Credit risk and its systemic effects*

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Abstract

Interconnectedness among financial institutions has been recognized as one of the most important factors for the amplification of the Global Financial Crisis (GFC). Since then, many different research approaches have been developed to study systemic risk and its relationship with interconnectedness. In this work, we propose to use a systemic risk metric for an extended network which includes the interbank network, the banks-firms bipartite network and the intra-firm exposures network in Uruguay. This is one of the first works, to the best of our knowledge, in which the intra-firm exposures network is estimated with such an accuracy by using information from a firm survey and is used for the computation of a systemic risk metric. Given that the survey only asks for the three most relevant debtors and creditors, we have to complete the full intra-firm exposures matrix by resorting to two well-known methods: the Maximum entropy and the Minimum Density; additionally, we use an additional method which takes into account the known entries of the matrix obtained from the survey. Our results show an important underestimation of systemic risk if the information of intra-firm exposures is ignored. Moreover, even if the marginal liabilities or assets are used as an indicator of systemic importance for firms, important network effects are ignored. The paper has several contributions among which the most important one is the precise estimation of the contribution of intra-firm exposures to the overall systemic risk.

1 Introduction

The increasingly complex and interrelated connections in the financial system are considered to be one of the main sources of risk amplification and propagation of shocks. This was made evident in the worst possible way during the GFC after the fall of Lehman Brothers.

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These interconnections among financial entities have been modelled by resorting to network theory and models. Since then, we have been modeling financial entities and their relationships by financial networks. There is an extensive literature on the structure of these networks and the effects of these structures on the propagation of both microeconomics and macroeconomic shocks, see Battiston and Martinez-Jaramillo (2018a) and Martinez-Jaramillo et al. (2019) for an introduction.

Nevertheless, contagion through commercial indebtedness among firms or economic sectors has deserved less attention, Acemoglu et al. (2016), mainly due to an important lack of information. Fortunately, this has been changing very recently and now, it is possible to find some works that include the real sector of the economy that is related with the banking system: Poledna et al. (2018) and T. C. Silva et al. (2018).

This work aims to contribute in filling that gap by building a commercial and financial debt network for Uruguay resorting to a survey on firms which includes questions on the main debtors and creditors for each firm that participates in the survey (see Baron et al. (2020)). Additionally, the links to the banking system are also known from the credit registry. Finally, the interbank exposures network is also used in this work. Uruguay has a small interbank market. As a consequence. in the stress testing exercises, with the information at hand, it is commonly found that contagion through the interbank market is low.

There is some evidence about the effect that the default by firms may have on banks in the case of Uruguay:

- Directly, through financial credit it is quite well understood.
- Indirectly, through common assets holdings, is not studied due to lack of information but it is perceived to be small.
- Indirectly, through commercial credit linkages, there is less empirical evidence but has been recently explored in Baron et al. (2020).

In this paper, we want to provide an empirical quantification on the latter type of effects. For this purpose we are using information from a survey applied to commercial firms. The central bank of Uruguay conducts a survey on commercial debt to a sample of firms which:

- Representative of the universe of firms with more than 50 employees.
- Does not include firms belonging to the primary activity sector, financial intermemdiation, public sector or real state activities.
- For these sectors connections are inferred.

Here, we combine this information with balance sheet and credit registry data to build a commercial and financial debt network. In doing so, we also provide a series of measures of interconnectedness and topology of the networks and we produce a systemic risk metric for the banking system and the firms with direct links to this system.

As a result we identify the most central firms in terms of commercial debt, and the most central banks in the network. We also quantify the exposure of banks to credit risk originated in firms. More importantly, we accomplish our final goal: Perform a stress test exercise consisting in the propagation of a default shock in order to analyze the vulnerability of the network and the systemic impact of the individual participants.

Nevertheless, the full structure of intrafirm exposures is not known. The sample of firms reporting in the survey only report the total amount lent and borrowed and the three most important debtors and creditors. In addition, the firms also report the total number of debtors and creditors. Given this rich but partial information, we have to estimate the full matrix of intrafirm exposures. For this purpose we will resort to two well-known estimation methods: the maximum entropy, Upper and Worms (2004) and the minimum density, Anand et al. (2014). Additionally, we resort to a method which also takes into account the known entries of the intra-firm exposures matrix which uses first the fitness model proposed in Musmeci et al. (2013a) and then uses a constrained version of the RAS algorithm to distribute the remaining amount for which the counterparties are not known.

We will perform a similar analysis to Poledna et al. (2018), the main difference is that the intrafirm exposures network is not estimated from balance sheet data but it is estimated by using partial information from the firm survey. Given this additional piece of information we are able to estimate how much the intrafrim exposures network contributes to overall systemic risk. We conduct this quantification for the three different estimation methods of the intrafirm exposures network.

The contributions of this paper are several: first, we estimate the intrafirm exposures network by resorting to three different methods; second, we estimate the contribution to systemic risk of the information contained on the intrafirm exposures network; third, we are able to identify systemically important firms on the basis of their impact of their failure on banks and other firms taking into account contagion (network) effects; fourth, we compute the DebtRank centrality metric for both, the interbank and for an extended network of exposures, including firms; and fifth, we measure the importance of effective exposures from banks to firms taking into account the exposures relationships among firms.

The remaining part of the paper is organized as follows: in Section 2 we place our work in the broader context of financial networks and systemic risk measurement. Section 3 describes all the different data sources used in this research project. Section 5 presents the results; finally, Section 6 concludes.

2 Literature Review

Interconnectdness has been recognized as one key element in the GFC; interconnectedness per se is not necessarily harmful, Martinez-Jaramillo et al. (2019), however under certain circumstances it can work as an amplifier of some negative shocks to the system, as it was the case with the GFC.

The effects of micro and macroeconomic shocks are related to the topology of the network. Elliott et al. (2014) find that integration (greater dependence per agent or node), and diversification (more counterparties per agent or node) have different, non monotonic effects on the extent of cascades.

Diversification connects the network initially, permitting cascades to travel; but as it increases further, agents or nodes are better insured against other participant's failures. Integration also faces trade-offs: increased dependence on other organizations versus less sensitivity to own investments.

Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) relate shock magnitude and network structures. According to their results, if negative shocks affecting financial institution are sufficiently small, then a more densely connected financial network enhances financial stability. However, when shocks are large enough, a more dense network serves as a mechanism for the propagation of shocks, leading to a more fragile financial system.

There is a strand of theoretical and empirical literature that studies contagion in the banking system or the financial system at large (Souza et al. (2014); T. Silva et al. (2016); Battiston and Martinez-Jaramillo (2018b); Calomiris et al. (2019)). Because the information on financial interconnections is usually non public and only partial information is available on the total debt between financial institutions (aggregate positions), a series of methodologies have been developed to complete the interconnection matrix between financial institutions.

Anand et al. (2018), conduct a horse race of network reconstruction methods using network data obtained from 25 different markets spanning 13 jurisdictions. They consider seven reconstruction methods, and find that the best methods depends on the final purpose of the network reconstructed.

In this paper we use three of the methods proposed by Anand et al. (2018) to reconstruct the firm to firm network. The first method is the minimum density introduced my Anand et al. (2014). This reconstruction method minimizes the number of links necessary to distribute a given amount of loans. The second method is the maximum entropy reconstruction method proposed in Upper and Worms (2004). Opposite to the minimum density reconstruction method this one will result in a more dense interconnection matrix.

Additionally, we use the fitness model based on Musmeci et al. (2013b) which is a novel model to reconstruct global topological properties, in particular, of the intrafirm network from known information (information from the survey and credit registry databases). This methodology uses partial information and an auxiliary non-topological property, which is interpreted as the fitness associated to each node.

Research on the effects of contagion through commercial indebtedness among firms, industries and economic sectors has deserved less attention (see, for instance, Acemoglu, Akcigit, and Kerr (2015)). This has changed in recent years where we can find some empirical and theoretical work that studies the interconnection between the real sector and the financial system mainly through bipartite networks between banks and firms.

Lux (2014) proposes a model of a bipartite network between banks and the non-bank corporate sector and concludes that contagion due to joint exposures to counterparty risk via loans to firms is more important for contagious spread of defaults than the interbank credit channel.

De Masi et al. (2009) and De Masi and Gallegati (2007) analyzes the bank-firm network in Italy and Japan. Poledna et al. (2018) reconstruct and analyze the financial liability network combining firm-bank network and interbank network in the Austrian banking system. They find that all firms together create more systemic risk than the entire financial sector.

Risk imposed by firms to the financial system seems to be important according to the results in the most recent empirical work. This risk is not considered when the analysis of the network is performed only in the interbank network.

Aoyama et al. (2013) analyzes the bipartite credit network of the lending/borrowing relationship between banks and firms in Japan. They performed an stress test exercise, introducing distress to firms and banks nodes and evaluating the propagation through the network.

Baron et al. (2020) build a commercial and financial debt network at the sector level for Uruguay using the same data set that the one used in this paper. They provide a series of measures of the indebtedness structure, and identify the most central sectors in terms of commercial debt, as well as the most central banks in the network. They also perform a stress test consisting in the propagation of a default shock to analyze the vulnerability of the network.

Our work includes a new layer to this analysis and considers the firm to firm network. To our knowledge is the first work that includes simultaneously banks-firms network, interbank network and intrafirms network at an institution level of granularity.

3 Data

In this section we present the details of the data used to conduct the systemic risk evaluation for the Financial System in Uruguay. One of the most novel data sets used in this paper consists of a firm level survey conducted to 240 firms by the Central Bank of Uruguay in October 2018 with information about:

- The amount of commercial debts and sales credit.
- The three main debtors and creditors for each firm.
- Sectorial information of firms, debtors and creditors.

The survey ask a sample of 240 Uruguayan firms, which is representative of the universe of firms with more than 50 employees, questions regarding their commercial debts and credits. With this information we can build a network of commercial debt between firms that is representative of firms with more than 50 employees.

From the survey we not only have information on the amount of commercial debts and credits, but also we are able to identify each firm three major debtors and creditors. We can also identify the sector of the economy to which each firm belongs to, allowing us to aggregate the results at the sectoral level.

The survey does not include firms belonging to the primary activity sector, financial intermediation, public sector or real state activities. Hence, the information about the connections with these sectors is incomplete, and connections are inferred from the answers of other firms that declare to have debt or been creditor of some firm from these activities.

We also use balance sheet data from the last Central Bank economic activity survey from 2015. To update balance sheet information until October 2018 we use the consumer price index.

Another important data set is the Central Bank Credit Register database containing all the loans given to firms by banks, this data set allows us to identify all the credit lent by financial institutions to companies and construct the bank-firm network.

As a result of the combination of these data set we obtain three networks.

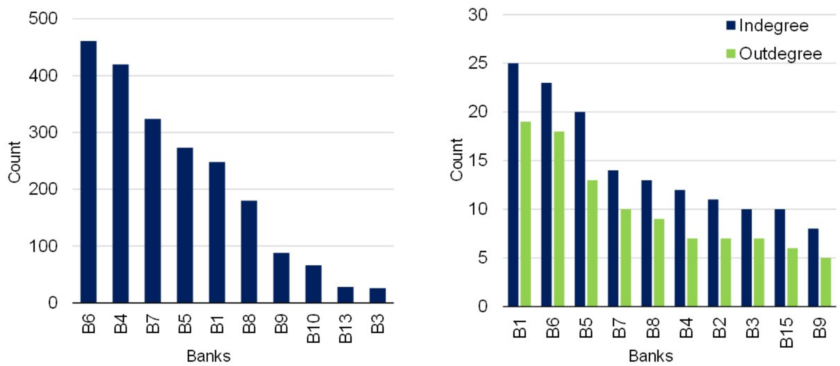
Available data	
Banks	11
Other financial institutions	15
Survey firms	240
Survey firms + main creditors + main debtors	1073
Firms with bank credit	613

Table 1: Banks-Firms Network

- Firm-Bank network: in Uruguay there are in total 11 banks but one of them only provides mortgage credit to families. With the data we can build the network of credit lend by banks to firms. The total number of firms considered includes the 240 surveyed plus the three main debtors and creditors declared by those firms. The final network has 1073 firms and 10 banks.

- Financial institutions network: We also build a network for interbank loans with data provided by the Superintendency of Financial Services from October 2008. In this network we consider the loans and derivative expositions between banks and other financial institutions. The network has in total 26 institutions, 15 that are other financial institutions in the Uruguayan Financial System and 11 are banks. 10 of these banks lends to firms while there is one bank that provides mainly mortgage credit to families. According to the results presented in Table 1 57% of the firms in the network have credit from the banking system.

- Firm-Firm network: These network is constructed considering commercial lending between the 240 surveyed firms and their three major creditors and debtors. In total there are 1073 firms in this layer.



(a) Degree banks-firms

(b) Degree banks-financial institutions

Figure 1: Financial institutions' degree

4 Methodology

In this section, we will describe some basic concepts related to graphs and network. This concepts will be the building blocks for the rest of the paper. We will follow a similar notation used in Martinez-Jaramillo et al. (2014).

We define a graph as an ordered pair $G = (V, E)$ where V refers to the set of vertices and E is the set of edges, which are unordered pairs (simple graph or undirected graph) from V.

A directed graph or digraph is a graph $D = (N, A)$ where N refers to the set of nodes and A is the set of arcs, those are ordered pairs from N. For directed graphs the direction is relevant.

For the above defined mathematical objects there are two additional object known as the undirected weighted graph and the directed weighted graph. For these objects the important additional element is the function weight, $w : R \rightarrow E$. This function assigns a number to each edge or arc.

In the rest of the paper we adopt the convention of calling networks to the weighted versions of graphs and digraphs.

The undirected networks are represented as $N = (V, E, w)$, where N refers to the network, V refers to the vertices or nodes and is a finite set different from zero. E are the unordered pairs set and w as we defined above, refers to the weight assigned to each edge or arc. (V, E) is an undirected graph.

A common mathematical representation of the networks comes in the form of a matrix.

In mathematical literature, there are two ways to describe an adjacency matrix. The first one is defining through a order list of arcs (i, j). For an undirected graph, the adjacency matrix is defined as

$$Aij = \begin{cases} 1 & \text{if } (i,j) \in E \; or \; (j,i) \in E, \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

We could take directions into account and define the following two matrices A^+_{ij} and A^-_{ij} which can be useful to perform some computations related to in and out degree.

$$A^+_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E, \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$A^-_{ij} = \begin{cases} 1 & \text{if } (j,i) \in E, \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Neighbour of a node i refers to the existence of an edge that connects the nodes. In particular, from the adjacency matrix we define the neighbours of a node i. For an undirected graph $G = (V, E)$, the vertex $j \in V$ is neighbour from the node $i \in V$ if there is an edge that connects them.

$$N(i) = (j \in V : a_{ij} = 1)$$

A weighted matrix represents a weighted networks. In the financial context, the weight of the arcs in the directed networks represent money flows, exposures, correlations, transaction values, etc.

We define a external weight matrix W^+ from a directed network $R = (N, A, w)$ as:

$$W^+_{ij} = \begin{cases} w^+_{ij} & \text{if } (i,j) \in A, \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

The internal weight matrix W^- is defined as:

$$W^-_{ij} = \begin{cases} w^-_{ij} & \text{if } (j,i) \in A, \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

The interpretation of money flow used in financial networks depends on the matrix where money belongs. For instance, to calculate losses from contagion in an exposure network, (i, j) from matrix W^+ refers to the quantity of money that institution i is exposed to institution j, in other words, the amount of money that j owes to i. For W^- is the same interpretation, but in the other direction.

We can define W=$W^+ + W^-$ as total weight matrix that is the connection between both institutions.

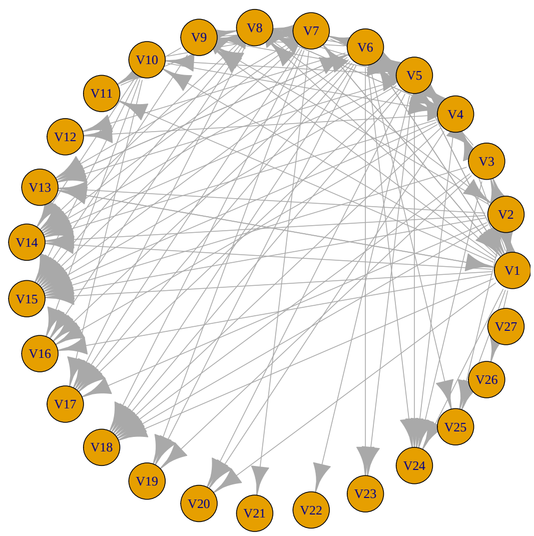
Figure 2 shows the network representation of the interbank exposures network

4.1 Network metrics

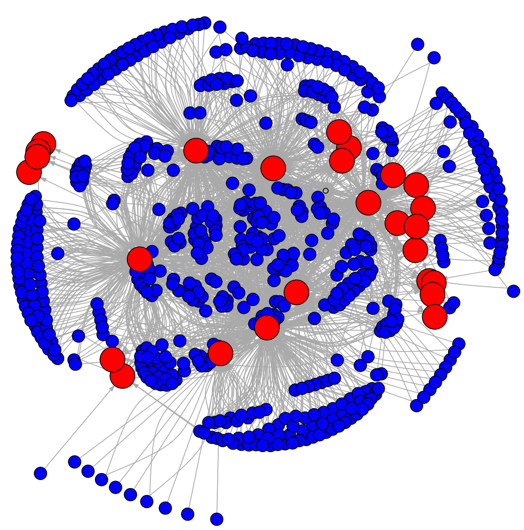
In order to characterize the network and identify the nodes (banks) that are more central we use conventional measures of centrality.

As our interest is in the bank credit risk exposition we also compare the differences in this measures when considering the Bank to Bank network and the Firm and Bank network (inferred).

Figure 2: Exposures Networks



(a) Interbank exposures



(b) Bank-firm exposures

T. C. Silva et al. (2016) divided the structural measures in three categories: local,quasi-locals and global. Local measures refer and describe only the node where these measures belong. They do not take into account either neighbors nodes or the rest of the nodes in the network. Quasi-locals measures refer describe only the node where these measures belong, but they take into account neighbours characteristics of each node to provide information. This measure is relevant to identify the nodes which a node is connected with and that bring useful information which complement with local measures. Global measures use global information from a network and is useful to identify the characteristics of the network as a system, not only for the individual nodes.

4.1.1 Centrality

The main objective of centrality metrics is to identify the most important or central node in the network. The measures differ in the concept of what we mean by important or central.

Borgatti and Everett (2006) develops a framework for the measurement of centrality. This assess the nodes involved in the walk structure of a network. In fact, they established four key dimensions: type of node, type of walk, property of walk and choice of summary measure. The authors defined a walk from node u to node v as "a sequence of adjacent nodes that begins with u and ends with v".

Centrality measures are an important tool for rank nodes according to their relevance in the network. The larger the centrality measure the greater importance a node has in a network (Martinez-Jaramillo et al., 2014). Some of those measures are describe below.

Degree centrality of a node defined as the number of edges attached to it. The greater degree centrality the greater number of institutions affected. Degree centrality is defined as the degree of each node.

$$C_d(i) = d_i$$

In the Firm-Bank network it's equivalent to the In-degree centrality.

In-degree centrality only considers the edges that go to node i.

Out-degree centrality only considers the edges that originate in the node i.

Banks ranking differs if we consider the interbank market or the financial credit between banks and firms. Moreover, we would like to investigate banks and firm centrality by using these three layers, including the firm to firm commercial debt layer to the multilayer network.

We have as starting point the 240 firms from the survey and the commercial debts between them if they are classified for the others as main debtor or creditor We also have the financial debt of this 240 firms.

4.2 DebtRank

DebtRank was introduced by Battiston et al. (2012) as a method to compute the systemic importance of financial institutions on the basis of their position within a network of interbank exposures.

The algorithm assumes that, when an institution suffer some losses, these are propagated to its creditors because of credit quality deterioration, and it can therefore account for the propagation of shocks before the default of financial institutions.

Here we consider the formulation of this method considered in Bardoscia et al. (2015).

Given a network W of interbank exposures, with W_{ij} the exposure of instituion i towards institution j, and denoting by E_i the equity of institution i and by h_i the relative loss of equity of bank i, the algorithm reduces to the calculation of the fixed point of the following map

$$h_i(t) = \min \left\{ 1, \sum_j \frac{W_{ij}}{E_i} h_j(t-1) + h_i(1) \right\}, \tag{6}$$

where $h_i(0) = 0$, and $h_i(1)$ is the relative loss of i associated with an exogenous shock. The above equation implements the idea that the loss of bank i is due to the exogenous shock plus a contribution that comes from the institutions i is exposed to.

DebtRank can be used to compute impact and vulnerabilities metrics for each institution in the network. The impact of institution i is the loss that an exogenous shock corresponding to the default of i would cause to the system. Conversely, the vulnerability of i is its average loss if the exogenous shock causes the default of another institution.

4.3 Beyond interbank exposures

In the network literature on financial contagion and systemic risk, interbank exposures have been the main object of study. Less work has been made on bank to firm networks and even less on firm to firm networks. There are a few exceptions on these extensions: Lux (2014), Lux and Luu (2019), Marotta et al. (2015), T. C. Silva et al. (2018), Poledna et al. (2018) and Baron et al. (2020). Nevertheless, only in Poledna et al. (2018) the firm to firm network is estimated from aggregated balance sheet data and in Baron et al. (2020) intrafirm exposure is estimated at the sectoral level.

In Poledna et al. (2018), the author characterize in a useful meta exposures matrix, the different exposures which link the banking system with the real economy, represented by the firms which have credits with the banking system. In their representation, the Austrian economy (at least the one with measurable links to the banking system) exposures network has two types of nodes, banks B and firms F, $|B| = b$, $|F| = f$, $n = b + f$. There are links between banks (interbank), links between banks and firms (firms deposits at banks and banks credits to firms), and links between firms (intrafirm). The mathematical representation of such pattern of interactions can be given in terms of a matrix with a block structure:

$$W_{n\times n} = \begin{pmatrix} BB_{b\times b} & BF_{b\times f} \\ FB_{f\times b} & FF_{f\times f} \end{pmatrix} \quad (7)$$

where BB is the interbank exposures matrix, BF is the bank-firms loans matrix, FB are the firms' deposits at banks and FF is the intra-firm exposures matrix.

As it was mentioned before, the BB relations have been extensively used for interbank contagion studies, and the BF block has also being used as a bipartite network (see e.g. Ramadiah et al. (2020). The FB block is harder to get and, to the best of our knowledge, there are no studies that use this data, with the exception of T. C. Silva et al. (2018). The FF block has also been used also in T. C. Silva et al. (2018) and Poledna et al. (2018), where it was however estimated from payment system data and from aggregated balance sheet data respectively. Finally, Baron et al. (2020) use the same survey data used in this paper but perform the analysis at the sectoral level.

To the best of our knowledge, this is the first time in which this estimation is being made at the firm level by resorting to survey data, which makes it far more accurate than in the previous cases.

In the following, we will perform a systemic risk analysis of the whole system represented by the matrix $W_{n\times n}$. To this end, we use an extension of the DebtRank algorithm that also accounts for the bank-firm and firm-firm interactions. The algorithm is the same as the one described in the previous section

$$h_i(t) = \min\left\{1, \sum_j \frac{(W_{n\times n})_{ij}}{E_i} h_j(t-1) + h_i(1)\right\}, \quad (8)$$

where we replaced W with $W_{n\times n}$, and where the index i runs over banks and firms, rather than only banks.

4.4 Network reconstruction methods

In this paper we will consider different methods to reconstruct the firm-to-firm network. In particular we will consider the Maximum Entropy Upper and Worms (2004), Minimum Density Anand et al. (2014), and Fitness model Caldarelli et al. (2002); Park and Newman (2004); Squartini and Garlaschelli (2011).

The different methods produce networks with different properties, and we will discuss how they impact the estimation of risk.

For instance, Maximum Entropy (ME) (Upper & Worms, 2004) tends to creates complete networks in which all entries are as homogeneous as possible while being compatible with the constraints provided by the total borrowing and lending of each individual institution. Initial guess

Minimum density (MD) (Anand et al. (2014)) , at the opposite end, tries to allocate the total amount lent to and borrowed from each bank while using as few links as possible, thus producing a very sparse network which represents a lower bound in terms of connectivity.

In the following we will also use a combination of a fitness model and maximum entropy. The fitness model can in fact be used to compute probabilities for links to exist in the network. These probabilities are computed such that, on average, the number of creditors and debtors of each individual institution is equal to the one observed empirically. The linking probabilities can then used to produce adjacency matrices that correspond to plausible network structures compatible with the number of counterparties of each institution. Like for the ME case, the RAS algorithm can then be used to assign weights to the existing links. This method allows to generate networks with a connectivity that is intermediate between those of the ME and MD.

These and other methods well documented in Anand et al. (2018).

4.5 Reconstruction of firm-firm network

We now provide more details about how we reconstruct the firm-to-firm network taking into account all the information available through the survey.

We have a system of N firms. For each firm i we know the total amount a_i of loans to other firms, the total amount ℓ_i of money borrowed from other firms, the number of creditors k_i^{out} and the number of debtors k_i^{in} (the convention we use is that a link goes from the borrower to the lender). We also know the identity of a subset of creditors and debtors. We denote these subsets respectively by ν_i^{out} and ν_i^{in}.

Given the information specified above, we have an incomplete matrix of interfirm exposures, which we need to fill by satisfying the constraint on the total in and out degree and in and out strength of each node (in and out strengths are ai and ℓ_i respectively). In order to achieve this goal, we proceed with a two-steps method: First, we reconstruct a binary adjacency matrix that satisfies (on average) the constraint on in and out degree using a fitness model. Then we assign weights to the links using the RAS method.

4.5.1 Binary adjacency matrix via the fitness model

According to the fitness model the probability that a link from node i to node j exists is given by ((Park & Newman, 2004; Squartini & Garlaschelli, 2011))

$$p_{i\to j} = \frac{x_i^{\text{out}} x_j^{\text{in}}}{1 + x_i^{\text{out}} x_j^{\text{in}}}, \tag{9}$$

where the set of variables x_i^{out} and x_i^{in}, called fitness, have to be computed in such a way that the constraints on the in and out degrees are satisfied, i.e. by solving the following set of equations

$$k_i^{\rm out} - |\nu_i^{\rm out}| = \sum_{j \notin \nu_i^{\rm out}} \frac{x_i^{\rm out} x_j^{\rm in}}{1 + x_i^{\rm out} x_j^{\rm in}} \quad \forall i \in \{1 \dots N\} \tag{10}$$

$$k_i^{\rm in} - |\nu_i^{\rm in}| = \sum_{j \notin \nu_i^{\rm in}} \frac{x_i^{\rm in} x_j^{\rm out}}{1 + x_i^{\rm in} x_j^{\rm out}} \quad \forall i \in \{1 \dots N\}, \tag{11}$$

where $|\nu_i^{\rm in}|$ and $|\nu_i^{\rm out}|$ represent the number of elements in sets $\nu_i^{\rm in}$ and $\nu_i^{\rm out}$ (these are the numbers of creditors and debtors for which we know the exposures).

Once we have solved the above set of equations to determine the values of the x_i's, we can generate an instance of a binary adjacency matrix by drawing each link $i \to j$ with probability $p_{i \to j}$.

4.5.2 Assigning weights via the RAS algorithm

Once we have determined which links are present in the network, we have to assign weights to those links. To those links we know the weight of from the data (i.e. those in the sets $\nu_i^{\rm in}$ and $\nu_i^{\rm out}$) we assign the known weights. To the other links we assign weights through the following iterative procedure (n denotes the iteration)

$$W_{i \to j}^{(2n)} = \frac{W_{i \to j}^{(2n-1)}}{\sum_{j \notin \nu_i^{\rm out}} W_{i \to j}^{(2n-1)}} \left(\ell_i - \sum_{j \in \nu_i^{\rm out}} W_{i \to j} \right). \tag{12}$$

$$W_{j \to i}^{(2n+1)} = \frac{W_{j \to i}^{(2n)}}{\sum_{j \notin \nu_i^{\rm in}} W_{j \to i}^{(2n)}} \left(a_i - \sum_{j \in \nu_i^{\rm in}} W_{j \to i} \right). \tag{13}$$

The idea of the above iterations is that in even (odd) steps we re-scale the unknown weights such that the sum of the elements in each row (column) is equal to the total amount of debt (credit). Clearly when we enforce the sum over the rows, the one over the columns will be off, and the other way around. The idea is to iterate the equations until we reach a given precision.

4.6 Building a network of effective exposures of banks towards firms

Let us denote by V_{ai} the exposure of bank a towards firm i. This is associated in our case with a loan from the bank to the firm. However, in the presence of credit relationships between firms, a bank can be exposed towards firms it did not directly lent to. This because if bank a lends to firm i and not to firm j, but firm i lends to firm j, the inability of firm j to pay its debt towards bank i may affect bank a. In Figure 3 we show a pictorial representation of how this can occur.

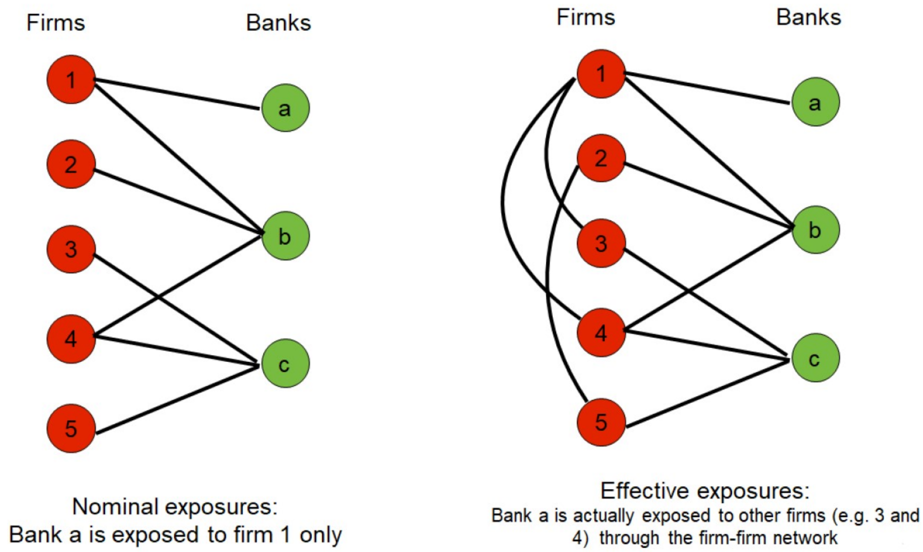


Figure 3: Effective exposures

Rather than trying to construct a micro-founded model of how these types of shocks propagate in the network of firms, we consider the existence of effective exposures of banks towards firms.

These can be computed as follows: Let us denote by W_{kj} the amount lent by firm j to firm i, and by D_k the total debt of firm i (thins includes debt to firms as well as to banks). We can say that firm j owns a fraction $\frac{W_{kj}}{D_k}$ of k's debt. If now firm k lent an amount W_{ik} to firm i, k owns a fraction $\frac{W_{ik}}{D_i}$ of i's debt. This means that firm j effectively owns a fraction $\frac{W_{ik}W_{kj}}{D_i D_k}$ of k's debt. This intuition can be extended to all types of paths in the network, so that we can say that bank a is effectively exposed to firm i by an amount equal to

$$\begin{aligned} \tilde{V}_{ai} &= V_{ai} + \sum_j \frac{V_{aj}}{D_j} \Pi_{ij} D_i + \sum_{jk} \frac{V_{aj}}{D_j} \Pi_{ik} \Pi_{kj} D_i + \ldots & (14) \\ &= \sum_j \left[(\mathbb{1} - \Pi)^{-1} \right]_{ij} \frac{V_{aj}}{D_j} D_i, & (15) \end{aligned}$$

where we have defined $\Pi_{ij} = W_{ij}/D_i$.

The intuition of the first line above is the following: The effective exposure of a towards i is equal to a sum that accounts for all possible paths in the network that connect a to i. The first term of the right-hand side is the direct exposure of a to i. In the second term, the factor D_i is the loss associated with the default of firm i. A fraction Π_{ij} of this loss is passed to firm j, which in turn passes a fraction V_{aj}/D_j to bank a. The sum over j accounts for all possible paths of length 2 from a to i in the network. In the third term of the right-hand side, the loss is passed from i to k, then from k to j and finally from j to a, so that paths of length 3 are considered. And so on.

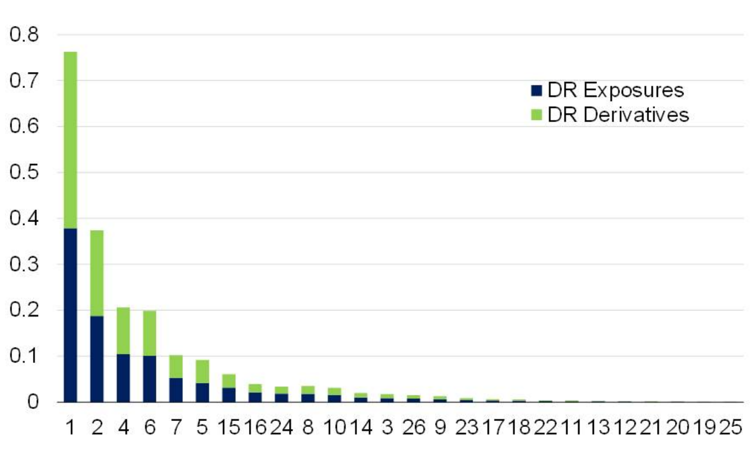
The underlying assumption is that if a firm defaults, its creditors (linearly) propagate some loss to their creditors, and so on. In practice, the propagation could stop if some creditors absorb the loss without passing it further on. Those calculated following are therefore only an upper bound to effective exposures, while nominal exposures are a lower bound.

5 Results

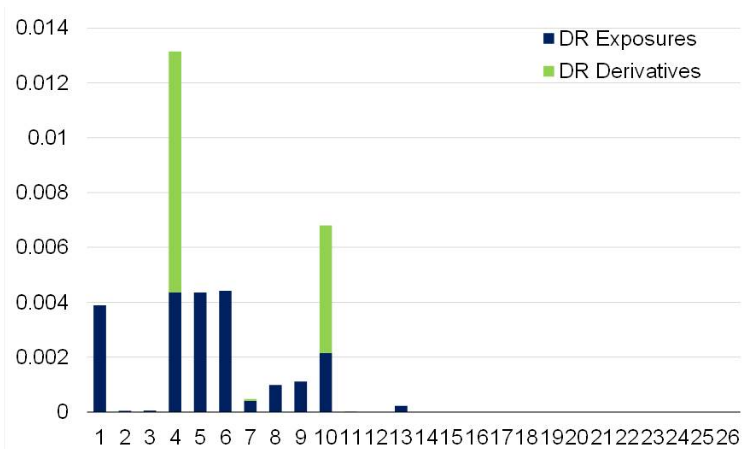
In this section we present the results. The first result is the application of the DebtRank algorithm to the interbank exposures network. This network consists of two layers: the unsecured lending layer and the derivatives layer. Figure 4 presents the systemic risk profile for the banking system in Uruguay. Neglecting the derivatives layer can underestimate bank systemic importance in the network.

The second set of results involves the bank-firms bipartite and the intra-firm exposures networks. In a similar way as we did for the interbank exposures, we compute the DebtRank for the network that includes all the three different methodologies. In this way we can find not only the systemically important banks, but also systemically important firms.

Figure 4: Banking system's risk profile.



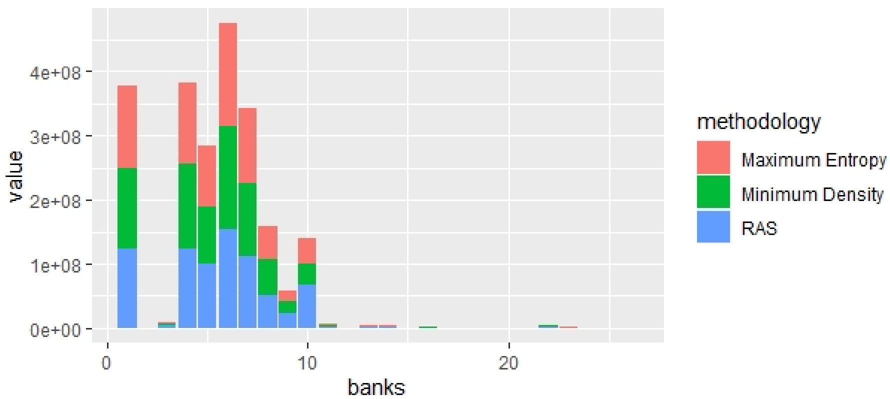
(a) With initial shock

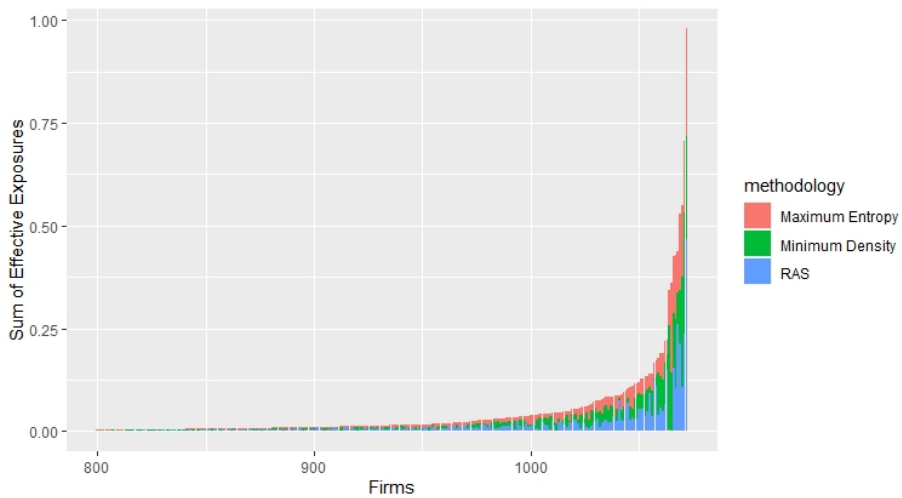


(b) Without the initial shock

Figures 5a and 5b show the difference between nominal and effective exposures. In particular, Figure 5a shows the effective exposures effect of the interbank network. Each bar for each entity is the sum of effective exposures estimation for all methodologies. The graph shows that the effect is similar for each methodology, but for some entities the effective exposure is slightly larger in the RAS methodology, for instance in entities 5 and 10. Figure 5b shows the difference of effective exposure's effect of each methodology for the intrafirm network.

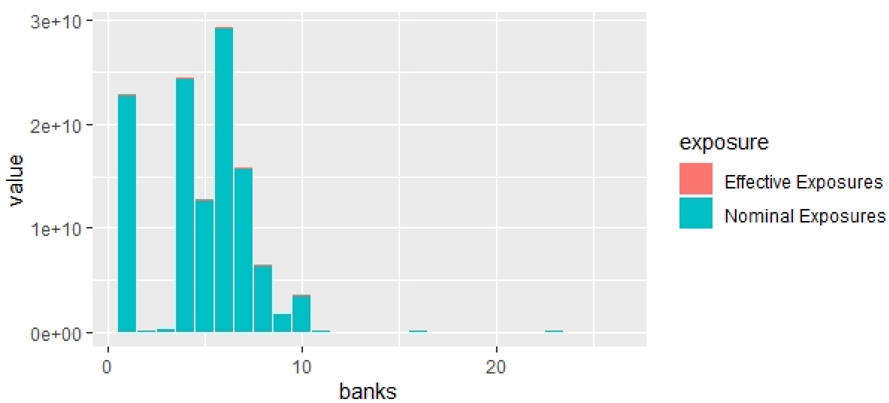
Figure 5: Banks and firms Effective Exposures





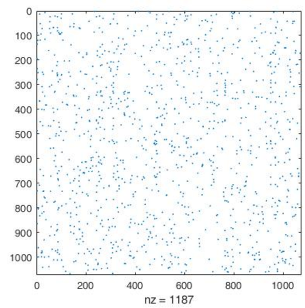
In Figure 6, we show an aggregated view for the nominal an effective exposures in the interbank network for the RAS Methodology. We find the effective exposures are not much larger than nominal ones for this system, however it is important take into account this effect and do not underestimate interbank network and its effects on systemic risk.

Figure 6: Effective vs Nominal Exposures
RAS Methodology

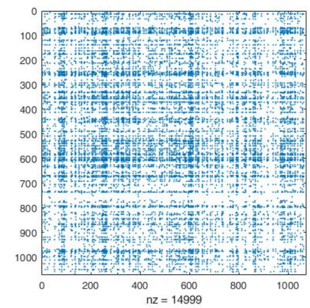


The information from the survey is about 1,187 observations (Figure 7a) which presents a sparse matrix, when we applied the RAS algorithm to complete the matrix from the known information (Figure 7b) we increase the non zero elements to 14,999 observations. This means that RAS algorithm give us more useful information for vulnerability and impact analysis. In Figure 7c, we use Maximum Entropy methodology which shows a plenty matrix with 430,487 observations. On the other hand, with minimum density, the number of observations decrease to 1,184 almost the same number of observations from the survey (Figure 7d). In Figure 8 we show the intersection between the survey observations and Anand Minimum Density methodology. We found only four intersections between the two matrices.

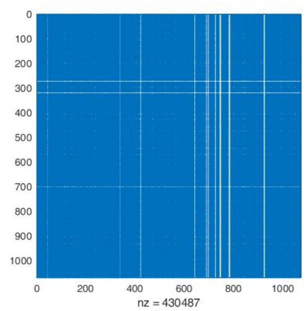
Figure 7: matrix of exposures estimation methods.



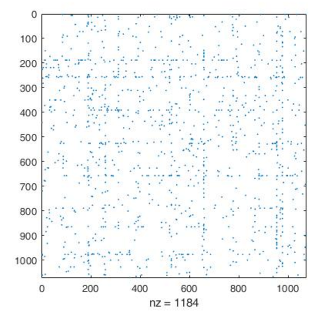
(a) Survey matrix



(b) Constrained RAS matrix

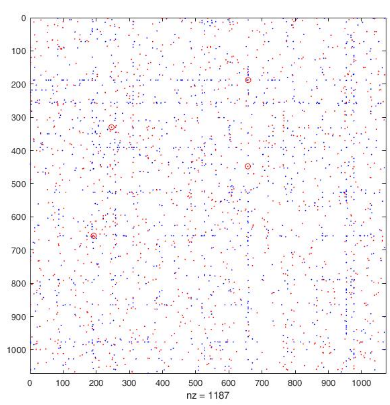


(c) Maximum entropy matrix



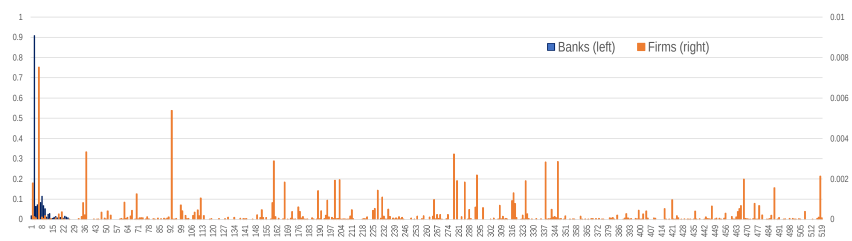
(d) Minimum density matrix

Figure 8: Intersection Survey and Anand matrix



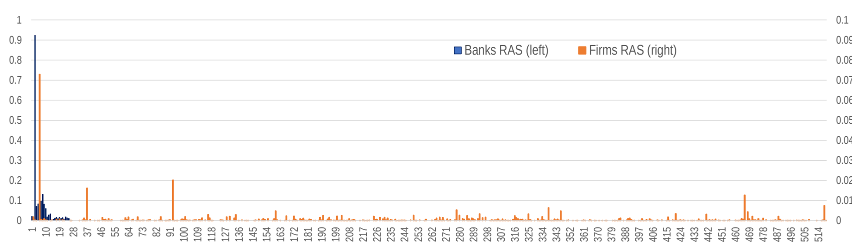
In this section, we include the intrafirm exposures information to analyze the change in vulnerability in banks and firms. In the base case, we found that at least one bank is importantly vulnerable when we include intrafirm exposures information, its vulnerability reaches 90%, an important effect on the interbank network. Concerning firms, we find that in some cases the vulnerability goes from 0.1% to 0.8% (Figure 9).

Figure 9: Base case



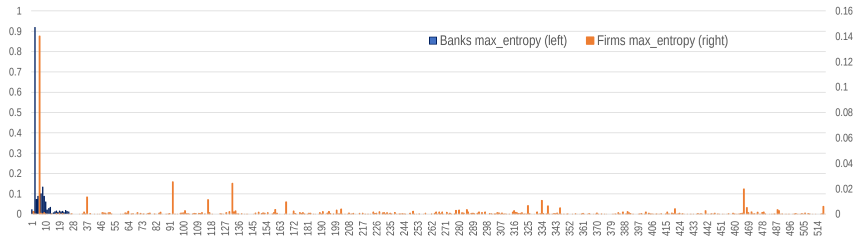
If we reconstruct the intrafirm matrix using RAS methodology, we find a slightly increase in the vulnerability of some banks. The same applies to firms, whose vulnerability goes from 1% to 7%. At the aggregate level, the increase in vulnerability of firms including intrafirm exposures is around 21% in contrast to the base case (Figure 10). The 21% results from computing the aggregated sum of the differences for each firm between intrafirm network from RAS methodology and the base case.

Figure 10: RAS



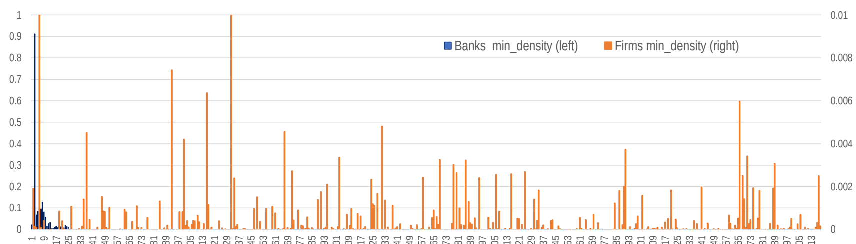
Maximum entropy analysis shows more firms with a higher vulnerability than in the base case and RAS methodology. The vulnerability of firms goes from 1% to 14%, and the increase in aggregated vulnerability for firms including intrafirm exposures is around 37 percent in contrast with the base case (Figure 11).

Figure 11: Maximum entropy



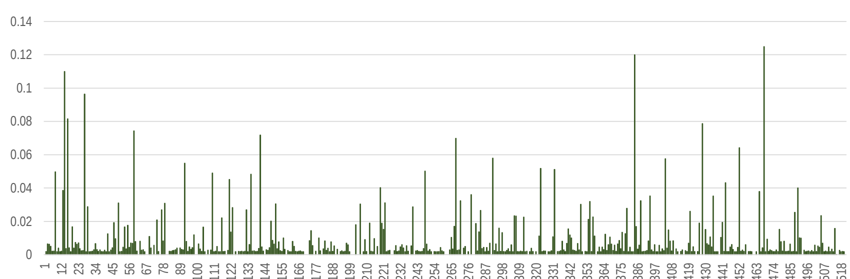
On the other hand, the Minimum density approach shows lower levels of vulnerability in firms, from 0.1% to 1.0% in contrast with maximum entropy and RAS algorithm. While the increase in aggregated vulnerability for firms including intrafirm exposures is around 11 percent in contrast with the base case (Figure 12).

Figure 12: Minimum density



In the next section, we analyze the changes in firm impact due the inclusion of intrafirm exposures. We quantify the aggregated firm impact through the different methodologies. In Figure 13 we have the case zero, in which the firm impact does not include intrafirm exposures information. Some entities reach an impact of around 12 percent.

Figure 13: Firm impact without firm to firm exposures



In Figure 14 intrafirm exposures increase aggregated firm impact around 18%. The following figures refer to each methodology (maximum entropy, minimum density and RAS). Intrafirm exposures with the Maximum Entropy show an increase of the aggregate impact, measured by the aggregated losses, of around 84% (Figure 15). The 84% results from computing the aggregate sum of the difference between including the intrafirm exposures and without them for the RAS methodology. In Figure 16, intrafirm exposures with minimum density increase aggregated firm impact around 62%. Finally, with RAS methodology, the intrafirm exposures increase aggregated firm impact around 63% (Figure 17).

Figure 14: Firm impact-Base case

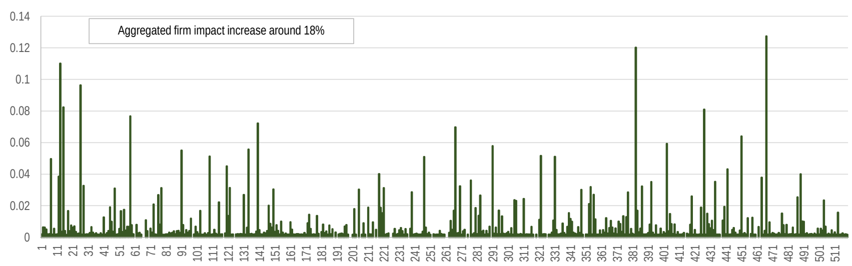


Figure 15: Firm impact with Maximum entropy

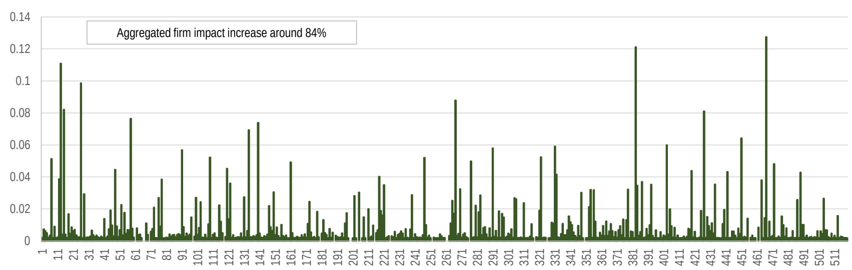


Figure 16: Firm impact with Minimum density

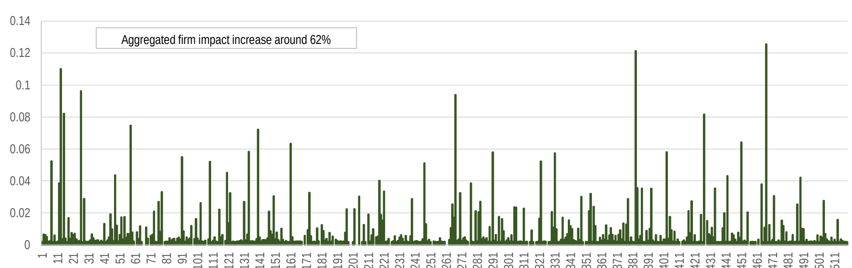
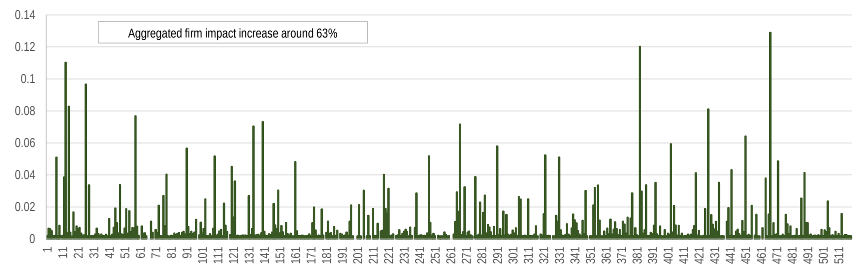
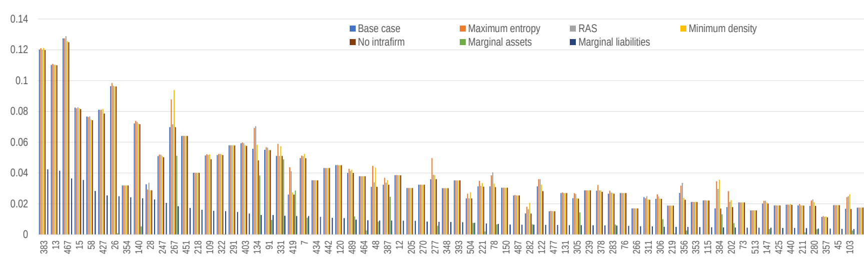


Figure 17: Firm impact with RAS methodology



In Figure 18, we order vulnerability including intrafirm exposures information by marginal liabilities. For each entity we have the information for the three different methodologies (RAS, maximum entropy, minimum density), marginal assets, marginal liabilities, vulnerability without intrafirm exposures information (0), and DebtRank (1). It is important to highlight that we order the ranking by marginal liabilities because is the way where the contagion propagates. According to RAS and DebtRank methodology most of the entities are not order in the same way that marginal liabilities rank the entities. For instance, third firm can affect around 13 percent of equity in firms network.

Figure 18: Ranking by Marginal Liabilities



In Table 2 we illustrate the differences for the first 10 ranking order for each entity regarding the methodology. We take as pivot the Survey information and we found the principal differences when we order by marginal assets and marginal liabilities, and with maximum entropy and minimum density. For the first 10 entities, RAS and Survey information have the same order of entities.

Survey	Max Entropy	RAS	Min Density	Base Case	Marg assets	Marg liab
467	467	467	467	467	267	383
383	383	383	383	383	140	13
13	13	13	13	13	26	467
26	26	26	26	26	13	15
15	267	15	267	15	427	58
427	15	427	15	427	383	427
58	427	58	427	58	467	26
140	58	140	58	140	15	140
267	140	267	140	267	58	267
451	451	451	451	451	451	451

Table 2: Top 10 Ranking table by different methodologies

6 Conclusions and further work

In this work there are many interesting takeaways but the most important is that that ignoring the intrafirm exposures underestimates systemic risk. Moreover, the most important part of the vulnerability of Uruguayan banks to financial contagion comes from the real sector of the economy, in contrast to the well-studied interbank exposures.

In the first part of our work we compute the DebtRank for banks considering the only direct inter-bank credit and derivatives exposures. The results showed a small impact from such direct exposures. Then, by using information from the credit registry we built the bank-firms bipartite network in order to study the impact of firm failures in the banking system, considering network effects from the interbank exposures network. However, the most novel part of this work relies on the estimation of the intrafirm exposures network and its contribution on the systemic risk faced by the banking system.

We estimated the intrafirm exposures network by resorting to three different methods and document the differences; then, we estimated the contribution to systemic risk of the information contained on the intrafirm exposures network; finally, we were able to identify systemically important firms on the basis of their impact on banks and other firms taking into account contagion (network) effects. In order to achieve these goals, we computed the DebtRank centrality metric for both, the interbank and for an extended network of exposures, including firms.

Before concluding we would like to highlight two side products of this work: i) the comparison of three different methods to reconstruct the firm-firm exposures networks in terms of systemic risk (the ME, the MD and a constrained version that uses the RAS algorithm), and ii) the computation of effective exposures which show that banks are exposed among them beyond their direct credit lines given to firms through the firm-firm lending relationships.

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