This is a common assumption in the literature on fire-sales (Huang et al., 2013; Greenwood et al., 2015; Cont and Schaanning, 2017), and it has recently been shown (Schaanning, 2017) to be a good approximation of the behavior of large banks.
- ❑ A further assumption we make is that each bank knows the value of the equity of its counterparties at each step of the dynamics. This is required because DebtRank assumes banks to compute the value of their interbank assets using an ex-ante mark-to-market valuation, according to which the value of an interbank asset depends on the value of the capital of the borrower (Battiston et al., 2012; Bardoscia et al., 2015; Barucca et al., 2016; Roncoroni et al., 2019).

Price Impact Function: Assumptions

To consider contagion from asset liquidation we calculate the DebtRank of the indirect exposure network X_{ij}^{OP} ,

$$R_i^{OP} := R_i^{OP}(X_{ij}^{OP}, C_i, v_i^{OP})$$

where C_i is i 's capital and v_i^{OP} i 's economic value. Given the current value of assets a in i 's investment portfolio, we define its economic value as:

$$v_i^{OP} = \frac{\sum_a p_a S_{ia}}{\sum_j \sum_a p_a S_{ja}}$$

i.e. the fraction of i 's investment portfolio from the total investment portfolios of all banks.

R_i^{OP} measures the fraction of the total economic value ($V^{OP} = \sum_i \sum_a p_a S_i^a$) that is affected by the distress of a bank i from indirect exposures, i.e. from overlapping portfolios.

Data

The financial system as a (multilayer) network

- Data were collected and are owned by Banco de México, contains detailed information about various types of daily exposures between the major Mexican financial intermediaries (banks) over the period 2004-2013:
- **Securities holdings** of Mexican financial intermediaries by containing the International Securities Identification Number (ISIN) that uniquely identifies every security.
- **Capitalization** of banks at every month and the market data (prices) for the various securities.
- **Complete information about securities holdings** of major financial intermediaries and the ability to uniquely identify securities in the portfolios allows us to represent the Mexican financial system as a bipartite network of securities and financial institutions.

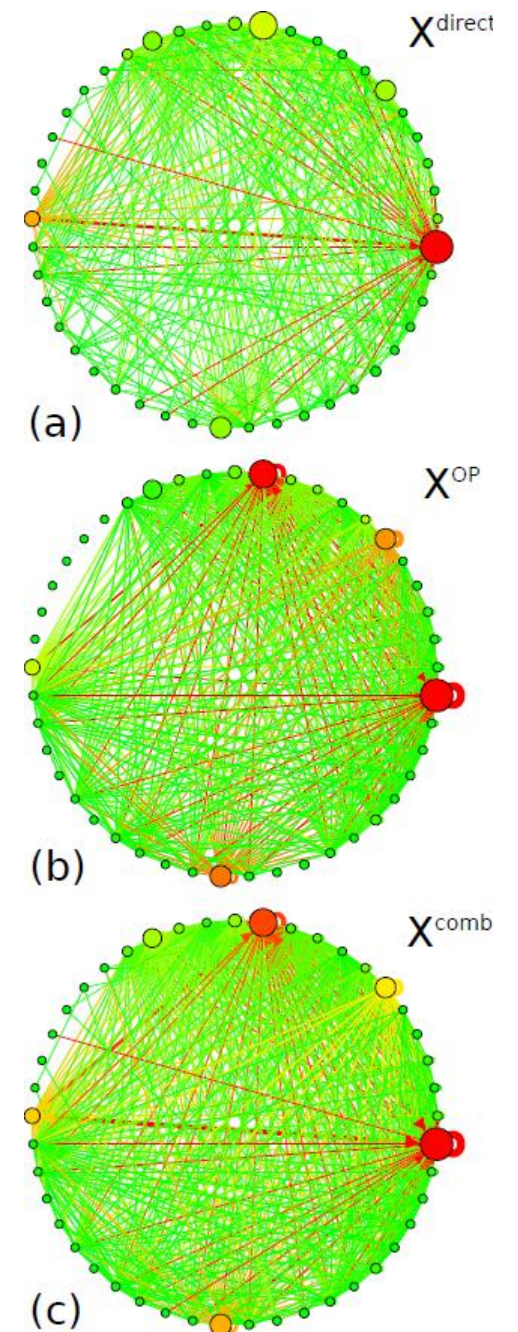
Results

Mexican multi-layer banking network

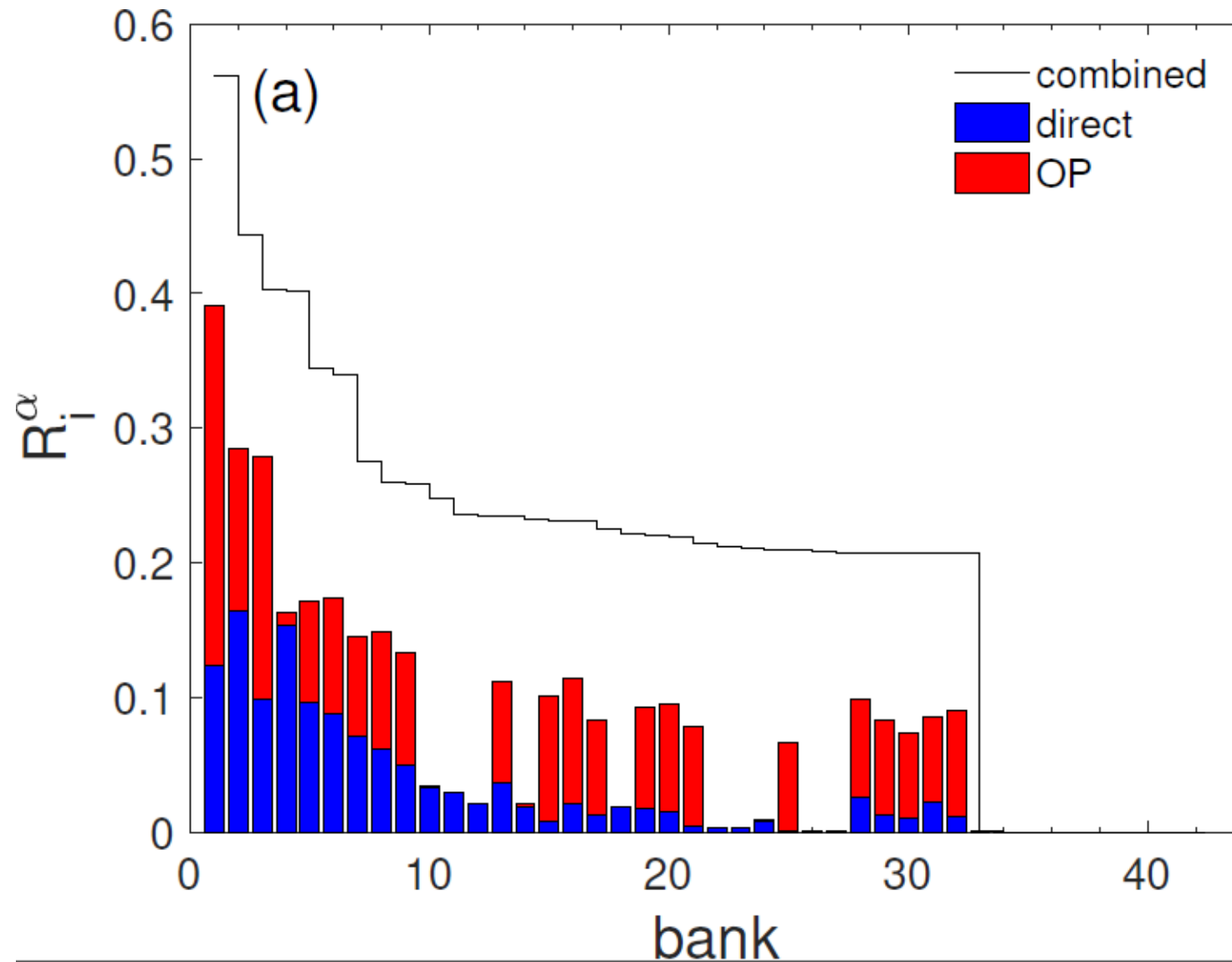
Node size represents the size of banks in terms of total assets. The important banks are red; unimportant ones are green, the width of links represents the size of the exposures in the layer, link color is the same as the counterparty's node color (DebtRank).

Diagonal elements represent the loss for a bank itself from liquidating its portfolio and are typically larger than the indirect exposure to other banks with similar portfolios. The different layers of exposure of the Mexican financial system are rather dense.

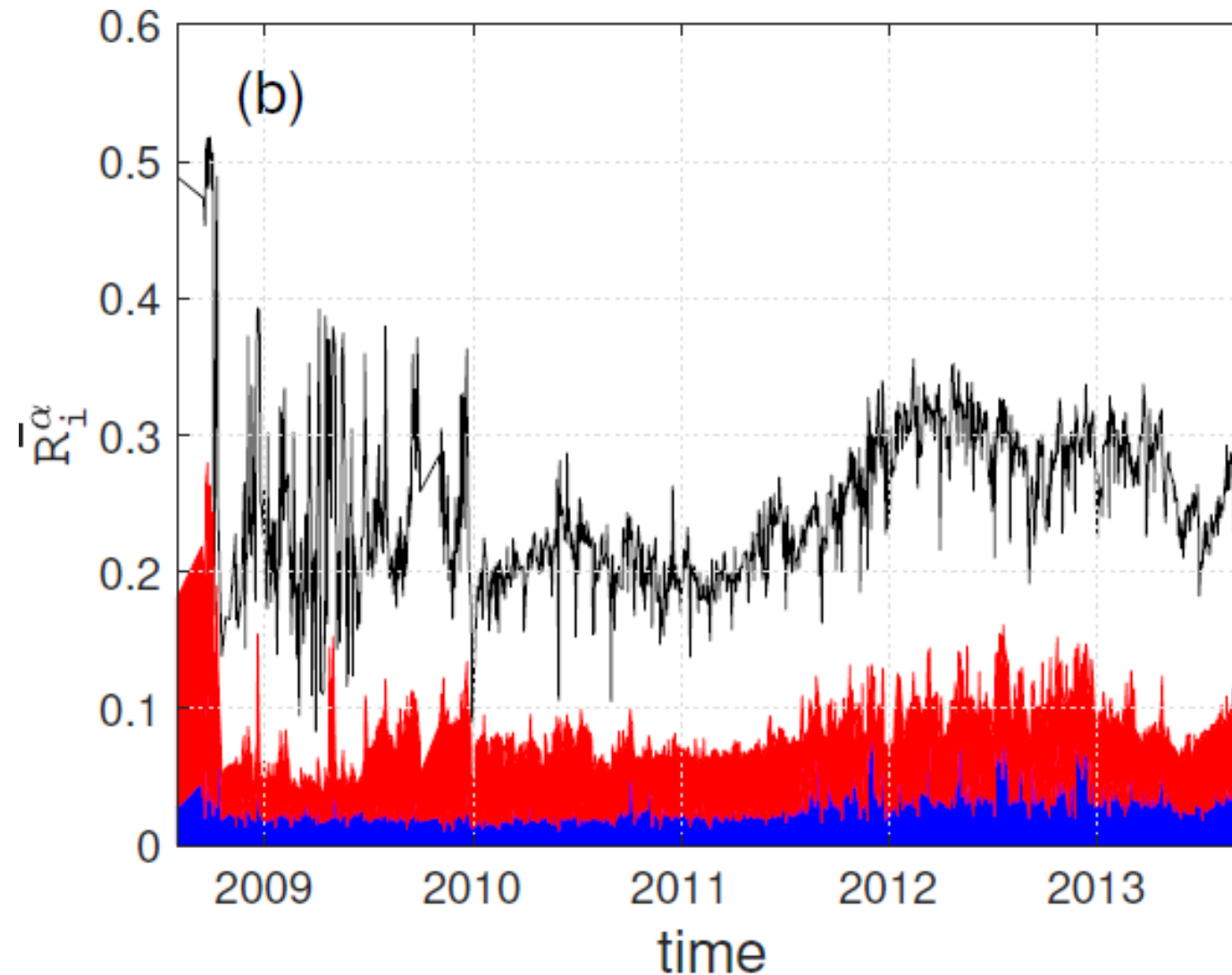
- (a) Network of direct interbank exposures. The density of this layer is 0.23.
- (b) Network of indirect external exposures from overlapping portfolios. The density of this layer is 0.43.
- (c) Combined banking network. The density of this layer is 0.49.



Systemic Risk profile for the different layers



Time series for the average DebtRank from 31 July 2008 to 30 September 2013



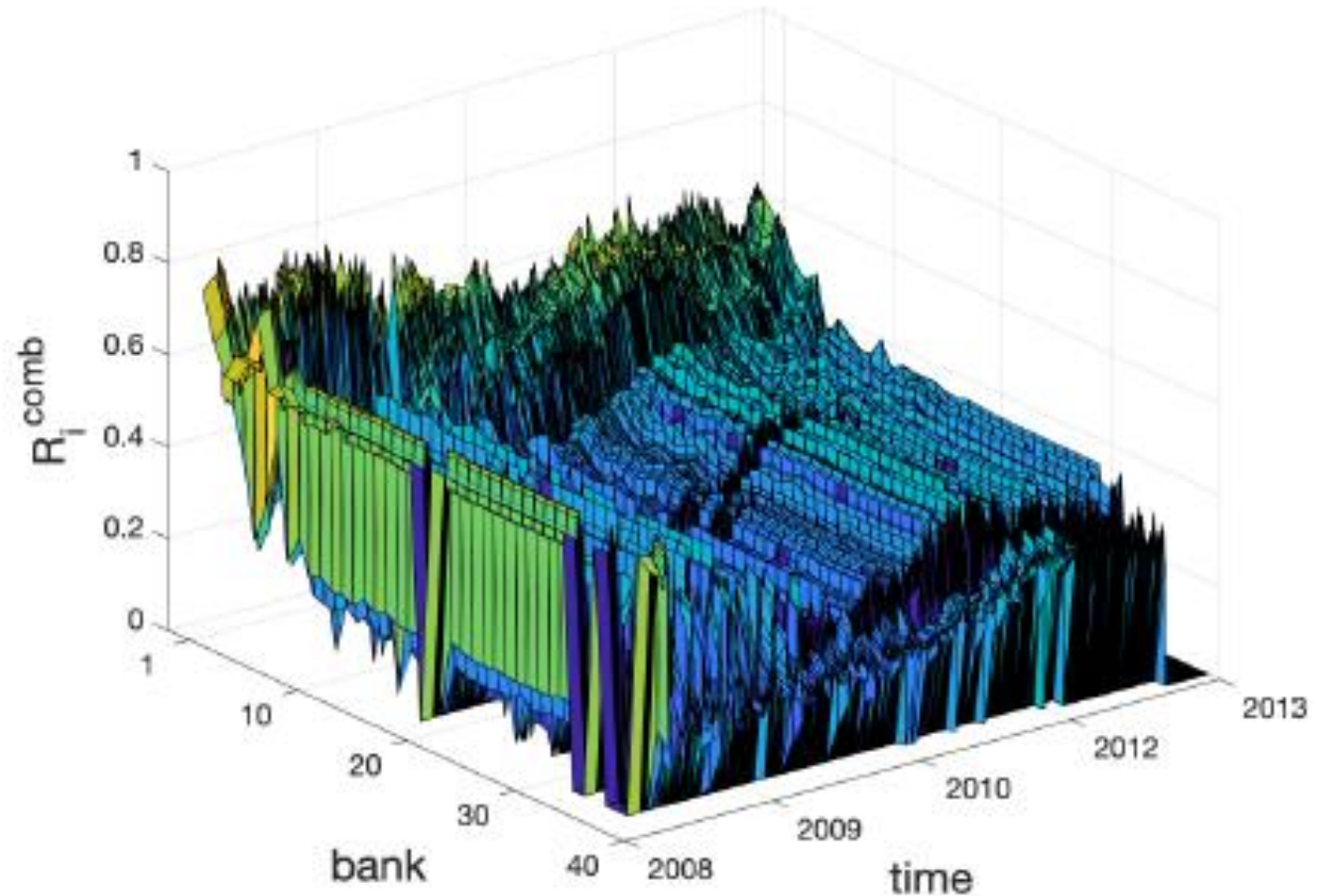
Systemic Risk surface for the combined network from all layers, from 31 July 2008 to 30 September 2013.

In this figure, we show the daily DebtRanks in the combined network from all layers for each bank from 2008 to 2013.

The most systemically important banks do not change too much over time.

Systemic Risk was higher for almost all banks at the beginning of the measurement period (2008 financial crisis).

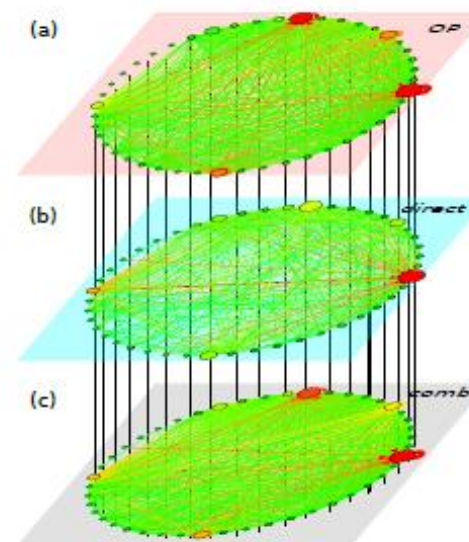
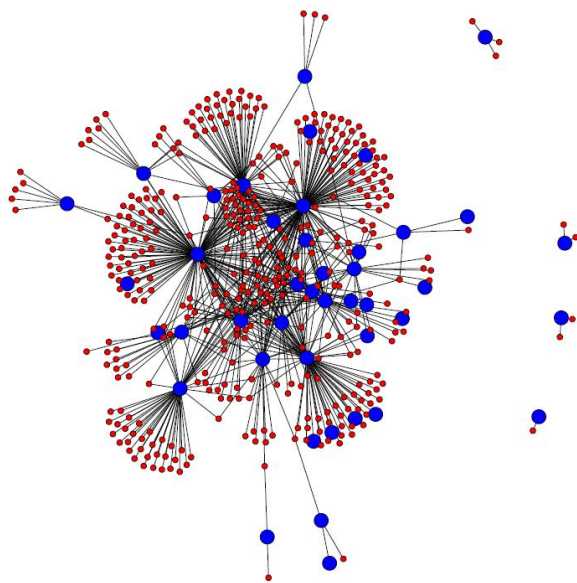
After the height of the financial crisis, there is a group of banks that are basically flat in terms of SR and over time.



Sovereign and financial risk

- Banks are exposed to systemic risk directly and indirectly
 - Propagate through different mechanisms and channels of contagion.
- Overlapping portfolios
 - Indirect interconnection. Financial institutions invest in common assets. An important source of contagion and systemic risk.
 - *Poledna et al (2019)*¹ propose a network model to quantify systemic risk from direct and indirect exposures.

Red: assets
Blue: banks



Multi-layered Mexican banking network

Conclusions

- Systemic risk (SR) arises from indirect interconnections that occur when financial institutions invest in common assets (overlapping portfolios).
- Mutual influence of different channels of contagion were represented by a financial system as a multi-layer network of direct interbank exposures (default contagion) and indirect external exposures (overlapping portfolios).
- Indirect exposures represents an important form of financial contagion.
- Direct interbank exposures underestimates total systemic risk levels by up to 50 percent.
- There are many more aspects of the modeling of financial stability and systemic risk which can be tackled by using network theory and models.

Conclusions

- We find that financial markets systematically underestimate SR.
- In recent years various studies using multiplex network analysis have demonstrated that trying to understand a system from a single network layer can lead to a fundamentally wrong understanding of the entire system and that the dynamics of multiplex systems can be very different from single-layer networks.
- The multilayer analysis of financial networks points in a similar direction, namely, that there might be much higher SR levels present in the financial system than previously anticipated or than markets assume.

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