

# Humans Against Virus or Humans Against Humans:

## A Game Theory Approach to the COVID-19 Pandemic

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### Abstract

Externalities and private information are key characteristics of an epidemic like the Covid-19 pandemic. We study the welfare costs stemming from the incomplete information environment that these characteristics foster. We develop a framework that embeds a game theory approach into a macro SIR model to analyze the role of information in determining the extent of the health-economy trade-off of a pandemic. We apply the model to the Covid-19 epidemic in the US and find that the costs of keeping health information private are between USD 5.9 trillion and USD 6.7 trillion. We then find an optimal policy of disclosure and divulgation that, combined with testing and containment measures, can improve welfare. Since it is private information about individuals' health what produces the greatest welfare losses, finding ways to make such information known as precisely as possible, would result in significantly fewer deaths and significantly higher economic activity.

**Keywords**— COVID-19, epidemic, game theory, information asymmetries, macroeconomics, testing, containment policies, disclosure, divulgation, optimal policies

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# 1 Introduction

A pandemic caused by a virus is a health shock that may induce consumers to reduce their activities to protect themselves and reduce the probability of contagion. It may also induce governments and policy makers to implement restrictive measures to slow down and control the spread of the virus. Both private and government responses produce a tradeoff between economic and health outcomes (Eichenbaum et al. (2020a)). A characteristic of a pandemic like COVID-19, is that there are externalities (for example the infection externality), and information asymmetries (a person may know she is sick, but others may not), which sets up an environment of strategic interaction among consumers. Clearly, the fact that people lacks information about other people’s health complicates the containment of the virus, magnifies the externalities and affects how people choose the extent of their social and economic activity.

However, concerns about the collection and use of private information about the health of individuals have been important in the efforts to control the COVID-19 disease, particularly the use of apps and technology to trace and track infected individuals. The following guidelines for contact tracing in the COVID-19 pandemic are provided by the The Centers for Disease Control and Prevention (CDC) in its website: “All public health staff involved in case investigation and contact tracing activities with access to such information should sign a confidentiality statement acknowledging the legal requirements not to disclose COVID-19 information. Efforts to locate and communicate with clients and close contacts must be carried out in a manner that preserves the confidentiality and privacy of all involved. This includes never revealing the name of the client to a close contact unless permission has been given (preferably in writing), and not giving confidential information to third parties (e.g., roommates, neighbors, family members).”<sup>1</sup>

Given the importance of information and the limitations from privacy, in this paper we develop an analytical framework that combines a game theory set-up and the Macro-SIR model proposed in Eichenbaum et al. (2020a) to make explicit how information influences the spread of an epidemic and quantify its importance. We also extend the model to include asymptomatic infected people, an important characteristic of the COVID-19 pandemic as argued by Berger et al. (2020), and one that certainly entails a key source of information loss. Asymptomatic individuals increase infections, but as long as they do not die, taking them into account will change quantitatively the predictions of the model about deaths and the fall in economic activity with respect to a classic SIR. Our framework allows to understand and quantify the costs of privacy from a microeconomics perspective. It also provides a way to study how different degrees of information can determine how an infectious disease spreads and evolves over time, how it affects economic outcomes, and what the optimal mix of policy tools could be to reduce its negative effects.

As a case study, we apply the model to the US and analyze the recent COVID-19 crisis. We show that the lack of both private and common information generates relevant welfare losses, albeit the greater losses are associated with the latter. Accordingly, we study and quantify the effects of a policy of disclosure and divulcation of private health information about individuals in alleviating the negative health and economic effects of a pandemic. We argue that disclosure and active divulcation of precise information about who is infected can have large welfare effects, especially when combined with the more traditional policy tools of testing and containments. We find that what we label the

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<sup>1</sup><https://www.cdc.gov/coronavirus/2019-ncov/php/contact-tracing/contact-tracing-plan/Confidentiality-Consent.html>

“Optimal Mix” of policies, calls for the use of containment and testing, but the welfare gains from these two policies are overshadowed by the gains from precise divulgation.

In our setup, we vary the information available to individuals. In a first case, which we call Total Incomplete Information (TII), agents do not know their own or other people’s health condition. In the second case, Partial Incomplete Information (PII), people know their own health condition but are ignorant of the infection status of others. Finally, in the third case, one of Complete Information (CI), everything is known. These different information worlds produce widely different welfare outcomes. Total Incomplete Information is of course the worst of the worlds, and it is arguably the closest to reality.

We study optimal policies in these different information worlds and how they allow us to improve outcomes and eventually go from one world to the next. We begin with containment policies, which have been the main policy tools used during the COVID-19 pandemic. We generally find that containment generates little welfare gains, and that the ability to make them targeted (so called conditional) and their optimal extent depends on the available information. In general, the scarcer information is, the more stringent and generalized optimal containments must be. The economic gains from general containment improve consumption in about USD 3.5 trillion when information is gathered and incorporated allowing for conditional quarantines. Despite this improvement, conditional containments are still insufficient to compensate the welfare losses from the disease under incomplete information.

We then study testing as a tool to gather information and produce better aggregate estimates of the extent of the infection in the economy. Testing allows people to learn about their health status. We show that testing alone improves welfare in a rather modest amount. The reason is that it is a double-edge sword. On the one hand testing improves private information about the disease, helping tested people improve their decision making. On the other hand, while testing improves aggregate information it also creates an information asymmetry, which, in the absence of any other policy, results in infected asymptomatic people not reducing consumption and work, and thus increasing the spread of the infection. Nevertheless, a combination of testing and targeted quarantines could generate better results as seen in Eichenbaum et al. (2020b).

We subsequently analyze a policy of disclosure and divulgation of private information about people’s health status at an individual level. Depending on the costs of making this information public and of people being able to use it, such policy is the game changer. A disclosure and divulgation policy can be optimally used accompanied by policies of testing and containment. First, containment helps internalize the infection externality, which is still present in a world of complete information. Second, together with testing, divulging will flatten the infections curve with a much smaller economic downturn. Unlike containments and testing only, it does so by enabling mutually beneficial transactions with no contagion risk to take place normally. Ultimately, this relaxes the trade-off between economic activity and public health. We estimate the potential gains of frequently divulging precise information to fall between USD 5.9 trillion and USD 6.7 trillion in dollars of 2019.

Altogether, our paper illustrates that the COVID-19 crisis can be thought about as an information problem,

rather than a problem that needs to be controlled through stringent containment. Consequently, an appropriate policy response to the epidemic should aim at closing information gaps. We are aware that a full disclosure of private information brings out several considerations about privacy rights, however, thinking about how to make precise information available is worthwhile. Paraphrasing Dr. Tedros Adhanom Ghebreyesus, the President of the WHO, when he recommends “Testing, Testing and Testing”, we encourage authorities to do “Divulcation, Divulcation and Divulcation”.

Our paper makes two contributions. First, a framework to analyze an epidemic and its economic consequences as the result of many strategic interactions where information is of the essence. An advantage of such framework is that information’s importance can be quantified to guide policy decisions. In this sense, our paper is broadly related to the literature that links epidemiological models and macroeconomics to think about optimal policies as in Eichenbaum et al. (2020a), Acemoglu et al. (2020), Rowthorn and Toxvaerd (2020), Alvarez et al. (2020), Jones et al. (2020), Farboodi et al. (2020), and Garriga et al. (2020). Our second contribution is to highlight the importance of disclosing and divulgating more precise and disaggregated information about people’s health status, which can become a powerful policy tool to reduce the economic health trade-off of an epidemic like the COVID-19. Since we make more explicit the role of information in the analysis of pandemic dynamics, our work contributes more closely related to Argente et al. (2020), Eichenbaum et al. (2020b) and Berger et al. (2020). A key difference with these papers is that we microfound how information affects economic decisions.

The rest of the paper is organized as follows. In Section 2 we describe the model and how it changes under different information contexts. In Section 3 we study the ability of containment policies to improve health and economic outcomes in the face of different information structures. Then, in Section 4, we analyze the effects of testing and disclosure and divulcation as policy tools that, by reducing the effects of information deficiencies and asymmetries, can potentially achieve better health and economic outcomes. In Section 5 we conduct some exercises to consider the possibility that beliefs are not “right”, which is a possibility in the presence of poor information about the disease. Finally, Section 6 concludes.

## 2 Model

Our model builds on the framework of Eichenbaum et al. (2020a), in which we couple an economic structure together with the epidemiological model of Kermack and McKendrick (1927). We depart from Eichenbaum et al. (2020a) in two important ways. First, we extend the epidemiology block to include asymptomatic cases. Second, the economic structure is built using game theory. That approach is motivated by people’s high interdependence when engaging in economic activities with contagion risk and the existence of information asymmetries. Together, these changes give information a key role in the model. This helps us incorporate and underscore the idea that people’s economic choices and how they get infected critically hinges on health information available to them.

The economy is populated by three classes of agents: the government, a continuum of identical firms, and representative households. The government collects taxes from consumption and redistributes them among the

population. Firms produce a consumption good,  $C_t$ , choosing how many hours of labor to hire,  $N_t$ , and using a linear production technology in order to maximize profits  $\Pi_t$  :

$$\Pi_t = AN_t - w_tN_t$$

Households make decisions on consumption and work hours in a strategic environment, which we model as a game. These decisions are strategic because the health status of individuals and the information that they have about it, matters for an economic transaction to happen and for the potential health consequences of the interaction.

## 2.1 Game Setup

The players of this game are households who choose how much to consume and work at every moment of time  $t$  to maximize utility, which takes the following form:

$$u(c_t^i, n_t^i) = \ln(c_t^i) - \frac{\theta}{2}(n_t^i)^2$$

where  $i$  indexes the health status. Throughout the model section we will refer to players as agents. Given that the game is set up in an economy during an epidemic, agents know there is a risk of getting infected when interacting with others. Thus, their economic decisions become intertwined with the health status of others and of themselves.

At the beginning of the game, Nature plays first and randomly chooses two agents  $(i, j)$  from the population  $P_t$  to interact with each other in an economic transaction.<sup>2</sup> Agents are *ex-ante* identical and they will differ only by their type, which is the health status they get assigned by Nature. These types are drawn from the set  $T_i = \{S, I^E, I^A, R^E, R^A\}$ , where  $S$  is Susceptible,  $I^E$  is Symptomatic Infected,  $I^A$  is Asymptomatic Infected,  $R^E$  is Symptomatic Recovered and  $R^A$  is Asymptomatic Recovered<sup>3</sup>. After Nature's move, the two players have to choose (actions) consumption  $c_t^i$  and work hours  $n_t^i$  simultaneously. In the baseline version of the model, the main difference between  $I^A$  and  $I^E$  individuals is that only the latter will have their productivity negatively affected by the shock.

Players' payoffs are given by value functions that depend on their types and actions. Clearly, being in a strategic environment means that players strategies will depend upon each player's information set, particularly what they learn about their health and others' health after Nature moves. In particular, we will study the game under three information worlds, in which we vary the information assumptions of the game: 1) Complete information (CI); 2) Partial incomplete information (PII); 3) Total incomplete information (TII). Since agents are *ex-ante* identical, hereafter we study the game only from player  $i$ 's perspective without loss of generality.

<sup>2</sup>Throughout the paper, we assume all interactions occur only between two people at a time.

<sup>3</sup>Symptomatic infected are people who exhibit symptoms. We assume that these symptoms are observationally unique and thus, the virus cannot be confused with another disease. Symptomatic Recovered people got infected, had symptoms and recovered while Asymptomatic Recovered got infected, did not have symptoms and recovered

## 2.2 Complete Information

Our first world is one where players know their own type and the type of each player they face. We will call this world the Complete Information (CI) case. Even though this may be an unrealistic scenario it will serve as the ideal benchmark.

Given the game set up, player  $i$ 's strategy is contingent on both player's types. This can be described by a tuple of dimension 25 (all combinations of the 5 types in  $T_i$ ), which contains, for each combination of player types, a pair of actions for consumption and hours worked. Since what is relevant for the consumption and labor decision is the chance of getting infected, to solve the game we group the subgames into two categories: 1) No contagion risk for player  $i$  and 2) Positive contagion risk for player  $i$ .

### 2.2.1 No Contagion Risk for player $i$

The interactions  $T_i \times T_j$  where player  $i$  faces no risk of getting infected are:

$$\{S\} \times \{S, R^E, R^A\} \cup \{I^E, I^A, R^E, R^A\} \times T_j.$$

Player  $i$  chooses consumption and hours worked to maximize her value function, subject only to her budget constraint. Since in these interactions player  $i$  can safely disregard player  $j$ 's type, she has a dominant strategy. Nonetheless, player  $i$  is affected by her own type through her budget constraint due to the productivity shock associated with the virus. In other words, player  $i$ 's dominant strategy will vary depending on her health status. We now find the strategies for the different cases.

**Player  $i$  is Infected:**  $T_i = \{I^E, I^A\}$

Player  $i$ 's type is  $I^Z$  with  $Z \in \{E, A\}$ , and takes actions  $(c_t^{I^{Z*}}, n_t^{I^{Z*}})$  by solving:

$$\begin{aligned} \max U_t^{I^Z} &= u(c_t^{I^Z}, n_t^{I^Z}) + \beta \left[ (1 - \pi_d^Z - \pi_r^Z) U_{t+1}^{I^Z} + \pi_r^Z U_{t+1}^{R^Z} \right] \\ \text{s.t.} & (1 + \mu_t) c_t^{I^Z} = w_t \phi^{I^Z} n_t^{I^Z} + \Gamma_t \end{aligned}$$

where  $u(\cdot)$  is the instant utility function,  $\pi_d^Z$  is the mortality rate,  $\pi_r^Z$  is the recovery rate, and  $\phi^{I^Z} \in [0, 1)$  is a parameter that captures the fall in infected people's labor productivity. We assume that for Asymptomatic Infected ( $I^A$ ) mortality rate is zero ( $\pi_d^A = 0$ ) and that their productivity does not get affected  $\phi^{I^A} = 1$ . For the Symptomatic Infected ( $I^E$ ), we have  $\pi_d^E = \pi_d$  and  $\phi^{I^E} = \phi^I$ . The government enters the problem in the budget constraint through lump-sum transfers,  $\Gamma_t$ , and through a containment rate,  $\mu_t$ , which affects consumption. It is worth noting that the value function reflects the assumption that the cost of death is the foregone lifetime utility.

Optimal levels of consumption and hours are such that:

$$\frac{\partial u(c_t^{I^{Z*}}, n_t^{I^{Z*}})}{\partial c_t^{I^{Z*}}} = \lambda_t^{I^Z} (1 + \mu_t) \quad (1)$$

$$\frac{\partial u(c_t^{I^{Z^*}}, n_t^{I^{Z^*}})}{\partial n_t^{I^{Z^*}}} = -\lambda_t^{I^Z} w_t \phi^{I^Z} \quad (2)$$

**Player  $i$  has Recovered**  $T_i = \{R^E, R^A\}$

In these cases the optimal choices  $(c_t^{R^{Z^*}}, n_t^{R^{Z^*}})$  with  $Z \in \{E, A\}$  come from the solution to the optimization problem:

$$\begin{aligned} \max U_t^{R^Z} &= u(c_t^{R^Z}, n_t^{R^Z}) + \beta U_{t+1}^{R^Z} \\ \text{s.t.} &(1 + \mu_t)c_t^{R^Z} = w_t n_t^{R^Z} + \Gamma_t \end{aligned} \quad (3)$$

So that consumption and hours worked satisfy the optimality conditions:

$$\frac{\partial u(c_t^{R^{Z^*}}, n_t^{R^{Z^*}})}{\partial c_t^{R^{Z^*}}} = \lambda_t^{R^Z} (1 + \mu_t) \quad (4)$$

$$\frac{\partial u(c_t^{R^{Z^*}}, n_t^{R^{Z^*}})}{\partial n_t^{R^{Z^*}}} = -\lambda_t^{R^Z} w_t \quad (5)$$

**Player  $i$  is Susceptible**  $T_i = S$

If player  $i$  is Susceptible of getting infected but player  $j$ 's type belongs to  $\{S, R^E, R^A\}$ , there is no risk of contagion. Then player  $i$  solves the optimization problem below to find her actions  $(c_t^{S, NI^*}, n_t^{S, NI^*})$ .

$$\begin{aligned} \max U_t^{S, NI} &= u(c_t^{S, NI}, n_t^{S, NI}) + \beta U_{t+1}^S \\ \text{s.t.} &(1 + \mu_t)c_t^{S, NI} = w_t n_t^{S, NI} + \Gamma_t \end{aligned} \quad (6)$$

With the optimal levels of consumption and hours worked satisfying:

$$\frac{\partial u(c_t^{S, NI^*}, n_t^{S, NI^*})}{\partial c_t^{S, NI^*}} = \lambda_t^{S, NI} (1 + \mu_t) \quad (7)$$

$$\frac{\partial u(c_t^{S, NI^*}, n_t^{S, NI^*})}{\partial n_t^{S, NI^*}} = -\lambda_t^{S, NI} w_t \quad (8)$$

### 2.2.2 Contagion Risk for Player $i$

Player  $i$  faces a risk of contagion as long as she is susceptible and Player  $j$  is of type  $I^Z$  with  $Z \in \{E, A\}$ . In these interactions, player  $i$  will choose the pair  $(c_t^{S, I^{Z^*}}, n_t^{S, I^{Z^*}})$  by solving:

$$\max U_t^{S, I^Z} = u(c_t^{S, I^Z}, n_t^{S, I^Z}) + \beta \left[ (1 - \tau_t^{I^Z}) U_{t+1}^S + \tau_t^{I^Z} U_{t+1}^{I^Z} \right]$$

$$s.t.(1 + \mu_t)c_t^{S,I^Z} = w_t n_t^{S,I^Z} + \Gamma_t \quad (9)$$

$$, \wedge, \tau_t^{I^Z} = \pi_1 c_t^{S,I^Z} c_t^{I^Z} + \pi_2 n_t^{S,I^Z} n_t^{I^Z} + \pi_3 \quad (10)$$

With  $\tau_t^{I^Z}$  being the probability of Player  $i$  getting infected by Player  $j$ ,  $\pi_1$  the probability of getting infected from consumption interactions,  $\pi_2$  the probability of getting infected from work interactions, and  $\pi_3$  the probability of getting infected in any other way. Simultaneously, the Player  $j$  solves its own optimization problem and acts according to the pair  $(c_t^{I^Z*}, n_t^{I^Z*})$ . Thus, these actions influence Player  $i$ 's optimal decisions as follows:

$$\frac{\partial u(c_t^{S,I^Z*}, n_t^{S,I^Z*})}{\partial c_t^{S,I^Z*}} + \beta \pi_1 c_t^{I^Z*} (U_{t+1}^I - U_{t+1}^S) = \lambda_t^{S,I^Z} (1 + \mu_t) \quad (11)$$

$$\frac{\partial u(c_t^{S,I^Z*}, n_t^{S,I^Z*})}{\partial n_t^{S,I^Z*}} + \beta \pi_2 n_t^{I^Z*} (U_{t+1}^I - U_{t+1}^S) = -\lambda_t^{S,I^Z} w_t \quad (12)$$

### 2.2.3 Aggregates and Equilibrium

Given that the game is symmetric for players and that before Nature randomly selects their type they are identical, to find aggregates we can just aggregate over  $i$ . Aggregating over players  $i$  such that  $T_i \in \{I, R\}$  yields the following aggregate value functions:

$$\begin{aligned} R_t U_t^R &= R_t^E U_t^{R^E} + R_t^A U_t^{R^A} \\ I_t U_t^I &= I_t^E U_t^{I^E} + I_t^A U_t^{I^A} \end{aligned}$$

Aggregate consumption and hours for the infected and recovered population have analogous expressions.

When Player  $i$  is Susceptible, her value function, consumption and work take into account Player  $j$ 's type, so we need to integrate over all other individuals in the population ( $P_t$ ):

$$\begin{aligned} U_t^S &= \frac{1}{P_t} \int_0^{P_t} U_t^S(j) dj \\ &= \frac{1}{P_t} \left[ (S_t + R_t) U_t^{S,NI} + I_t^E U_t^{S,I^E} + I_t^A U_t^{S,I^A} \right] \end{aligned}$$

And consumption and hours can be found similarly to get:

$$\begin{aligned} c_t^S &= \frac{1}{P_t} \left[ (S_t + R_t) c_t^{S,NI} + I_t^E c_t^{S,I^E} + I_t^A c_t^{S,I^A} \right] \\ n_t^S &= \frac{1}{P_t} \left[ (S_t + R_t) n_t^{S,NI} + I_t^E n_t^{S,I^E} + I_t^A n_t^{S,I^A} \right] \end{aligned}$$

Finally, Susceptible aggregates are just  $S_t U_t^S$ ,  $S_t c_t^S$ , and  $S_t n_t^S$ .

### Government



Government may collect taxes on consumption ( $\mu_t$ ) to disincentivize interactions. This will capture the effect of lockdowns. The government also makes transfers  $\Gamma_t$  to households. The government's budget constraint is given by:

$$\mu_t(S_t c_t^S + I_t c_t^I + R_t c_t^R) = \Gamma_t(S_t + I_t + R_t) \quad (13)$$

### Market clearing

Merging together consumers' and government's budget constraints and using the production function we obtain the market clearing condition in the good and services markets:

$$S_t c_t^S + I_t c_t^I + R_t c_t^R = AN_t \quad (14)$$

While market clearing in the labor market must satisfy:

$$S_t n_t^S + I_t^A n_t^{I^A} + I_t^E \phi^I n_t^{I^E} + R_t n_t^R = N_t \quad (15)$$

### Population Dynamics

New infection cases  $T_t$  come from interactions between players  $i, j$  when there is risk of contagion:

$$\begin{aligned} T_t &= \int_0^{S_t} \int_0^{I_t^A} \tau_t^{I^A} dj di + \int_0^{S_t} \int_0^{I_t^E} \tau_t^{I^E} dj di \\ &= \pi_1 c_t^{S, I^A} S_t c_t^{I^A} I_t^A + \pi_2 n_t^{S, I^A} S_t n_t^{I^A} I_t^A + \pi_3 S_t I_t^A + \pi_1 c_t^{S, I^E} S_t c_t^{I^E} I_t^E + \pi_2 n_t^{S, I^E} S_t n_t^{I^E} I_t^E + \pi_3 S_t I_t^E \end{aligned}$$

Susceptible population evolves according to:

$$S_{t+1} = S_t - T_t$$

With the share of new infections that end up being asymptomatic given by  $\chi^A$ , and the probability that an asymptomatic infected recovers given by  $\pi_r^A$ , total Asymptomatic Infected people in period  $t + 1$  can be calculated as:

$$I_{t+1}^A = I_t^A + \chi^A T_t - \pi_r^A I_t^A$$

The number of Symptomatic Infected people in  $t + 1$  will be equal to:

$$I_{t+1}^E = I_t^E + (1 - \chi^A) T_t - (\pi_r^E + \pi_d) I_t^E$$

where  $\pi_r^E$  and  $\pi_d$  are the probabilities of a symptomatic infected recovering and dying, respectively.

In period  $t + 1$  the total infected, asymptomatic recovered, symptomatic recovered, and recovered populations are respectively:

$$I_{t+1} = I_{t+1}^A + I_{t+1}^E$$

$$R_{t+1}^A = R_t^A + \pi_r^A I_t^A$$

$$R_{t+1}^E = R_t^E + \pi_r^E I_t^E$$

$$R_{t+1} = R_{t+1}^A + R_{t+1}^E$$

Total deaths will accumulate over time according to:

$$D_{t+1} = D_t + \pi_d I_t^E$$

Finally, the economy's total population in  $t + 1$  will be diminished by deaths occurred at time  $t$ :

$$P_{t+1} = P_t - \pi_d I_t^E$$

### 2.3 Partial incomplete information

In this second world, which we call Partial Incomplete Information (PII), every player knows her own type but ignores the type of others. Note that the number of subgames reduces to five in this information environment, because, for Player  $i$ , Player  $j$ 's type is actually one: unknown. Then a strategy for Player  $i$  is now a tuple of only five dimensions. In order to design a strategy Player  $i$  uses a Harsanyi prior  $F$  to assign probabilities to Player  $j$ 's potential types:  $p^S$  if  $T_j = S$ ,  $p^{I^A}$  if it is  $I^A$  ( $T_j = I^A$ ),  $p^{I^E}$  if it is  $I^E$  ( $T_j = I^E$ ),  $p^{R^A}$  if it is  $R^A$  ( $T_j = R^A$ ), and  $p^{R^E} = 1 - p^S - p^{I^A} - p^{I^E} - p^{R^A}$  if it is  $R^E$ ,  $T_j = R^E$ .

Notwithstanding the uncertainty about Player  $j$ 's type, optimization problems and solutions for subgames where Player  $i \in \{I^E, I^A, R^E, R^A\}$  are identical to those already presented in the CI case (section 2.2), because the probability of getting infected is zero. Since this is not the case when Player  $i$  is Susceptible we write it here explicitly:

$$\begin{aligned}
\max U_t^S &= u(c_t^S, n_t^S) + \beta \left[ \left(1 - p_t^{I^E} \tau_t^{I^E} - p_t^{I^A} \tau_t^{I^A}\right) U_{t+1}^S + \left(p_t^{I^E} \tau_t^{I^E} + p_t^{I^A} \tau_t^{I^A}\right) U_{t+1}^I \right] \\
s.t. \quad &(1 + \mu_t) c_t^S = w_t n_t^S + \Gamma_t \\
&\tau_t^{I^E} = \pi_1 c_t^S c_t^{I^E} + \pi_2 n_t^S n_t^{I^E} + \pi_3 \\
&\tau_t^{I^A} = \pi_1 c_t^S c_t^{I^A} + \pi_2 n_t^S n_t^{I^A} + \pi_3
\end{aligned}$$

Strategically, Player  $i$ 's decisions take into account Player  $j$ 's best response and as such, her optimal consumption and hours worked are:

$$[c_t^{S^*}] : \frac{\partial u(c_t^{S^*}, n_t^{S^*})}{\partial c_t^{S^*}} + \beta \pi_1 \left( p_t^{I^E} c_t^{I^{E^*}} + p_t^{I^A} c_t^{I^{A^*}} \right) (U_{t+1}^I - U_{t+1}^S) = \lambda_t^S (1 + \mu_t)$$

$$[n_t^{S^*}] : \frac{\partial u(c_t^{S^*}, n_t^{S^*})}{\partial n_t^{S^*}} + \beta \pi_2 \left( p_t^{I^E} n_t^{I^{E^*}} + p_t^{I^A} n_t^{I^{A^*}} \right) (U_{t+1}^I - U_{t+1}^S) = -\lambda_t^S w_t$$

In terms of finding economic aggregates one can follow the same process as in the CI case (section 2.2). Nevertheless, note that in this case Susceptible people behave the same no matter who they interact with. Government budget constraint and market clearing conditions remain the same.

The total number of new infections is given by:

$$T_t = \int_0^{S_t} \int_0^{I_t^{I^E}} \tau_t^{I^E} dj di + \int_0^{S_t} \int_0^{I_t^{I^A}} \tau_t^{I^A} dj di = \pi_1 c_t^S S_t c_t^I I_t + \pi_2 n_t^S S_t n_t^I I_t + \pi_3 S_t I_t$$

while all other population dynamics behave as in the CI case (section 2.2).

It is worth noting that, the Macro-Sir Model in Eichenbaum et al. (2020a) is nested in our model. In fact, it is a particular case of the PII world, in which there are no asymptomatic infections (i.e.  $\chi^A = 0$ ).

### 2.3.1 Beliefs Dynamics

When information is partially incomplete, every interaction features an information asymmetry. However, we assume that this private information is collected by the government and made public as population aggregates. Later, we will explore the benefits from disclosing and divulging disaggregated information. For now, we assume all players can access this public aggregate information through government reports. Once they are informed, players go on to form their beliefs about the probabilities that the player they interact with is either  $I^E$  or  $I^A$ . Since population groups by health status changes over time, beliefs become dynamic:

$$\begin{aligned}
p_t^{I^A} &= \frac{I_t^A}{P_t} \\
p_t^{I^E} &= \frac{I_t^E}{P_t}
\end{aligned}$$

In section 5 we discuss in more detail the assumption that beliefs get these probabilities right.

## 2.4 Total incomplete information

The third world we study is one of Total Incomplete Information (TII), in which a player ignores other people's type and possibly her own type. In reality it is likely that an asymptomatic infected person does not know her health status. In our setup we assume this is the case. The uncertainty about one's own health status will affect consumption and work decisions and, in the aggregate, the pandemics dynamics will be different. The strategic behavior of the types Asymptomatic Infected, Asymptomatic Recovered and Susceptible will now be the same: in the absence of symptoms they behave as though there is always the risk of getting infected.

In the case of the Symptomatic Infected and Symptomatic Recovered we assume that because they exhibit or have exhibited symptoms, they do know their health status. The government can learn about it and publish aggregate statistics, but other players can not identify them individually. As they did in the previous two information cases, these types pick dominant strategies.

In this environment, the number of subgames becomes three. The two subgames that arise when Player  $i$  is  $I^E$  or  $R^E$  entail optimization problems and solutions that are the same as in the CI case (section 2.2). The subgame that arises when Player  $i$  is either susceptible, asymptomatic infected or asymptomatic recovered is the one we focus on now.

As in the PII case (section 2.3), Player  $i$  uses the Harsanyi prior  $F$  to assign probabilities to Player  $j$ 's possible types. To deal with the additional uncertainty about her own type, Player  $i$  when  $T_i = \{S, R^A, I^A\}$  now employs another Harsanyi prior,  $G$ , that assigns probabilities:  $q^S$  if her type is  $S$  ( $T_i = S$ ),  $q^{I^A}$  if it is  $I^A$  ( $T_i = I^A$ ) and  $q^{R^A}$  if it is  $R^A$  ( $T_i = R^A$ ). Player  $i$  will then solve (here  $A$  stands for asymptomatic including types  $S, R^A, I^A$ ):

$$\begin{aligned}
\max U_t^A &= q_t^S U_t^{A,S} + q_t^{I^A} U_t^{A,I^A} + q_t^{R^A} U_t^{A,R^A} \\
s.t. (1 + \mu_t) c_t^A &= w_t n_t^A + \Gamma_t \\
\wedge \tau_t^{I^E} &= \pi_1 c_t^A c_t^{I^E} + \pi_2 n_t^A n_t^{I^E} + \pi_3 \\
\wedge \tau_t^{I^A} &= \pi_1 c_t^A c_t^{I^A} + \pi_2 n_t^A n_t^{I^A} + \pi_3
\end{aligned}$$

With

$$\begin{aligned}
U_t^{A,I^A} &= u(c_t^A, n_t^A) + \beta \left[ (1 - \pi_r^A) U_{t+1}^{A,I^A} + \pi_r^A U_{t+1}^{A,R^A} \right] \\
U_t^{A,R^A} &= u(c_t^A, n_t^A) + \beta U_{t+1}^{A,R^A} \\
U_t^{A,S} &= u(c_t^A, n_t^A) + \beta \left[ (1 - p_t^{I^E} \tau_t^{I^E} - p_t^{I^A} \tau_t^{I^A}) U_{t+1}^{A,S} + (p_t^{I^E} \tau_t^{I^E} + p_t^{I^A} \tau_t^{I^A}) U_{t+1}^I \right]
\end{aligned}$$

Player  $i$ 's first-order conditions for consumption and hours worked are:

$$[c_t^A] : \frac{\partial u(c_t^{A^*}, n_t^{A^*})}{\partial c_t^{A^*}} + q_t^S \beta \pi_1 (p_t^{I^E} c_t^{I^E*} + p_t^{I^A} c_t^{I^A*}) (U_{t+1}^I - U_{t+1}^{A,S}) = \lambda_t^A (1 + \mu_t)$$

$$[n_t^A] : \frac{\partial u(c_t^{A^*}, n_t^{A^*})}{\partial n_t^{A^*}} + q_t^S \beta \pi_2 (p_t^{I^E} n_t^{I^E*} + p_t^{I^A} n_t^{I^A*}) (U_{t+1}^I - U_{t+1}^{A,S}) = -\lambda_t^A w_t$$

### 2.4.1 Aggregates and Equilibrium

Aggregate economic variables are obtained by a process analogous to that of the PII case (section 2.3).

#### Government

The new government budget constraint is:

$$\mu_t ((S_t + I_t^A + R_t^A) c_t^A + I_t^E c_t^{I^E} + R_t^E c_t^{R^E}) = \Gamma_t (S_t + I_t + R_t) \quad (16)$$

#### Equilibrium

Market clearing conditions are:

$$(S_t + I_t^A + R_t^A) c_t^A + I_t^E c_t^{I^E} + R_t^E c_t^{R^E} = AN_t \quad (17)$$

$$S_t n_t^A + I_t^A n_t^A + I_t^E \phi^I n_t^{I^E} + R_t n_t^R = N_t \quad (18)$$

#### Population Dynamics

New infection cases are given by:

$$\begin{aligned}
T_t &= \int_0^{S_t} \int_0^{I_t^E} \tau_t^{I^E} dj di + \int_0^{S_t} \int_0^{I_t^A} \tau_t^{I^A} dj di \\
&= \pi_1 c_t^A S_t c_t^I I_t + \pi_2 n_t^A S_t n_t^I I_t + \pi_3 S_t I_t
\end{aligned}$$

The rest of the population dynamics remain unchanged relative to what was explained in the CI case (section 2.2).

### 2.4.2 Beliefs Dynamics

Contrary to the PII case (section 2.3), here players do not know their own health status when they have not exhibited symptoms. We will assume that all of those players believe they are susceptible, so that  $q_t^S = 1$ . The lack of private information for the asymptomatic also means that people cannot observe population aggregates about Asymptomatic Infected. Thus, players must form their beliefs about the probability of encountering Asymptomatic Infected players differently to how they did in the PII case (section 2.3). One can have beliefs that lie inside a neighborhood of the true probability:

$$p_t^{IA} = \frac{I_t^A}{P_t} * (1 + \varepsilon_t^{IA})$$

We will initially assume that this error in assessing the true probability,  $\varepsilon_t^{IA}$ , is zero. This assumption can be relaxed, something we discuss in Section 5.

## 2.5 Calibration

Since our model follows the economic structure of Eichenbaum et al. (2020a), we take some parameters directly from them. Such is the case of  $A$ ,  $\theta$  and  $\beta$ . This parameters are set so that in the pre-epidemic steady state the model is able to match some relevant economic statistics of the US economy.

Similarly, our calibration of the epidemiological parameters is also based in Eichenbaum et al. (2020a). We take the value of the parameter  $\phi^I$  exactly from their model and maintain their assumptions about the herd immunity threshold (60% of initial population) and the time it takes for an infected person to either recover or die (14 days).

The other epidemiological parameters cannot have the same values because we incorporate asymptomatic infections. Nonetheless, we do use their method to calibrate such parameters. In particular,  $\pi_1, \pi_2, \pi_3$  and  $\pi_d$  are set to match the same aggregate transmissions and mortality patterns as in Eichenbaum et al. (2020a). We assume that symptomatic and asymptomatic infected people share these transmission parameters. Finally, we use the 40% estimate of the CDC (2020) for the share of total infections that are asymptomatic and calibrate  $\chi^A$  to match this. The table below summarizes the calibration used in our model.

Parameter	Value
$A$	39.835
$\beta$	$0.96^{\frac{1}{52}}$
$\theta$	0.0013
$\phi^I$	0.8
$\pi_1$	$7.8408e^{-8}$
$\pi_2$	$1.2442e^{-4}$
$\pi_3$	0.3902
$\pi_d$	0.0032
$\pi_R^A$	0.3889
$\pi_R^E$	0.3857
$\chi^A$	0.3993

## 2.6 Welfare Analysis

In sections 2.2, 2.3 and 2.4, we study the influence of public and private information in players' decisions. In particular, we show how their optimal decisions on consumption and work are modified in response to changes in their information sets. This section evaluates the effects of such changes in the aggregate social welfare of the economy during a span of five years. Our simulation technique follows the algorithm exposed in Eichenbaum et al. (2020a), where value functions are iterated backwards and the epidemiological block forward. Given that we simulate our model in a deterministic fashion, the relevant welfare indicator is the weighted sum of the value function of each type of player,  $U_t$ , at the initial period:

$$U_t = S_t U_t^S + I_t U_t^I + R_t U_t^R$$

This indicator summarizes the two forces in action during the pandemic's evolution: economic activity, and people's health status and deaths. Table 1 contains the value of this indicator across the three cases considered so far. These results show that the Complete Information Case is our best possible scenario, followed by the Partial Incomplete Information and the Total Incomplete Information cases. Hence, one can see how losing information completeness gradually worsens welfare. This happens because poorer information prevents players from understanding the nature of their interactions and making the proper choices.

Furthermore, the welfare losses in each scenario can be understood through two different channels. First, the fall in consumption can be used as a proxy of the size of the recession induced by the epidemic. Our calibration implies that the pre-epidemic per capita annual consumption is 58.000 USD. We take this value and multiply it by the cumulative fall in aggregate consumption to obtain the monetary economic loss of the epidemic during the five years horizon. Second, our calibration also implies the statistical value of life is 9.3 million US dollars of 2019, so we can use this figure to quantify the costs of the deaths caused by the epidemic<sup>4</sup>. Let's recall that the cost of a death in the model is equal to the present value of foregone utility. For the CI case these losses are equal to 169.676.100.000

<sup>4</sup>The Unites States current GDP was 21.433.000.000.000 for 2019 <https://data.worldbank.org/country/united-states>

USD and 2.117.610.000.000 USD, respectively.

However, we are not only interested in quantifying the costs of the epidemic per se, but the costs or benefits of different information settings. Thus, we find that the absence of commonly known information about other players' health statuses (i.e. the PII case) causes additional losses of 1.025.808.300.000, due to lower economic activity, and of 6.273.036.000.000 associated with deaths. Analogously, we see that when private information is also absent (i.e. the TII case), the economy suffers an additional loss of 287.004.300.000 with respect to the PII case, while the cost of deaths actually decreases in 220.968.000.000 USD.

Figure 1 shows how the Complete Information case has a considerably lower number of deaths than the other two. This is because the public and total availability of private information about other's health status allows each player to reduce the intensity of interactions that bear contagion risk, flattening thusly the infections curve. Nevertheless, it is worth noting that this flat curve implies that Herd Immunity is not reached within the horizon considered, despite the epidemic is controlled and dies out as a consequence of the information completeness. The infection curves of the other two cases considered are quite similar, although it is slightly lower in the TII case. Here the difference is explained by the presence of a positive externality of losing private information about a player's type. Particularly, when asymptomatic infected players ignore their type, they reduce their economic activity as a result of their false perception of being vulnerable to the virus, a behavior that ultimately reduces the propagation of the virus.

As we can see in Figure 2, the fall in economic activity is smallest in the CI case. In this scenario, susceptible agents only reduce their economic activity in risky interactions, which in turn implies a minor aggregate contraction. When susceptible agents are no longer able to make this distinction (i.e. the PII case) they reduce more aggressively their economic activity to avoid getting infected. Nonetheless, their efforts are not very effective and the infection curve rises considerably. This further decreases economic activity due to the greater aggregate loss of productivity when there are more symptomatic infected. Finally, aggregate economic variables also fall because deaths increase.

If agents are also unable to know their health status, the same channel that generated the positive externality in infections through asymptomatic infected, implies a negative effect on economic activity. This is magnified by the additional reaction of recovered asymptomatic agents that further reduces their economic activity as a result of their perception of contagion risk. The joint reaction of asymptomatic infected and recovered explain the greater fall in economic activity compared with the PII case.

### 3 Containment Measures

Containment measures, such as lockdowns, school closures, restrictions on gatherings, and other mobility restrictions, have been the primary policy intervention put in place by most countries in their attempt to limit the effects of the COVID-19 on fatalities and their health systems. The results have been heterogenous, with some countries apparently being more successful than others (Deb et al. (2020)).



These measures prevent many transactions from taking place, thereby reducing economic activity and creating a trade-off between economic and health outcomes. This trade-off is the motivation behind the literature that studies the interaction between an epidemic and the macroeconomy. Eichenbaum et al. (2020a) model a quarantine as a tax to consumption and find the optimal path for a simple containment policy in which everybody is taxed, such that the benefits of lives saved outweigh the costs of worsening the recession. Eichenbaum et al. (2020b) study “smart” containment measures in which only sub-groups of the population are quarantined in the search for improving the health-economy trade-off. Acemoglu et al. (2020) and Berger et al. (2020) also study the gains from establishing quarantines for particular population groups.

In this section we ask, how does information affect the ability of both general and conditional containment policies to improve the health-economy trade-off?

A general containment policy is one that applies to all the population. In the model it is instrumented through a consumption tax. The revenue raised by the government is then rebated back to all population groups by means of a lump-sum transfer. This type of containment is the one that is presented explicitly in the model of Section 2.

A conditional containment policy seeks to exploit information (kept private from other players) about the health status of people to establish focused quarantines, avoiding the confinement of people who have achieved immunity or who do not have the virus. Under this policy, only people who generate the negative externalities from contagion are put in lockdown. This is implemented in the model through a consumption tax rate  $\mu_t^E$  for the symptomatic infected patients and  $\mu_t^A$  for the asymptomatic infected people. At the same time, the government only makes transfers to people affected by the externality: the susceptible population in CI and PII cases. This conditional containment is in fact an imperfect compensation mechanism, given that the tax collection and the transfers do not occur by interaction, but by player’s type.

We now compare the level of containment that maximizes the discounted social welfare as defined in Section 2.6 for the different information cases. Figure 3 shows that in the Complete Information world optimal general containment is zero and there is only a small positive conditional containment. Notice that conditional containment is still desirable in this world, because the externality generated by infected people still exists. Hence, in a world where information about the pandemic is fully available to everybody and everybody is able to process it efficiently, containment measures provide marginal gains. As shown in Table 1, this is true even if containments is conditional and its size is calibrated optimally, improving relative welfare to 0.0003%. The reason is that agents in the economy use the information to minimize market interactions where there is risk of contagion and engage normally in all other transactions. See Figures 4 and 5.

When information about people’s health status is kept private and agents cannot identify the contagion risk-free transactions, quarantine-type measures become optimal. As Table 1 shows, conditional containment policies yield better welfare outcomes vis-a-vis general containment, from  $-0.1741\%$  to  $-0.164\%$  relative welfare losses. This is the result of both higher aggregate consumption and hours worked ( $-1.14\%$  vs.  $-4.97\%$ ). In turn, this is explained by

1) consumption of recovered patients does not fall; 2) due to the more-targeted transfers in conditional containment, susceptibles' consumption does not fall as much; 3) despite the more pronounced decline in consumption of the infected, the flattening of the epidemic curve reduces the aggregate effect over time.

It is relevant to remark that conditional containment rates are orders of magnitude higher than those of general containment. To a large extent, this is due to the fact that asymptomatic infected people have a dominant strategy, which is to engage in all possible transactions, posing a large negative externality on others. Moreover, this sort of containment allows the government to only impose a cost on those that are propagating the virus. In order to reduce the virus propagation, the conditional confinement rates must be high, reducing the consumption of those infected (see Figure 6). When there is common-knowledge, complete information, the negative effects of such strategy are attenuated since susceptibles are able to reduce the intensity of interactions with asymptomatic patients.

In the TII scenario, the unavailability of private information on health to the government and players makes it impossible to establish conditional containment measures. In this case policy makers are left with the option of general containment policies which, as the literature has shown, exacerbates the health economy trade-off: the reduction of contagion and fatalities comes at the expense of larger declines in economic activity as shown in Figures 8 and 9.

There are couple of points worth mentioning. First, Figure 3 shows that regardless of the type of containment being considered, the less information is available the more aggressive optimal containment measures must be. In other words, more complete information helps players choose better their interactions, therefore reducing the volume of hazardous interactions that need to be avoided or diminished through containments.

Finally, Table 1 reports the welfare losses from all confinement measures. Even though they allow to mitigate the effects from the externalities stemming from contagion and improve welfare, they still exhibit a large gap with respect to the ideal complete information world, with more infections, deaths, and larger reductions in economic activity. This is due to the fact that instead of relaxing the trade-off between economic and health outcomes, containment exploits the trade-off to control the infection. Even when there are conditional, confinements impose a consumption cost because agents are forced not to engage normally in transactions even if they pose no risk for themselves or others. In contrast, when all information about each other's health is available to everybody the economic-health trade-off can be relaxed, so individuals are able to choose optimally the intensity of their interactions with each other and minimize thusly the risk of contagion without sacrificing their consumption.

## 4 Information Policy Tools

In the previous section we showed that optimal containment policies are not very effective in getting the economy close to a world of complete information in terms of welfare, deaths, and aggregate macroeconomic variables. In these section we consider policy tools that can actually fill the information gaps between the TII case and the first-best, so that the welfare gap between them closes.

In particular, we study two policy tools that can provide valuable information: testing and divulgation. Testing can fill the information gap that individuals have about their own health status. This information, gathered by health authorities, becomes privately known by the tested individual and disclosed at an aggregate level to all players. Divulgation makes this information publicly known, so that any given player can incorporate information on other people's health in her decision making. As a matter of fact, a policy implemented along these lines was seen in South Korea, where authorities disclosed detailed information on infected people to manage the epidemic. More specifically, the mechanism consisted of an intensive use of text messages to disclose and propagate information about the health status of infected individuals and the places they had recently visited. Argente et al. (2020) examined the effects of such policy on the epidemiological dynamics in Seoul and found that people modified their commuting patterns in response to the information, which resulted in turn in a reduction in infections and deaths. One can think of divulgation as "painting people's faces". Here we think of divulgation as a tool to provide individuals with better information at the interaction level, so that by completing players' information sets, we allow them to play out strategies where they can identify and engage normally in more mutually beneficial economic interactions (i.e. interactions with no contagion risk). The more information one provides, the lower the probability of engaging in risky interactions.

Starting from a world where information about health statuses is not common nor private, testing gets society closer to the PII case, whereas divulgation gets society closer to the CI case. We simplify the analysis by assuming that tests are performed only on asymptomatic people. In the model we do not test those who are already sick: we assume that the symptoms are enough to tell whether someone has the disease of interest. In this sense, testing serves the purpose of revealing private information to the agents about their own health status (type). With this in mind, the population subject to tests is given by  $As_t - R_{t-1}^X - I_{t-1}^X$ , where  $As_t = S_t + I_t^A + R_t^A$  is the asymptomatic population at time  $t$ . From this population we subtract those asymptomatic who have recovered  $R_{t-1}^X$  because, due to the immunity assumption, once they know their type it will not change. Similarly, we also subtract people who were asymptomatic infected since the person will know her health status in the future from the recovery dynamics of the virus disease itself. A number of  $X_t$  tests are performed at random on this population, such that in expectation:

$$\begin{aligned}
X_t &= X_t^S + X_t^I + X_t^R \\
&= X_t \text{Prob}(T_i = S) + X_t \text{Prob}(T_i = I^A) + X_t \text{Prob}(T_i = R^A) \\
&= X_t \frac{S_t}{As_t - R_{t-1}^X - I_{t-1}^X} + X_t \frac{I_t^A - I_{t-1}^X - \pi_r^A I_{t-1}^X}{As_t - R_{t-1}^X - I_{t-1}^X} + X_t \frac{R_t^A - R_{t-1}^X + \pi_r^A I_{t-1}^X}{As_t - R_{t-1}^X - I_{t-1}^X}
\end{aligned}$$

The population dynamics of the groups that get to learn their type, that is, the tested asymptomatic recovered  $R_t^X$ , the tested asymptomatic infected  $I_t^X$ , and the tested susceptible  $S_t^X$ , are given by:

$$\begin{aligned}
R_t^X &= R_{t-1}^X + X_t^R + \pi_r^A I_{t-1}^X \\
I_t^X &= I_{t-1}^X + X_t^I - \pi_r^A I_{t-1}^X \\
S_t^X &= X_t^S
\end{aligned}$$

Once players get information about their own health through testing, an asymmetry of information arises and with it a rationale to make this information publicly known at an individual level. Divulcation is an instrument that theoretically gives the susceptibles the possibility to distinguish between contagion risky versus a contagion riskless interaction. However, since not everybody is tested and the policy maker does not know who all the susceptibles are, the information is aimed at two groups of people: the tested susceptible  $S_t^X$  and people who do not know their type  $A_t^{NX} = S_t - S_t^X + I_t^A - I_t^X + R_t^A - R_t^X$ . The divulgation mechanism consists in giving the available information to a number of people belonging to each of these groups and which we denote  $Z_t^{SX}$  and  $Z_t^A$ , respectively. The information that is revealed are: the symptomatic infected and recovered, the tested asymptomatic infected, the tested susceptibles and the tested recovered. The people who receive the information are selected randomly and its number is given in expectation by:

$$\begin{aligned}
Z_t &= Z_t^{SX} + Z_t^A \\
&= Z_t^{SX} + Z_t^S + Z_t^I + Z_t^R \\
&= Z_t^{SX} + Z_t^A \text{Prob}(T_i = S) + Z_t^A \text{Prob}(T_i = I^A) + Z_t^A \text{Prob}(T_i = R^A) \\
&= Z_t^{SX} + Z_t^A \frac{S_t - S_t^X}{A_t^{NX}} + Z_t^A \frac{I_t^A - I_t^X}{A_t^{NX}} + Z_t^A \frac{R_t^A - R_t^X}{A_t^{NX}}
\end{aligned}$$

where  $Z_t^S$ ,  $Z_t^I$  y  $Z_t^R$  are the number of asymptomatic susceptibles, infected and recovered who do not know their type at time  $t$  and who are the receptors of the divulged private information.

In the model, the costs of testing and divulgation are financed through lump-sum taxes  $\Gamma_t^{Inf}$  levied on all agents in the economy, in such a way that:

$$\Gamma_t^{Inf}(S_t + I_t + R_t) = -mc_t^X X_t - mc_t^Z Z_t$$

The costs we consider have two components. The first component is the unit cost of a test (which we calibrate to be \$20). For simplicity, we abstract from any additional costs associated with testing. We also focus on the marginal cost, ignoring any initial investment required to set up testing infrastructure. The second component is the marginal cost of disclosing private information effectively at the individual level. This cost may include non-pecuniary costs such as ethical and regulatory restrictions on making public personal information, logistical and technological costs related to making the information available, and the capacity constraints that people may have when trying to process a high volume of information in an efficient way, such that they can use it to better choose how to act.

## 4.1 Modified Model

We now adjust the model to include the information tools introduced above. We do this for the TII case, so that this modified version can nest the three information cases explained in Section 2. When the cost of testing is zero, it is possible to get to the case of Partial Incomplete Information. Similarly, when the divulgation cost is zero and the information can be made available and processed perfectly, we can get to the Complete Information case.

In this version of the model we introduce new groups of agents. We will now distinguish the tested susceptibles,  $S_t^X$ , the tested asymptomatic infected,  $I_t^X$ , and the tested asymptomatic recovered,  $R_t^X$ . Additionally, due to divul-gation, we will now consider four possible interaction cases for asymptomatic and tested susceptibles. These are 1) when they do not know the other's type; 2) when they know the other's type and there is no risk of contagion; 3) when they know the other's type is asymptomatic infected and there is risk of contagion; and 4) when they know the other's type is symptomatic infected and there is risk of contagion.

We now present the key elements of the game that change in this set-up due to the new types and interactions mentioned above. Nonetheless, one can notice that the symptomatic infected and symptomatic recovered face the same optimization problems than they did in Section 2.2. Similarly, tested asymptomatic infected and recovered behave as the asymptomatic infected and recovered of the CI case explained in Section 2.2.

#### 4.1.1 Player $i$ is asymptomatic

An asymptomatic player solves the following problem:

$$\begin{aligned} \max U_t^{A^J} &= q_t^S U_t^{S,A^J} + q_t^{I^A} U_t^{I^A,A^J} + q_t^{R^A} U_t^{R^A,A^J} \\ \text{s.t. } (1 + \mu_t) c_t^{A^J} &= w_t n_t^{A^J} + \Gamma_t + \Gamma_t^{Inf} \\ \wedge \tau_t^{I^E J} &= \pi_1 c_t^{A^J} c_t^{I^E} + \pi_2 n_t^{A^J} n_t^{I^E} + \pi_3 \\ \wedge \tau_t^{I^A J} &= \pi_1 c_t^{A^J} c_t^{I^A} + \pi_2 n_t^{A^J} n_t^{I^A} + \pi_3 \end{aligned}$$

With  $J = \{U, NI, I^A, I^E\}$  indexing the different possibilities for information about player  $j$ 's type: unknown ( $U$ ), known and no contagion-risk ( $NI$ ), known and asymptomatic infected ( $I^A$ ), and known and symptomatic infected ( $I^E$ ). Also, the value functions on the right-hand side of the objective function are:

$$\begin{aligned} U_t^{I^A,A^J} &= u(c_t^{A^J}, n_t^{A^J}) + \beta \left[ (1 - \pi_r^A) (U_{t+1}^{I^A,A^J}) + \pi_r^A U_{t+1}^{R^A,A^J} \right] \\ U_t^{R^A,A^J} &= u(c_t^{A^J}, n_t^{A^J}) + \beta U_{t+1}^{R^A,A^J} \end{aligned}$$

The problem above states that, despite varying with player  $j$ 's type, the total value function of an untested asymptomatic player weights the different value functions ( $U_t^{S,A^J}, U_t^{I^A,A^J}, U_t^{R^A,A^J}$ ) according to her beliefs about her own health status ( $q_t^S, q_t^{I^A}, q_t^{R^A}$ ).

However, note that the value functions when player  $i$  beliefs she is  $I^A$  or  $R^A$  will not change with player  $j$ 's type. Nevertheless, when player  $i$  beliefs she is susceptible, even if she does it to a small degree, her value function and the probabilities of contagion will be influenced by her risk perception; that is, by her information about player  $j$ 's type. See the Appendix (10) to explore the specifics of this problem.

## Aggregation

The economic variables of the untested asymptomatic are given by two groups: those who received information about other players' types and those who did not. The former group is, in turn, subdivided into the different types she can encounter.

$$\begin{aligned}
As_t U_t^A &= \frac{Z_t^A}{P_t} \int_0^{P_t} U_t^A(j) dj + (As_t - Z_t^A) U_t^{AU} \\
&= \frac{Z_t^A}{P_t} \left[ (S_t^X + R_t^X + R_t^E) U_t^{ANI} + I_t^X U_t^{AI^A} + I_t^E U_t^{AI^E} + A_t^{NX} U_t^{AU} \right] \\
&\quad + (As_t - Z_t^A) U_t^{AU}
\end{aligned}$$

Aggregation for consumption and hours worked follows this same procedure yielding:

$$\begin{aligned}
As_t c_t^A &= \frac{Z_t^A}{P_t} \left[ (S_t^X + R_t^X + R_t^E) c_t^{ANI} + I_t^X c_t^{AI^A} + I_t^E c_t^{AI^E} + A_t^{NX} c_t^{AU} \right] \\
&\quad + (As_t - Z_t^A) c_t^{AU}
\end{aligned}$$

$$\begin{aligned}
As_t n_t^A &= \frac{Z_t^A}{P_t} \left[ (S_t^X + R_t^X + R_t^E) n_t^{ANI} + I_t^X n_t^{AI^A} + I_t^E n_t^{AI^E} + A_t^{NX} n_t^{AU} \right] \\
&\quad + (As_t - Z_t^A) n_t^{AU}
\end{aligned}$$

The equations above show that the representative decisions of an untested asymptomatic player are influenced by the information sets that they get, which are improved by the divulgation mechanism,  $Z_t^A$ . From this, it follows that the decisions of this representative player will end up being a weighted average of the decisions taken when she has common information and when she does not. This helps to see divulgation as a tool that improves, in the average interaction, the information sets with which players choose their actions.

### 4.1.2 Player $i$ is a tested susceptible

If player  $i$  is tested and knows she is susceptible, her optimization problem is:

$$\begin{aligned}
&\max U_t^{S^{X,J}} \\
&s.t. (1 + \mu_t) c_t^{S^{X,J}} = w_t n_t^{S^{X,J}} + \Gamma_t + \Gamma_t^{Inf} \\
&\wedge \tau_t^{IE S^{X,J}} = \pi_1 c_t^{S^{X,J}} c_t^{I^E} + \pi_2 n_t^{S^{X,J}} n_t^{I^E} + \pi_3 \\
&\wedge \tau_t^{IA S^{X,J}} = \pi_1 c_t^{S^{X,J}} c_t^{I^A} + \pi_2 n_t^{S^{X,J}} n_t^{I^A} + \pi_3
\end{aligned}$$

Player  $i$ 's value function and contagion probabilities now change with the information she has on player  $j$ 's type. Hence, this player faces four interaction scenarios just as it occurred with the untested asymptomatic. See the Appendix (10) to explore the specifics of this problem.

## Aggregation

The aggregate economic variables for those players who know themselves to be susceptibles is:

$$S_t^X U_t^{S^X} = \frac{Z_t^{S^X}}{P_t} \left[ (S_t^X + R_t^X + R_t^E) U_t^{S^X, NI} + I_t^X U_t^{S^X, IA} + I_t^E U_t^{S^X, IE} + A_t^{NX} U_t^{S^X, U} \right] + (S_t^X - Z_t^{S^X}) U_t^{S^X, U}$$

$$S_t^X c_t^{S^X} = \frac{Z_t^{S^X}}{P_t} \left[ (S_t^X + R_t^X + R_t^E) c_t^{S^X, NI} + I_t^X c_t^{S^X, IA} + I_t^E c_t^{S^X, IE} + A_t^{NX} c_t^{S^X, U} \right] + (S_t^X - Z_t^{S^X}) c_t^{S^X, U}$$

$$S_t^X n_t^{S^X} = \frac{Z_t^{S^X}}{P_t} \left[ (S_t^X + R_t^X + R_t^E) n_t^{S^X, NI} + I_t^X n_t^{S^X, IA} + I_t^E n_t^{S^X, IE} + A_t^{NX} n_t^{S^X, U} \right] + (S_t^X - Z_t^{S^X}) n_t^{S^X, U}$$

Note that, as seen in the aggregation of the untested asymptomatic players, here divulgation,  $Z_t^{S^X}$ , also acts as a tool that improves, in the average interaction of the tested symptomatic players, the information sets with which they choose their actions.

### 4.1.3 Final Aggregation and Market Clearing

The aggregation of the value functions for susceptibles, infected and recovered yields:

$$\begin{aligned} R_t U_t^R &= R_t^E U_t^{R^E} + (R_t^A - R_t^X) U_t^A + R_t^X U_t^{R^X} \\ I_t U_t^I &= I_t^E U_t^{I^E} + (I_t^A - I_t^X) U_t^A + I_t^X U_t^{I^X} \\ S_t U_t^S &= (S_t - S_t^X) U_t^A + S_t^X U_t^{S^X} \end{aligned}$$

Is easy to see that aggregate consumption and hours worked resemble this aggregate value functions.

#### Government

Government's set-up looks a bit different in the modified version of the model, in view of the new policy tools. To finance testing and divulgation, the government levies a lump-sum tax on all players.

$$\mu_t (S_t c_t^S + I_t c_t^I + R_t c_t^R) = \Gamma_t (S_t + I_t + R_t) \quad (19)$$

$$-\Gamma_t^{Inf} (S_t + I_t + R_t) = m c_t^X X_t + m c_t^Z Z_t \quad (20)$$

#### Equilibrium

Adding up the budget constraints of the players that populate the economy and those of the government, we get that

the aggregate budget constraint is:

$$S_t c_t^S + I_t c_t^I + R_t c_t^R = AN_t - mc_t^X X_t - mc_t^Z Z_t \quad (21)$$

The labor market clears so that:

$$S_t n_t^S + I_t^A n_t^{IA} + I_t^E \phi^I n_t^{IE} + R_t n_t^R = A_t N_t \quad (22)$$

### New Population Dynamics

The total number of newly infected people at time  $t$  comes from all the interactions between players  $i$  and  $j$  that entangle a risk of contagion for either one:

$$\begin{aligned} T_t &= \int_0^{S_t} \int_0^{I_t} \tau_t(i, j) \, di \, dj \\ T_t &= \left( S_t - S_t^X - Z_t^S \right) \left( \pi_1 c_t^{AU} c_t^I I_t + \pi_2 n_t^{AU} n_t^I I_t + \pi_3 I_t \right) \\ &+ Z_t^S \left[ I_t^A \left( \pi_1 c_t^{AIA} c_t^{IA} + \pi_2 n_t^{AIA} n_t^{IA} + \pi_3 \right) + I_t^E \left( \pi_1 c_t^{AIE} c_t^{IE} + \pi_2 n_t^{AIE} n_t^{IE} + \pi_3 \right) \right] \\ &+ \left( S_t^X - Z_t^{SX} \right) \left( \pi_1 c_t^{SXU} c_t^I I_t + \pi_2 n_t^{SXU} n_t^I I_t + \pi_3 I_t \right) \\ &+ Z_t^{SX} \left[ I_t^A \left( \pi_1 c_t^{SXI^A} c_t^{IA} + \pi_2 n_t^{SXI^A} n_t^{IA} + \pi_3 \right) \right. \\ &\left. + I_t^E \left( \pi_1 c_t^{SXI^E} c_t^{IE} + \pi_2 n_t^{SXI^E} n_t^{IE} + \pi_3 \right) \right] \end{aligned}$$

The rest of the population dynamics are remained unchanged with respect to what was shown in Section 2.2.

#### 4.1.4 Modified Beliefs

As in the case of Total Incomplete Information (Section 2.4), consumers form their beliefs over their own health status and the health status of people with whom they interact. However, now agents receive more information and use it to form their beliefs. The more information they receive, the closer their beliefs will be to the true probabilities.

On the one hand, individuals who have not been tested and have not experienced symptoms, form their beliefs using the aggregate information made publicly available by the policy makers. This aggregate information consists of the number of tests performed,  $X_t$ , and their results by type:  $X_t^I$ ,  $X_t^R$  y  $X_t^S$ . Under the assumption that agents in this economy know the testing technology, they use the tests results to gauge the probability of having a particular health status as follows:

$$q_t^{IA} = \frac{X_t^I}{X_t} = \frac{I_t^X - I_{t-1}^X + \pi_r^A I_{t-1}^X}{S_t^X + I_t^X + R_t^X - I_{t-1}^X - R_{t-1}^X}$$



$$q_t^{R^A} = \frac{X_t^R}{X_t} = \frac{R_t^X - R_{t-1}^X - \pi_r^A I_{t-1}^X}{S_t^X + I_t^X + R_t^X - I_{t-1}^X - R_{t-1}^X}$$

$$q_t^S = \frac{X_t^S}{X_t} = \frac{S_t^X}{S_t^X + I_t^X + R_t^X - I_{t-1}^X - R_{t-1}^X}$$

Here we assume that in absence of testing all non-symptomatic players believe themselves to be susceptible (i.e.  $q_t^S = 1$ ), like we did in the TII case (Section 2.4).

Additionally, by the assumption we have made throughout that aggregate information on symptomatic infected is publicly known, players form their beliefs as follows:

$$p_t^{I^E} = \frac{I_t^E}{P_t}$$

On the other hand, we assume that testing also affects the beliefs people have on the probability of meeting an asymptomatic infected person:

$$p_t^{I^A} = p_t^{I^A,PII} \frac{X_t}{As_t - R_{t-1}^X - I_{t-1}^X} + \left(1 - \frac{X_t}{As_t - R_{t-1}^X - I_{t-1}^X}\right) p_t^{I^A,TII}$$

where  $p_t^{I^A,TII}$  are the beliefs under Total Incomplete Information (Section 2.4) and  $p_t^{I^A,PII}$  are the beliefs under Partial Incomplete Information, just as they were shown in Section 2.3. Notice that under the assumption that  $\varepsilon_t^{I^A} = 0$ , which we also made in Section 2.4, this equation collapses to:

$$p_t^{I^A} = \frac{I_t^A}{P_t}$$

A more detailed discussion about this topic is provided in Section 5.

## 4.2 Testing

Testing is the first information instrument that one could consider to close the welfare gaps created by information incompleteness. Testing gives players private information about their own health and gives information to authorities that is usually communicated to the public as aggregate numbers on the disease.

In order to gauge the impact of testing in closing the welfare gap, we run an exercise to find the optimal path of testing to maximize the discounted aggregate welfare of the economy. We do so assuming that no disaggregated, private information is revealed to the public. Through this exercise we go from the world of Total Incomplete Information to the world of Partial Incomplete Information in which there is perfect private information.

Figure 10 shows the evolution of the population by epidemiological status under the optimal testing policy. As shown, testing by itself makes the population outcomes worse. The reason is that testing creates an information asymmetry that exacerbates the negative externalities imposed by the infected and reduces the positive externalities imposed by the asymptomatic who are uncertain about their health status. Specifically, the asymptomatic infected have a dominant strategy in which they favor their economic decisions, engaging in more transactions and thereby pushing up the infections curve above that of the TII world. This affects the welfare of the susceptible population. Similarly, the recovered asymptomatic also have a dominant strategy favoring their economic activity. As Figure 11 shows that, under the baseline calibration and optimal testing, the increased economic interactions of these groups are enough to produce improved economic aggregates with respect to the TII world.

Under the baseline calibration, welfare improves as the improvement in aggregate economic outcomes outweighs the negative effects of higher numbers of infections and deaths, as quantified in Table 2. Even though testing allows society to get closer (how close depends on the cost of testing) to the Partial Incomplete Information world, the gains from testing are modest vis-a-vis the losses stemming from the absence of common information. Despite this fact, is important to recall that testing is a necessary step to implement a more detailed divulgation of health statuses.

When testing is the only policy in action, its optimal path exhibits an accelerated behavior in the first twenty weeks, since the marginal gains from obtaining the information are large as cases soar. At the peak, 96% of the initial population will be tested in a week. Once the epidemic starts to recede, the gains from testing diminish and in the optimal it falls to 0% by week 50. On average, over the 5-year horizon of analysis, 11% of the population is tested per week, a number that is similar to the one found in Eichenbaum et al. (2020b) (see Figure 12 and Table 2).

### 4.3 Testing and Divulgation

In the previous section we quantified the positive effects of testing on aggregate welfare in the economy, under the assumption that no personal, private health information was made public. This way of handling the information has been the norm in most countries.

However, testing creates an information asymmetry and induces behavior that tempers its gains. Removing this asymmetry should be beneficial as the susceptible population could act optimally on it by reducing the intensity of those interactions where there is risk of contagion. As we show in Section 2.6, making all information public would imply gains of about \$6.3 trillion attributed to fewer deaths and of about \$1 trillion due to less pronounced falls in consumption. As found by Argente et al. (2020) in their study of the case of South Korea, the gains from making more detailed information publicly available are potentially large.

We now find the optimal paths for testing  $X_t$  and divulgation  $Z_t$  so as to maximize the discounted aggregate welfare of the economy. Since the information that is divulged to the public depends on the testing the optimization is performed over the two instruments simultaneously. A difficulty to produce an optimal path for divulgation is determining its marginal cost. This cost may include a variety of dimensions including some type of social cost due

to loss of privacy, the cost/difficuty of processing large amounts of information due to some capacity constraint, or monetary costs. In order to illustrate the results we use two cases. In the first case divulgation has zero marginal cost and in the second one it has a cost of \$10, half the cost of testing. Together these two cases allow us to analyze the marginal effect on the health and economic dynamics of adding costs to divulgation. Such analysis gives us a more comprehensible understanding of the extent to which divulgation can relax the health-economy trade-off and improve welfare if some or all of the costs aforementioned are present. Table 2 contains the welfare calculations. Even though these calculations depend on the value of the marginal cost of divulgation, the economic benefits are so large that dwarf the cost. In other words, the cost of divulging would have to be too large in order not to do it. The margin to get positive benefits from making private health information public is given by the gap between the Complete Information and the Partial Incomplete Information worlds.

The large benefits from disclosing private information come from reducing the information assymetries, which allows susceptible players to reduce intensity of risky interactions and dampen the effects of externalities. There are three sources of externalities: the symptomatic infected,  $I^E$ , who always know they have the virus and have no incentives to reduce their consumption, the non-tested asymptomatic infected,  $I^A - I^X$ , and the tested asymptomatic infected,  $I^X$ . The latter group learns about their health status once they get tested and modify their behavior imposing a negative externality on others. In the absence of testing, only the health status of those individuals who are symptomatic, either infected or recovered, could be potentially made public. Divulging private information from testing, makes testing more productive. At the beginning, as infections rise, it may be worth performing more tests as information becomes more valuable (see Figure 16). However, as Table 2 shows, the level of testing never reaches the levels of optimal testing of the no divulgation case, because the information that is released becomes very useful for players to make optimal decisions and avoid contagion. Some countries faced testing capacity constraints during the first stages of the COVID-19 pandemic. Divulgation lowers the capacity requirements and lowers the investment needed to set up such capacity.

Two more facts about testing and divulgation are illustrated in Figure 15. First, it shows that irrespective to its marginal cost, when divulgation is implemented, both testing and divulgation levels must be above zero during the entire horizon. The reason for this is that divulgating markedly flattens the infections curve and thus, herd immunity is never reached. If this is not attained, there is a latent risk of another outbreak. An alternative off-model benefit of this result is that an infection curve as flat as the one yielded by divulgation, buys authorities more time to find an effective vaccine or treatment with lower welfare losses.

Second, Figure 15 also shows that under costly divulgation there is a substitution between information policy tools. One reason for this substitution is that a different level of divulgation changes the infections curve and a higher peak of infections is reached faster. If there are more infections, testing brings larger gains and there is an incentive to increase it. Moreover, costly divulgation implies that the marginal benefit of divulgating is not necessarily greater than its marginal cost at all time. Notably, when there is not many people infected, revealing that information has low aggregate impacts, because there are not as much risky interactions but there are yet a lot of people to be informed.

Figure 13 shows and quantifies the flattening of the infections curve and the reduction in deaths under the COVID-19 calibration for two levels of the divulgation marginal cost. At the same time, the publicly known information allows susceptible agents to carry out more (potentially all) of their contagion-risk free transactions, which results in higher consumption. Figure 14 shows substantially higher levels of economic activity and smoother dynamics. Using the results from Table 2, under the baseline calibration, the gains with respect to the case in which only testing is used are quantified to be between US\$216 billion and US \$670 billion coming from higher consumption and between US\$5.7 trillion and US \$6 trillion coming from fewer deaths, depending on the two different levels for the marginal cost of divulgation.

The results aforementioned highlight the value of common information during a pandemic. It is worth mentioning that divulgation makes the pandemic last longer as the disease does not spread fast enough to reach the level of immunity as in the test-only case, but all outcomes exhibit smoother dynamics lowering the stress on testing facilities, health systems, and the economy.

#### 4.4 The "Optimal Mix"

In this section we put together the three policy tools that we have considered and that are available to control the pandemic, and find its optimal combination. There are at least a couple of reasons to study the interaction of the three instruments.

First, the marginal cost of divulgation is hard to pin down and measure, which means that there is uncertainty as to how close divulgation can get us to the complete information world. One can end up close to the PII scenario, in which containment measures can generate significant welfare gains, as shown in Section 3.

Second, not only it is the case that, as long as there are information gaps, containment measures may have an important role to play, but there may also be uncertainty about the parameters that determine the power of different policies. For example, lower risk aversion would increase the marginal benefit of containment policies even in the presence of a high degree of divulgation<sup>5</sup>. This is true because even in an ideal scenario of full testing and divulgation, which is identical to the CI world, the infection externality persists. Recall that in Section 3, it is shown that even when there is complete information welfare can be improved through conditional containments. It is possible then to study in the modified model the optimal mix under different parameter configurations.

With this in mind, we performed two simulation exercises combining the three policy instruments optimally to maximize aggregate discounted welfare. We will find the "Optimal Mix" of policies for two different values of the marginal cost of divulgation. For this marginal cost we used the same values of the previous section. Regarding containment policies we only consider conditional containments because, as we showed in Section 3, general containment policies make sense only in the world of total incomplete information. There will be two conditional containment rates, one for symptomatic patients ( $\mu_t^E$ ) and one for asymptomatic infected that get identified by testing ( $\mu_t^A$ ). The containment scheme will still involve an imperfect compensation mechanism, since the revenue collected from those

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<sup>5</sup>The model could be potentially extended to study optimal policies in the presence of heterogeneity. For example, the existence of a mass of low risk aversion individuals may result in higher gains from combining instruments than the gains one gets under the average risk aversion. See Brotherhood et al. (2020)

in lockdown is transferred to both the tested susceptible and the non-tested asymptomatic.

Under our baseline calibration, with costly divulgation, the gains from conditional containments are rather small, as shown in Table 2. The changes in both population outcomes (Figure 17) and aggregate economic variables (Figure 18) are marginal.

When the cost of divulgation falls to zero, there are important welfare gains but the outcome is still not quite the one of the CI case (see Table 2). Figure 19 illustrates the reason behind the small gains from adding containment policies to the mix. When divulgation is costly, the budget constraint of infected people becomes more stressed, which limits the room to impose more stringent lockdowns. Although optimal containment levels vary, the optimal paths for information instruments do not have significant changes because the tools are not substitutes.

## 5 Beliefs Discussion

In Sections 2.3.3, 2.4.3 and 4.1.4 we talked about players beliefs. Particularly, in Section 2.4.3 we made an explicit assumption about how players formed their beliefs in the absence of any information about asymptomatic population aggregates. The immediate consequence of this assumption is that players' beliefs are unbiased and reflect perfectly the real probabilities of facing asymptomatic infected players. Thus, the welfare loss between Partial Incomplete Information and Total Incomplete Information might be biased by this assumption. Nonetheless, our assumption is based in two factors: i) beliefs that are far from the real probabilities can induce inestability in the equilibrium of this type of games<sup>6</sup>; ii) choosing the sign and magnitude of the bias will in any case be a difficult task, since there is no enough studies or information about it.

Moreover, this assumption was maintained in Section 4.1.4 when we introduced information policy tools, even though we introduced a new mechanism. Specifically, in this section we specified a technology of beliefs where private information attained by the government through testing and later disclosed through aggregates publication, helps to improve the precision with which players form their beliefs. However, this channel was not explored in such section, since the beliefs under Total Incomplete Information were already unbiased. Consequently, players were still able to know the real probabilities irrespective of the level of testing. This means that the welfare gains of testing could be biased.

In the present section, we examine the welfare implications of deviating from this perfect beliefs formation assumption and analyze the mechanisms through which it can affect the model. For this purpose, we do two sensibility analyses, in which our benchmark scenario is the TII case. In both of them, we considered multiplicative constant biases present throughout the entire simulation horizon ranging between  $[-0.5, 1.5]$  with a granularity of 0.05.

We **first** analyzed the sensibility of the results of the Testing and Costly Divulgation Scenario. For this purpose,

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<sup>6</sup>This inestability arises because the game equilibria will no longer be sequentially rational or Bayes consistent.

we held constant the optimal paths of this scenario for both information policy tools across the different biases. This assumption enabled us to isolate the mechanism through which the bias distorts the model and its effects on welfare under two very different information environments.

We then did a **second** sensibility analysis in an information environment where there is no divulgation. However, we kept using the same path for testing as the one in the latter sensibility exercise. Removing divulgation in this fashion allowed us to analyze the marginal contribution of both testing and divulgation, while keeping the results as comparable as possible to the first sensibility analysis.

The results of this two exercises are shown in Figures 20 and 21 (7.6), respectively. The main conclusions that arise in these graphs are:

1. A positive bias worsens welfare and a negative bias improves it. This holds in every information policy mix, even in total absence of testing and divulgation.
2. Regardless of the sign or magnitude of the bias, the divulgation improves welfare with respect to the TII case. This further supports the usefulness of this information policy tool.
3. When there is a positive bias, given a combination of information policy tools, the marginal welfare gain of such policy is greater than with no bias.
4. A positive bias of any magnitude does not seem to change the policy recommendation of testing to improve welfare. However, this might not be the case if the bias is negative, since if it is sufficiently large, such bias can entail welfare losses when testing. A note of caution about this conclusion is that, although it is not possible to say that by itself testing is every time and everywhere desirable, it is possible to say that there cannot be any divulgation without testing.

## 6 Conclusions

In this paper, we develop an analytical framework that combines a game theory set-up and the Macro-SIR model proposed in Eichenbaum et al. (2020a), to understand how information influences the spread of an epidemic and to quantify its importance for economic welfare. As a case study, we applied the model to analyze from its outbreak the COVID-19 epidemic in the US and show that the lack of both private and common information generates relevant welfare losses, albeit the greater losses are associated with the latter. Accordingly, we propose disclosure and divulgation as a novel policy tool to alleviate the consequences for society from publicly available disaggregated information scarcity. This policy can be combined optimally with other policies, such as testing and containments, obtaining great welfare gains. However, we are aware that privacy is important but its costs are much too great in the presence of externalities and this paper is useful for the privacy debate.

# 7 Figures

## 7.1 Welfare

Figure 1: Population Dynamics

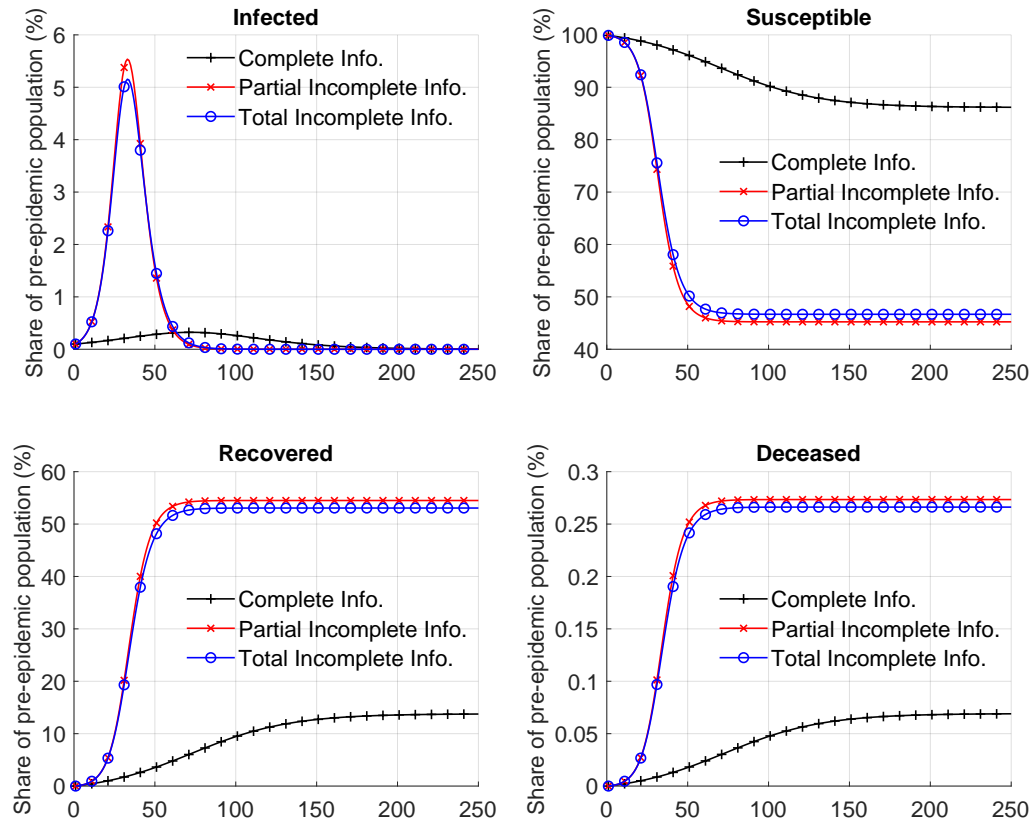
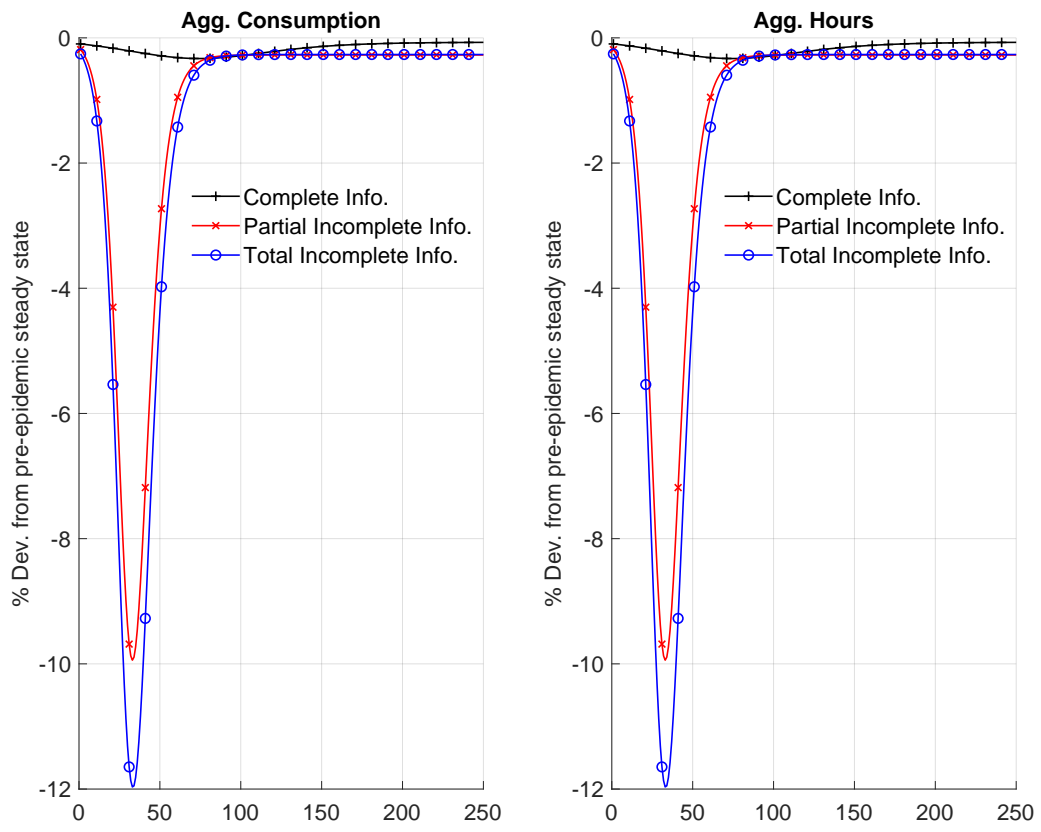


Figure 2: Economic Aggregates





## 7.2 Comparison

Figure 3: Optimal Containment Policies Compared

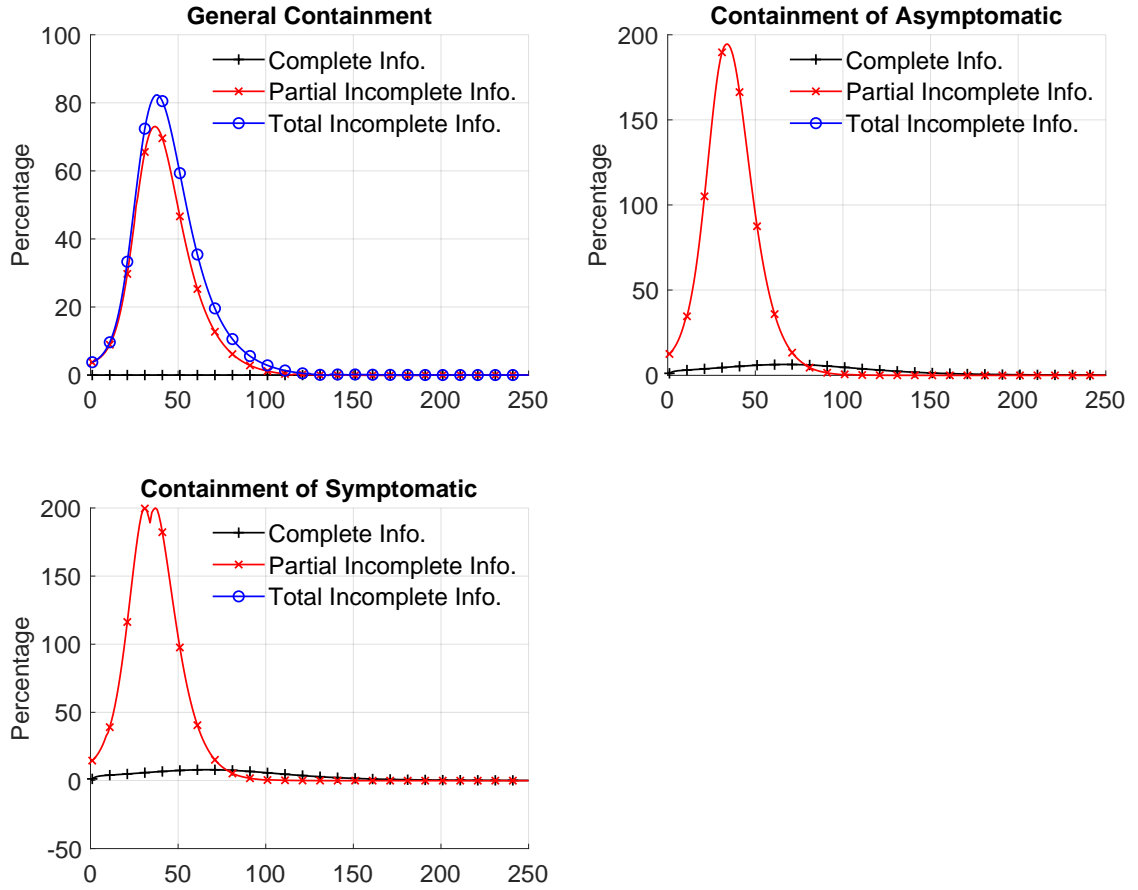


Figure 4: Population Dynamics - Comparison(CI)

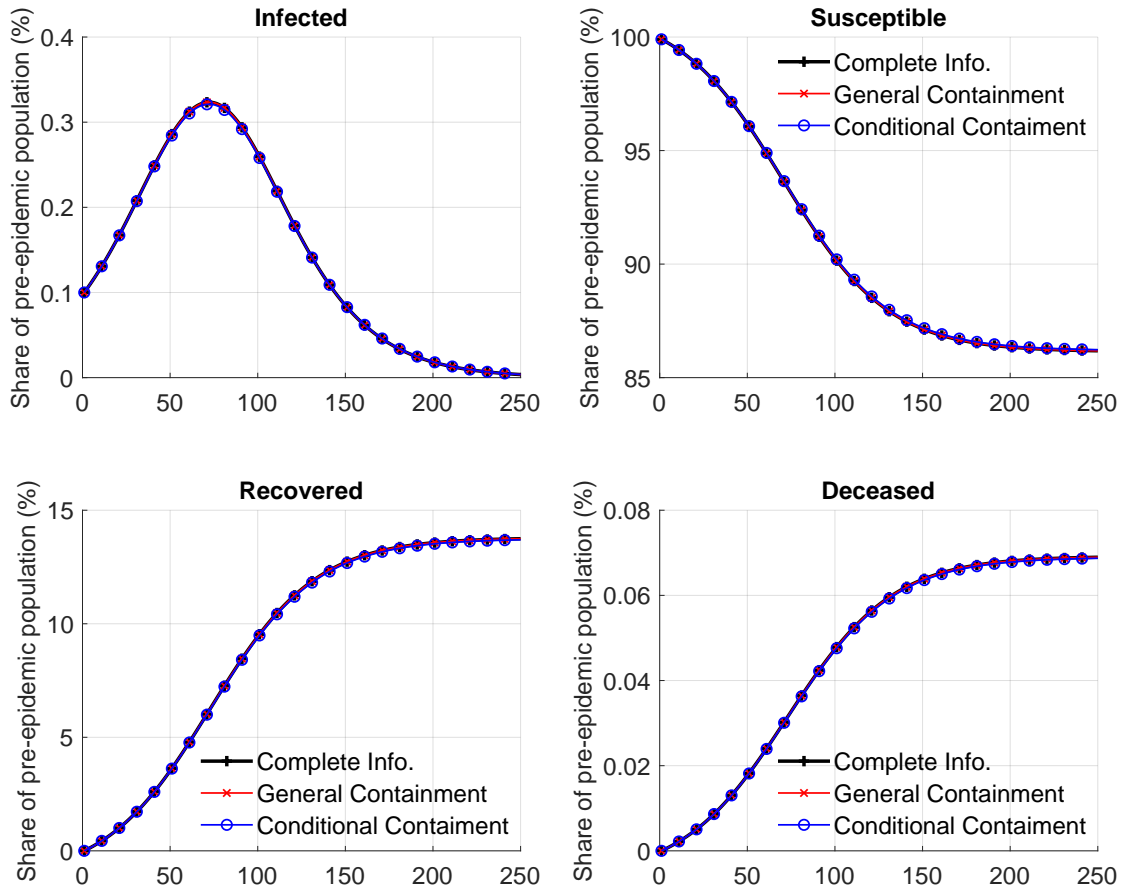


Figure 5: Economic Aggregates - Comparison(CI)

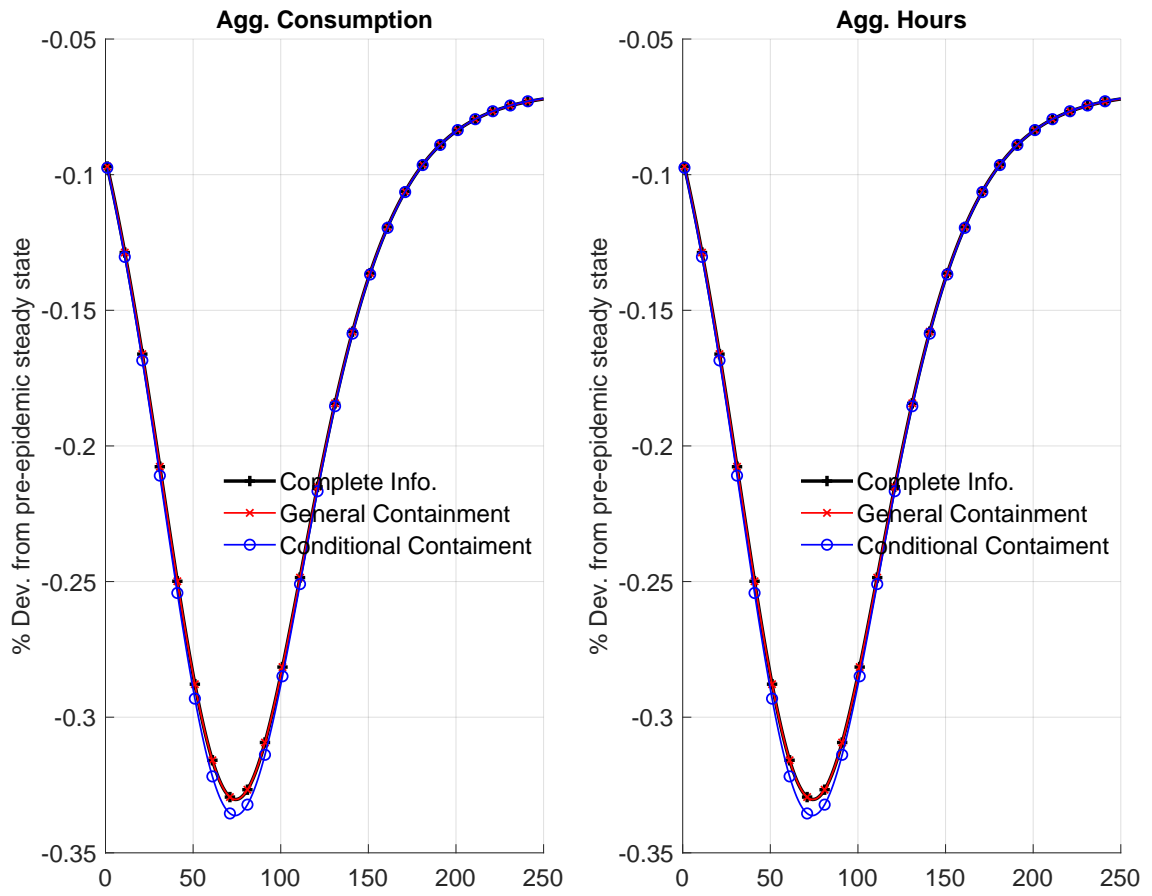


Figure 6: Population Dynamics - Comparison(PII)

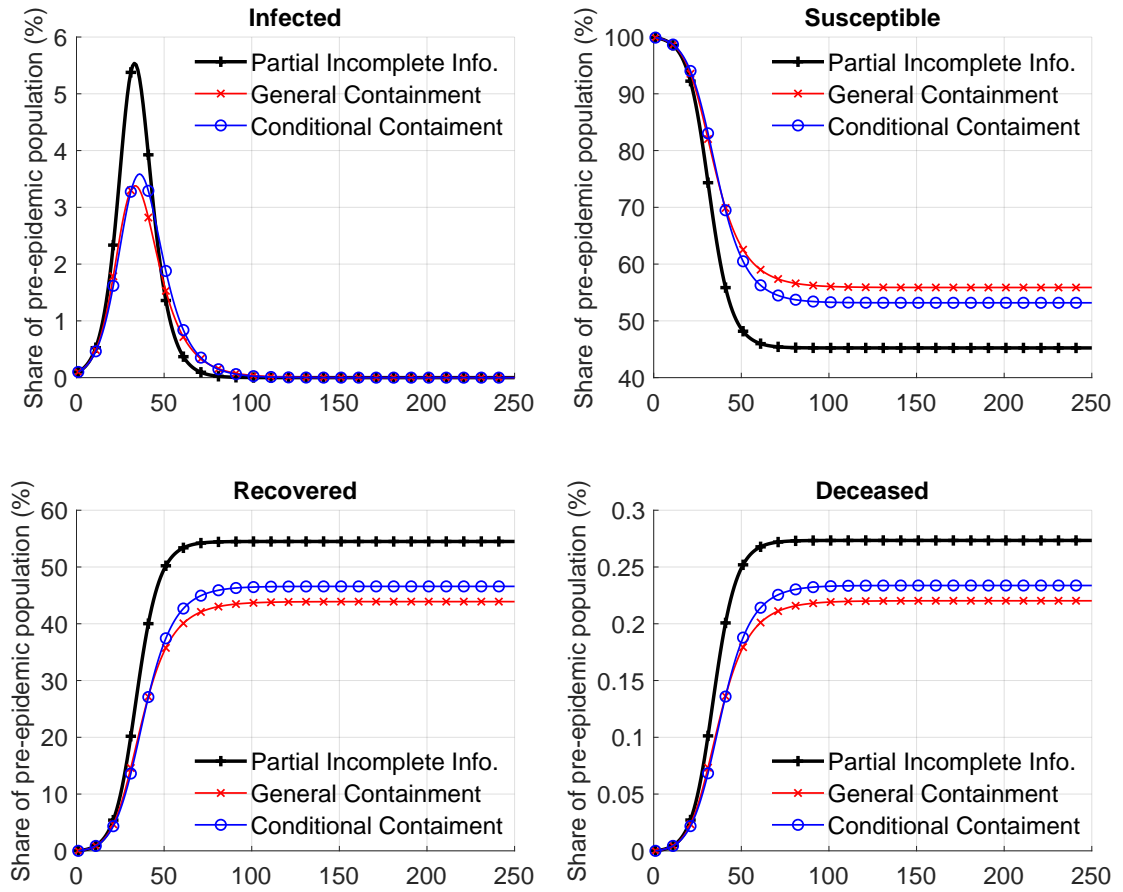


Figure 7: Economic Aggregates - Comparison(PII)

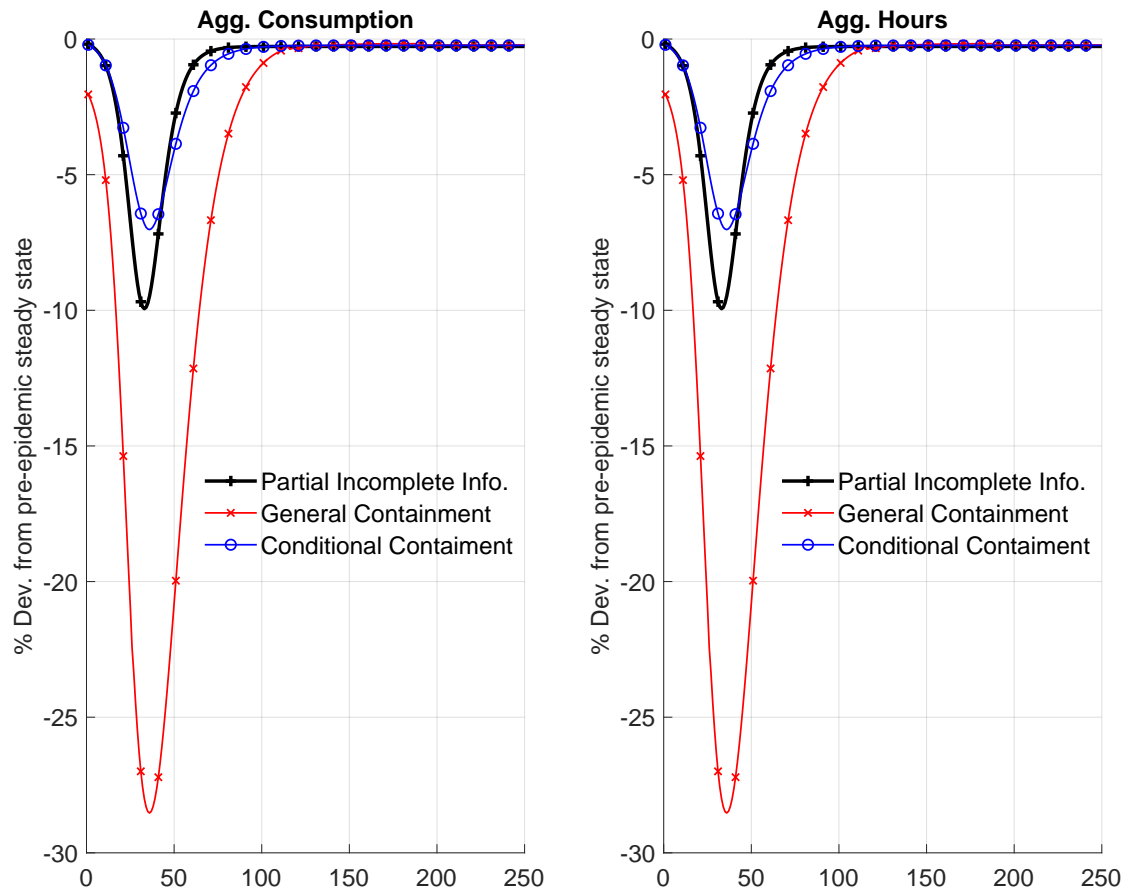


Figure 8: Population Dynamics - Comparison(TII)

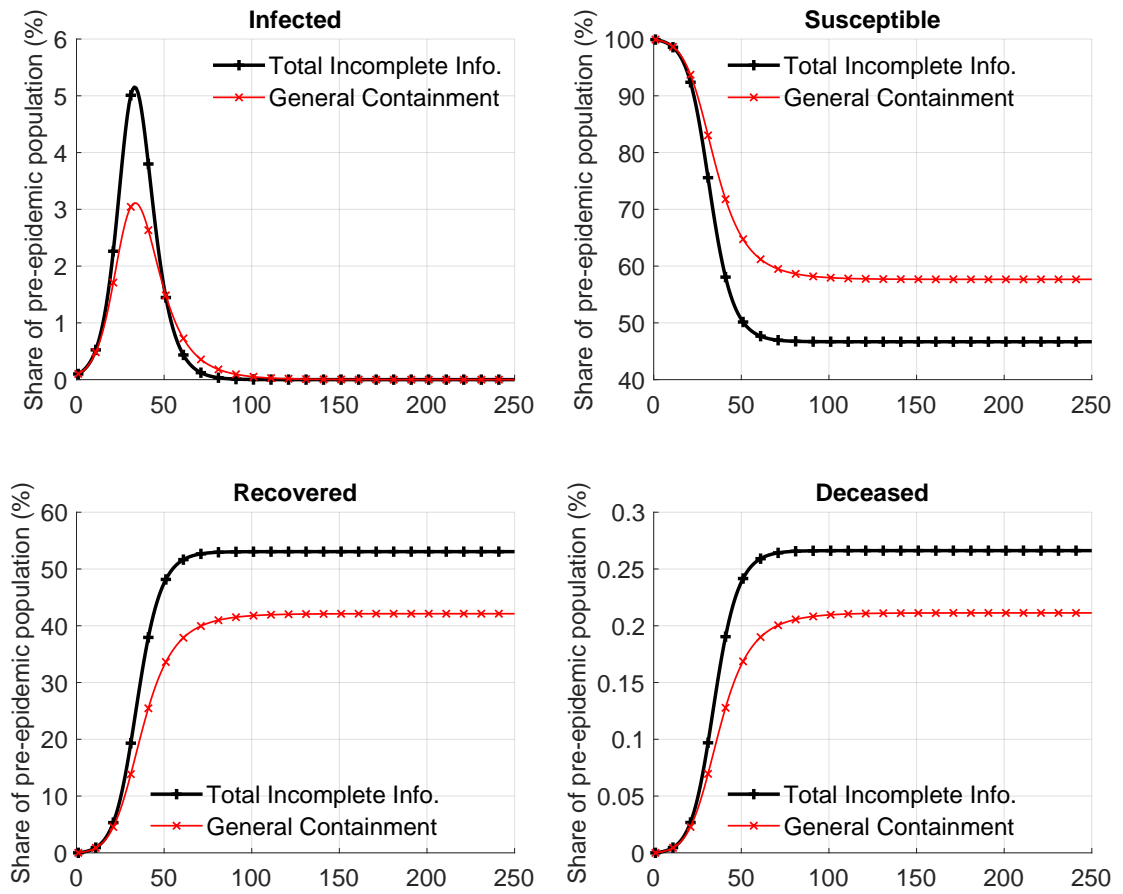
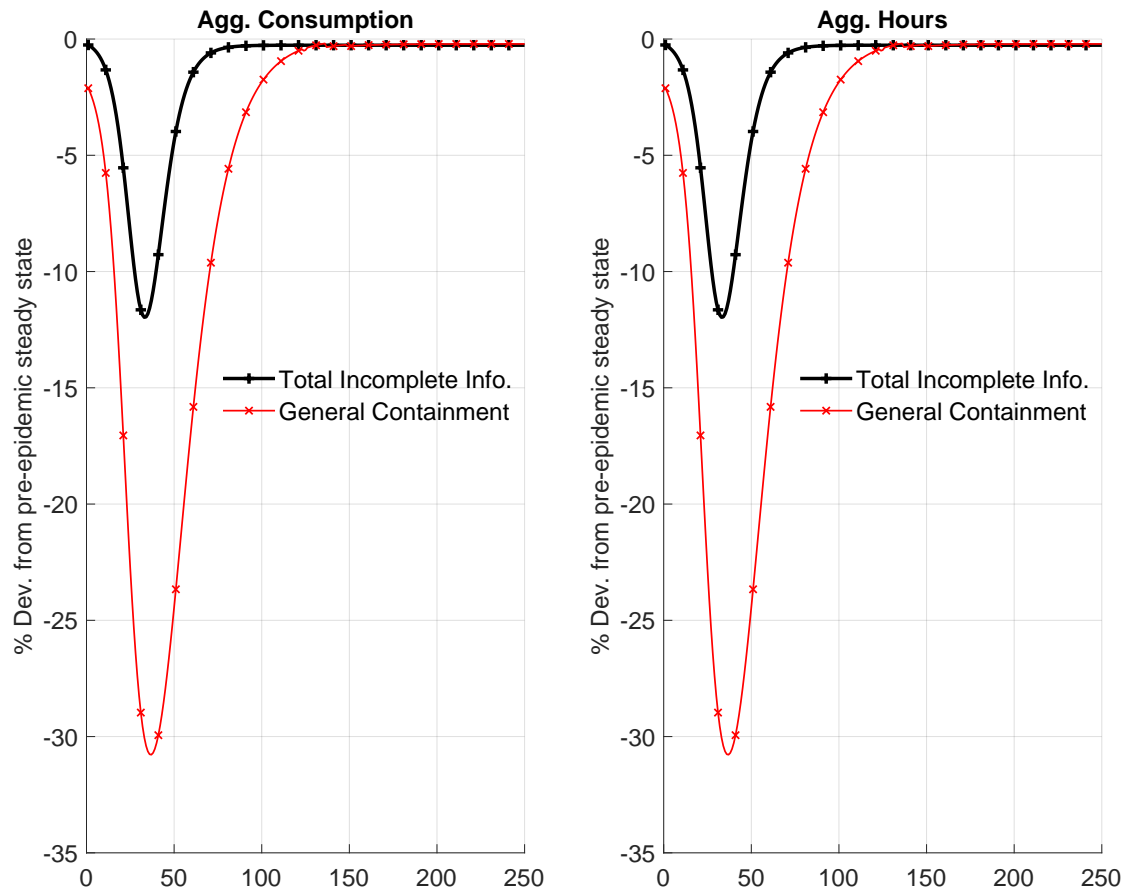


Figure 9: Economic Aggregates - Comparison(TII)



### 7.3 Testing

Figure 10: Population Dynamics - Testing

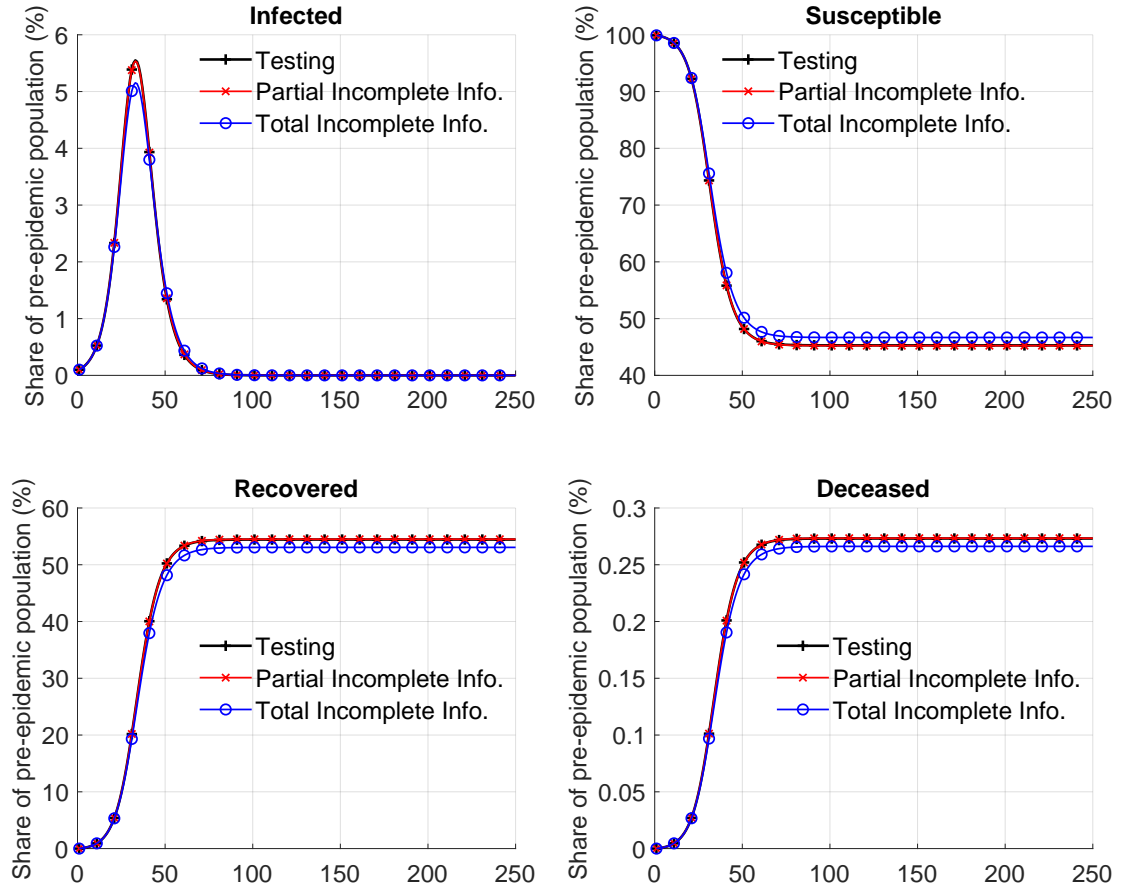




Figure 11: Economic Aggregates - Testing

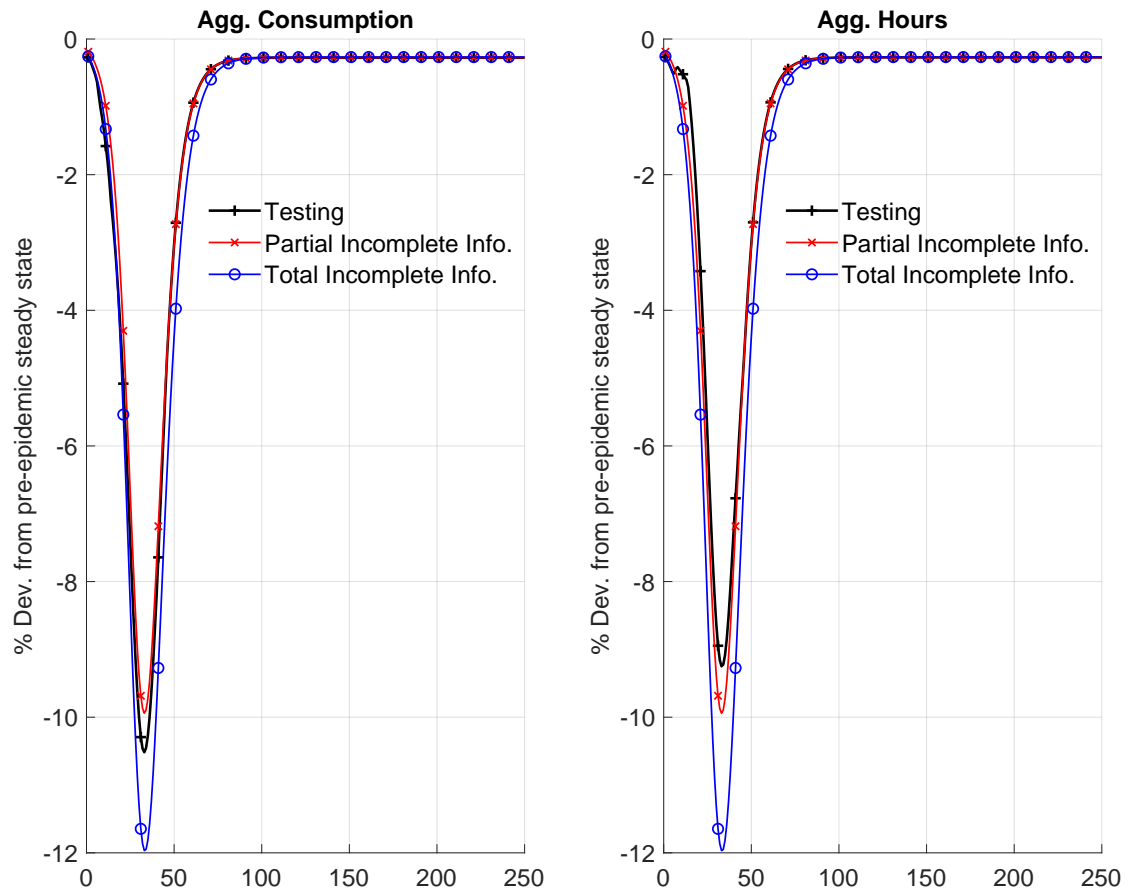
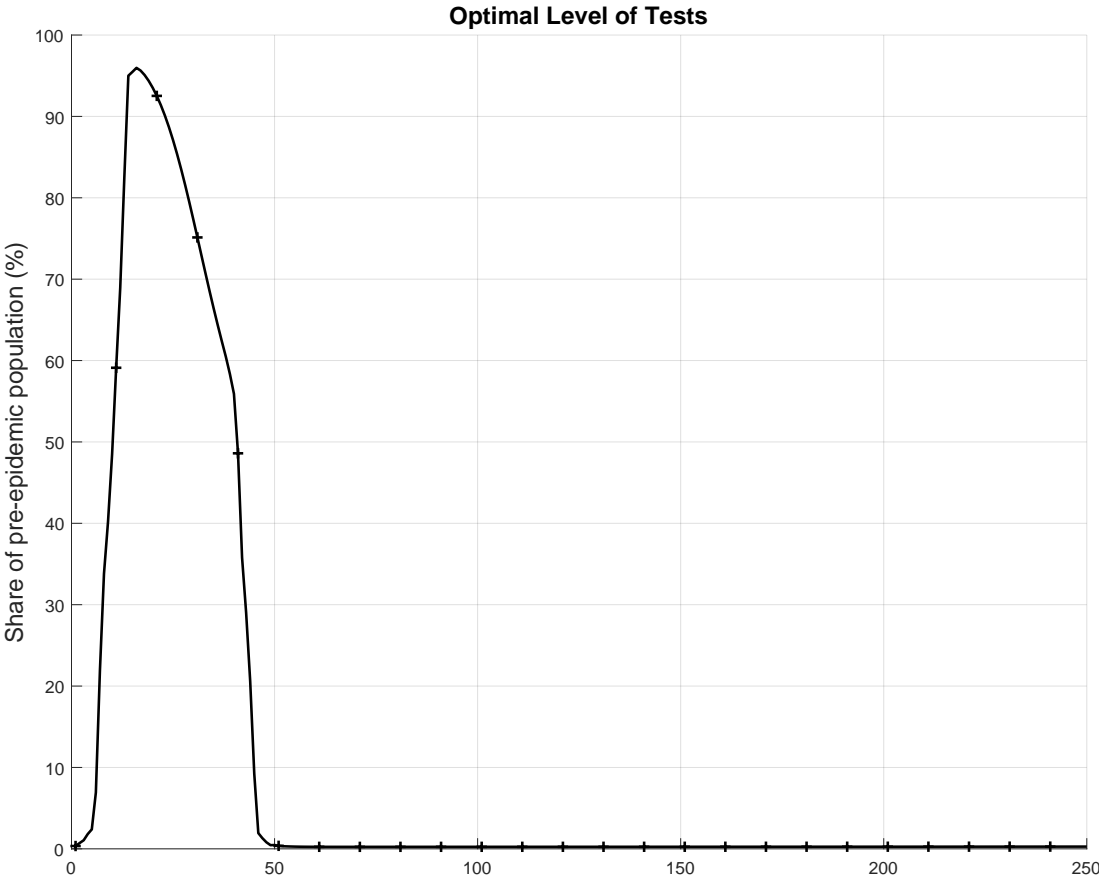


Figure 12: Optimal Tests - Testing



## 7.4 Testing and Divulgence

Figure 13: Population Dynamics - Testing and Divulgence

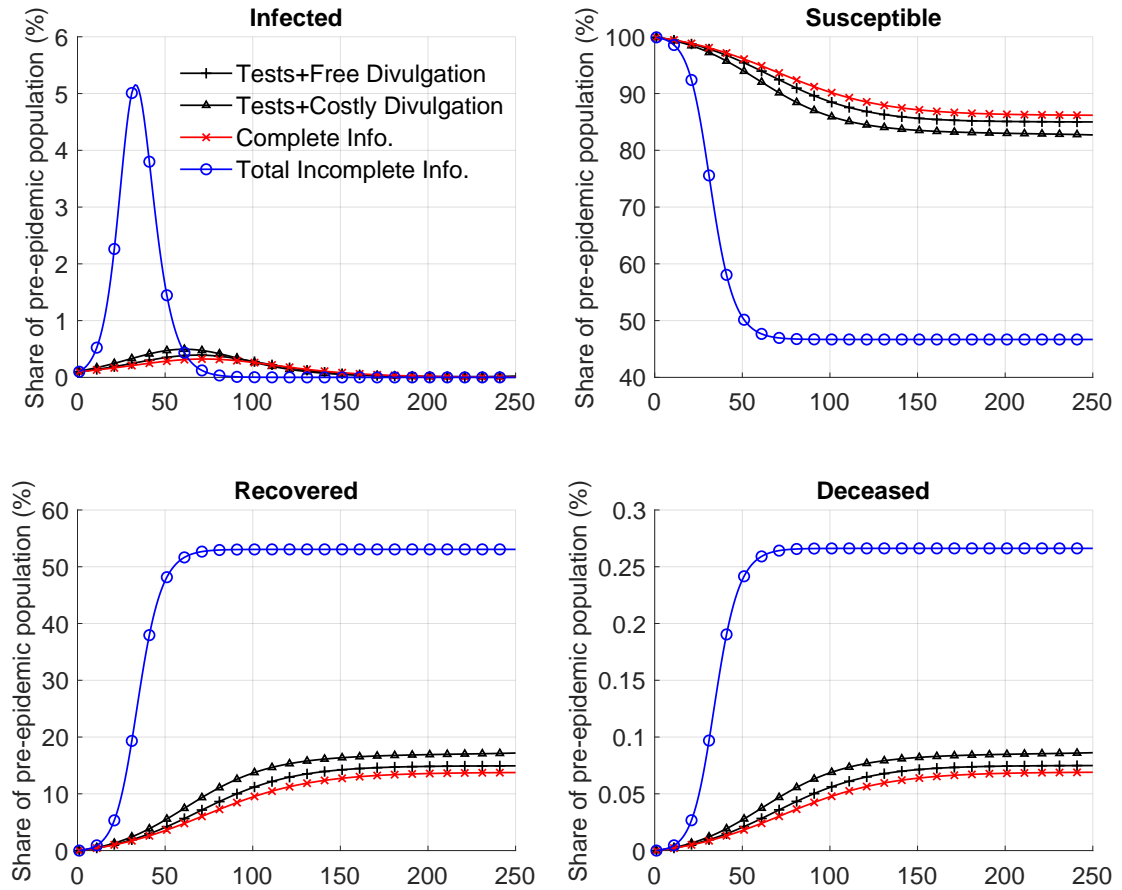


Figure 14: Economic Aggregates - Testing and Divulgence

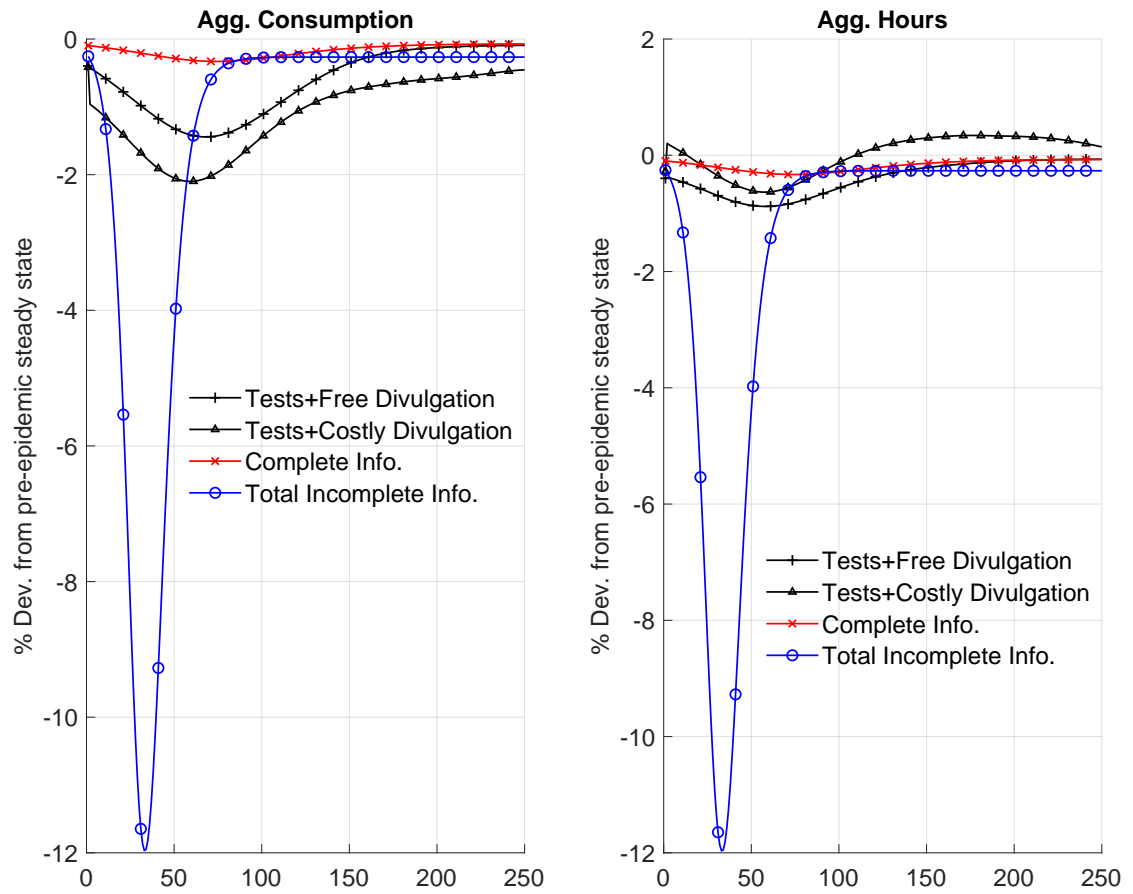


Figure 15: Optimal Tests - Testing and Divuligation

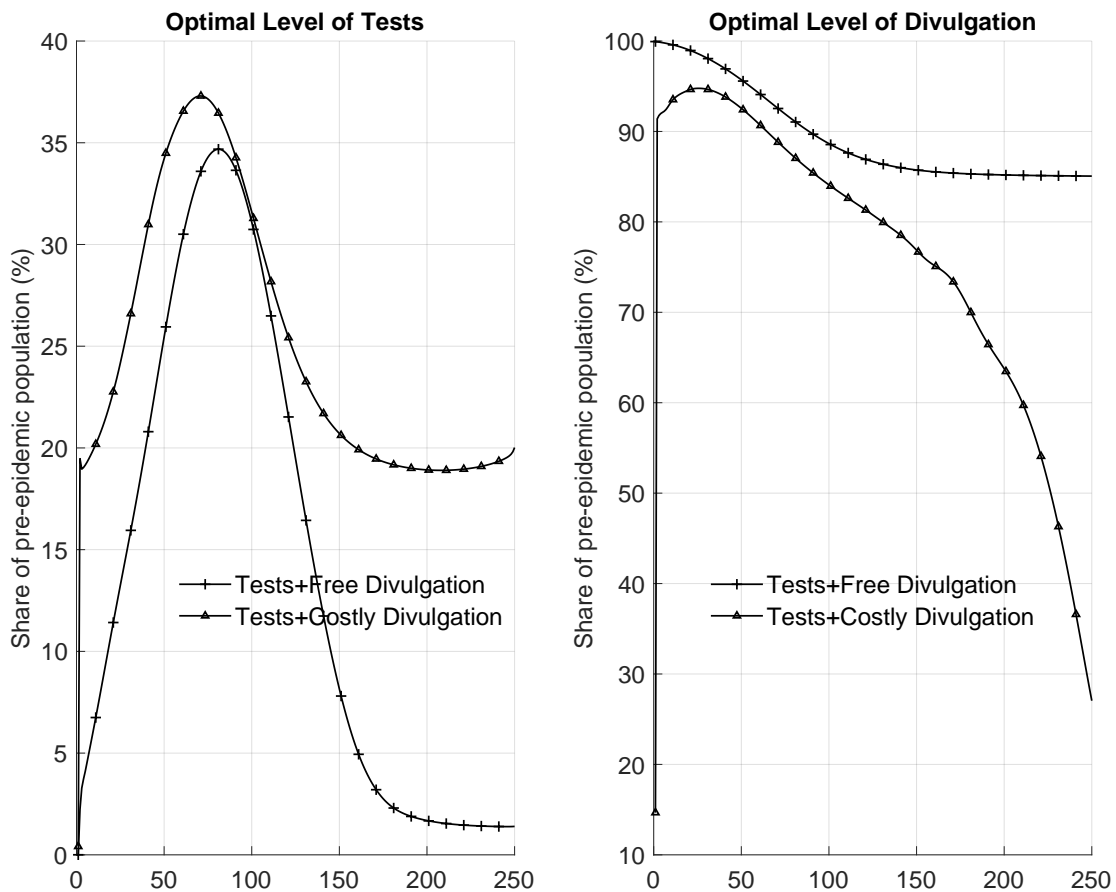
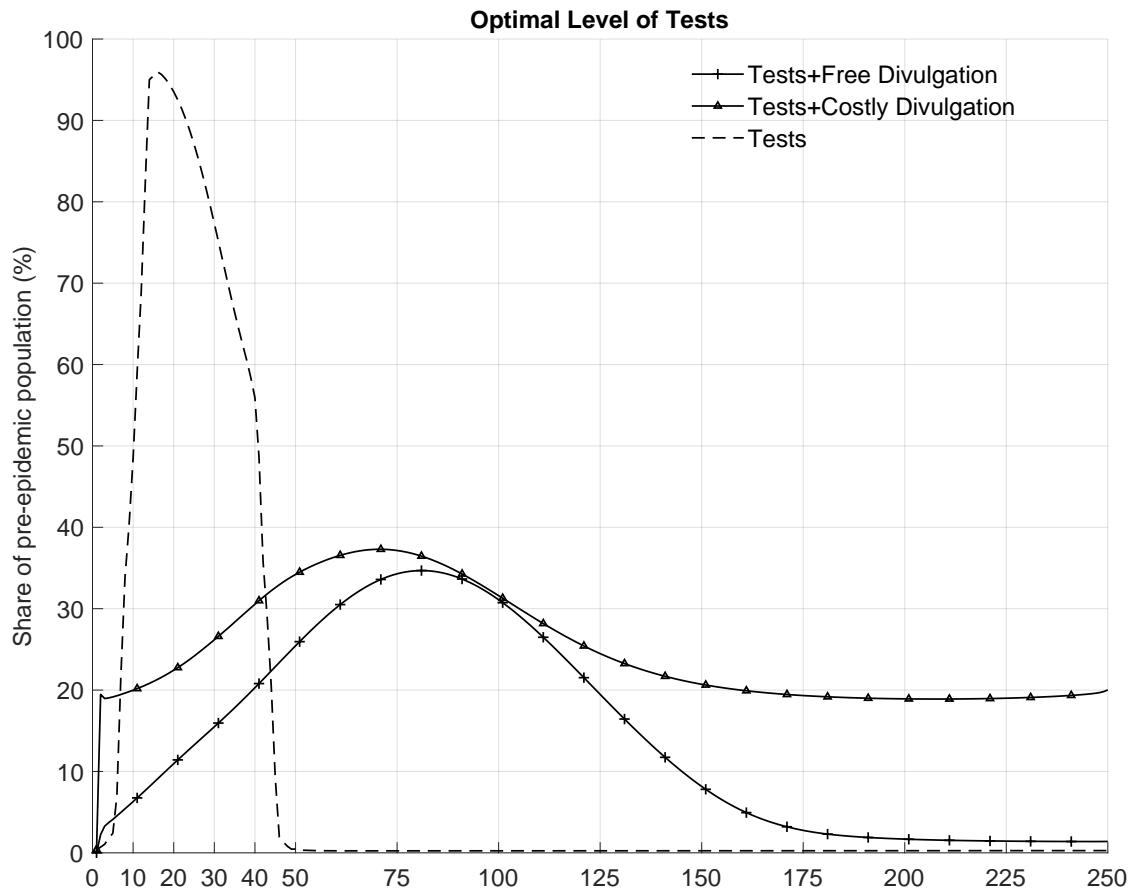


Figure 16: Optimal Tests - Testing vs Divuligation



## 7.5 Optimal Mix

Figure 17: Population Dynamics - Optimal Mix

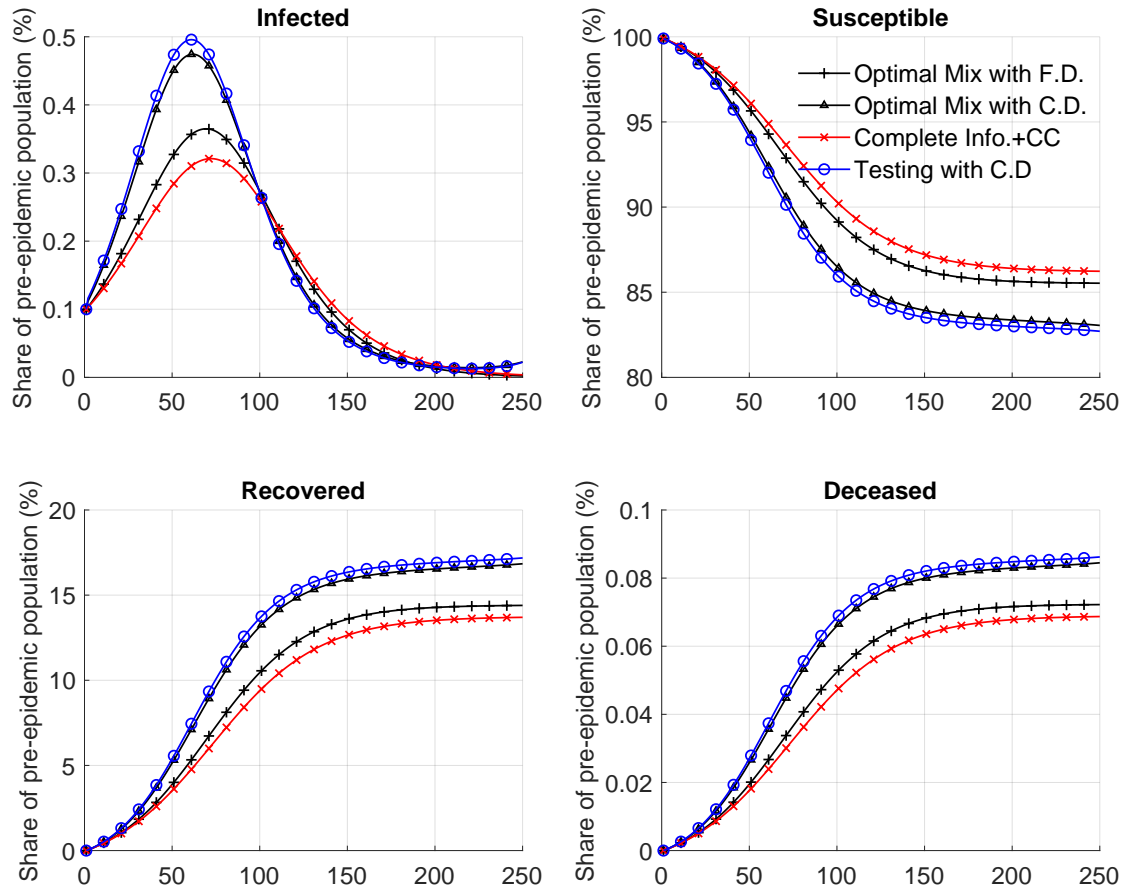


Figure 18: Economic Aggregates - Testing and Divulgence

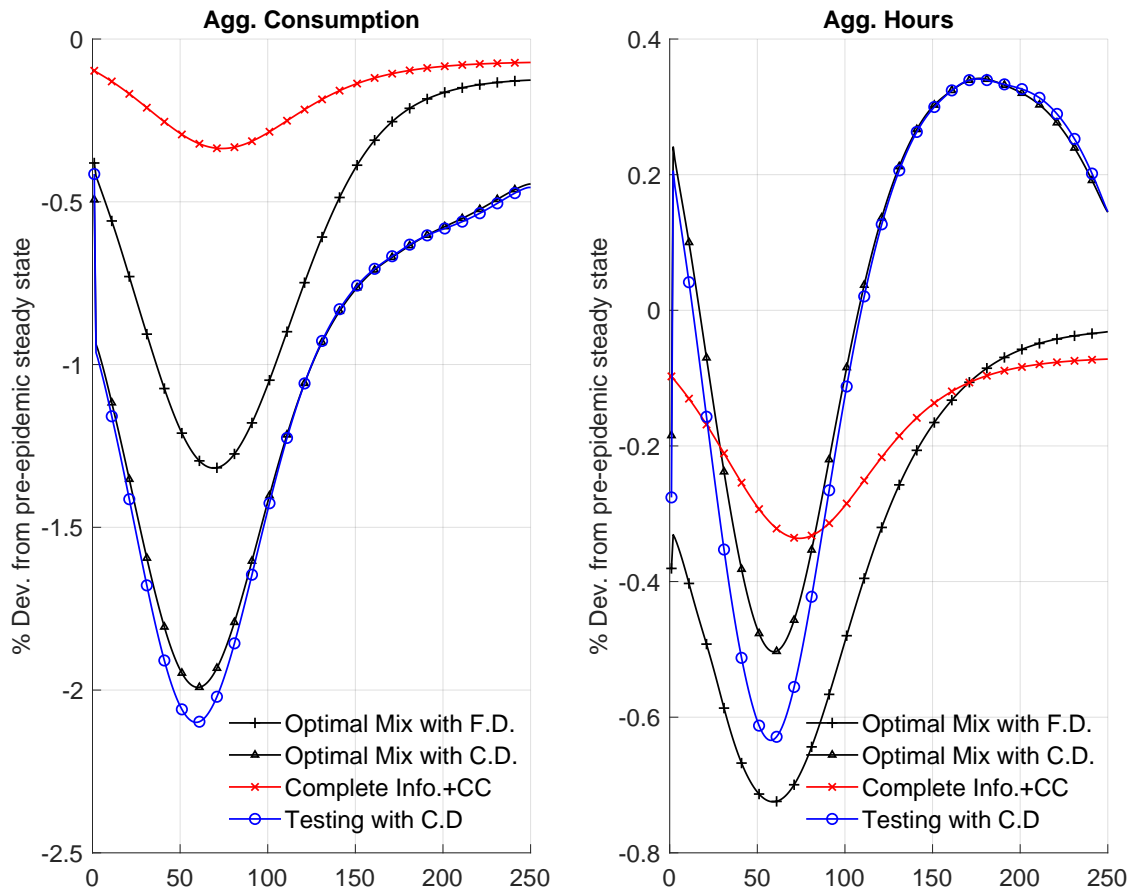
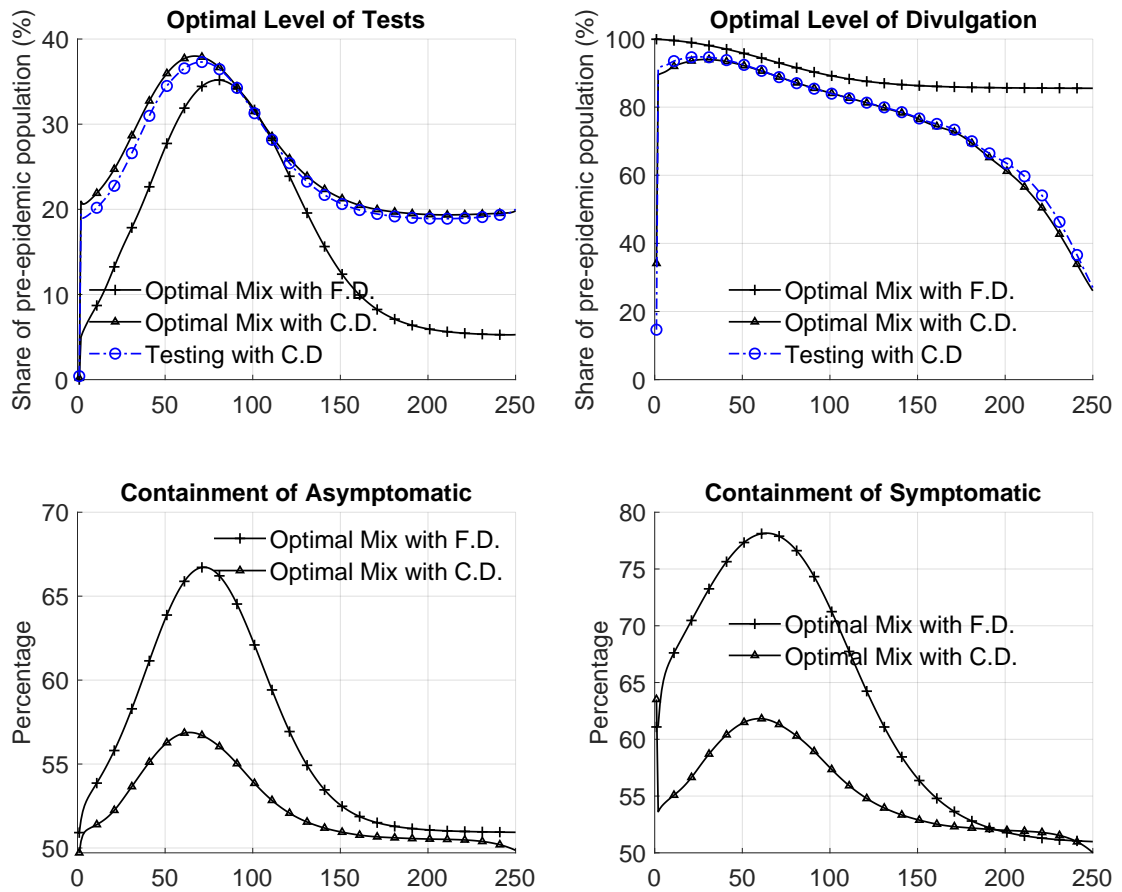




Figure 19: Optimal Mixes



## 7.6 Beliefs Discussion

Figure 20: Beliefs Biases: TII vs Testing and Divulgence

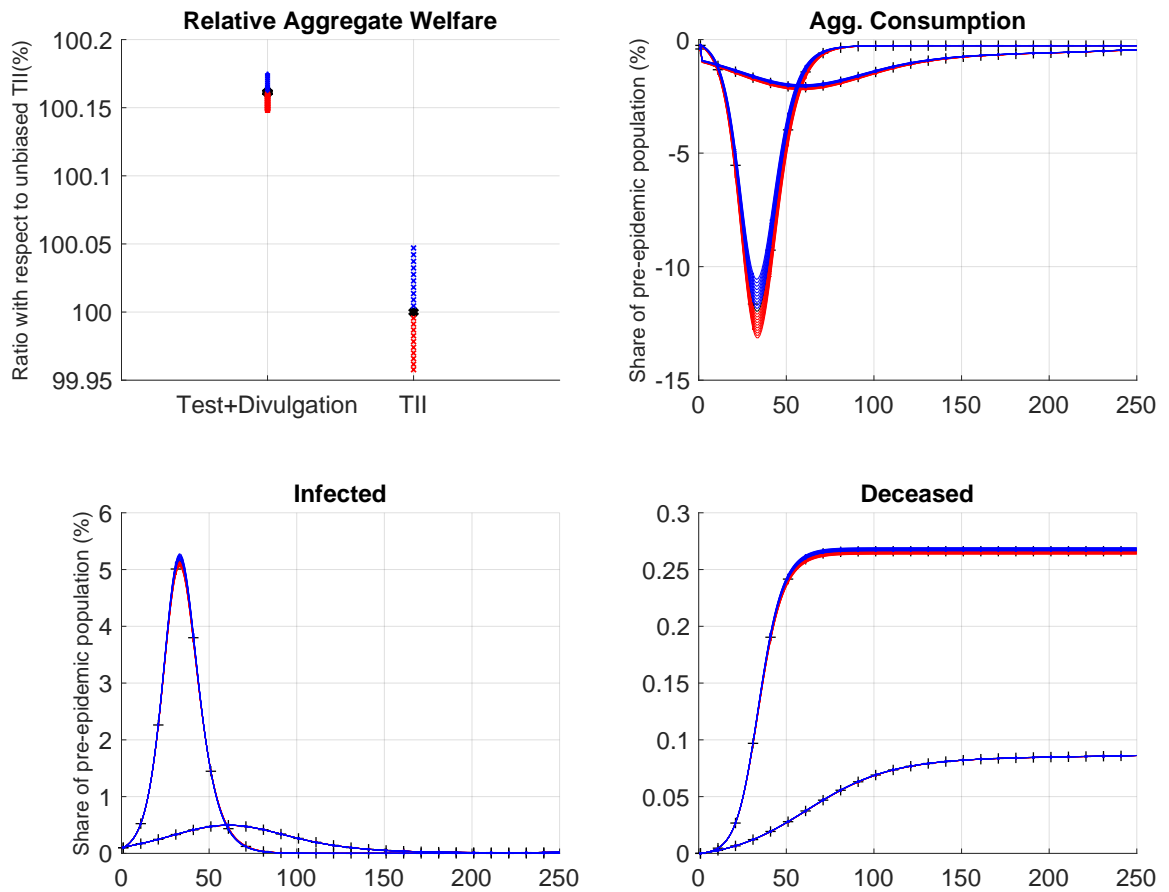
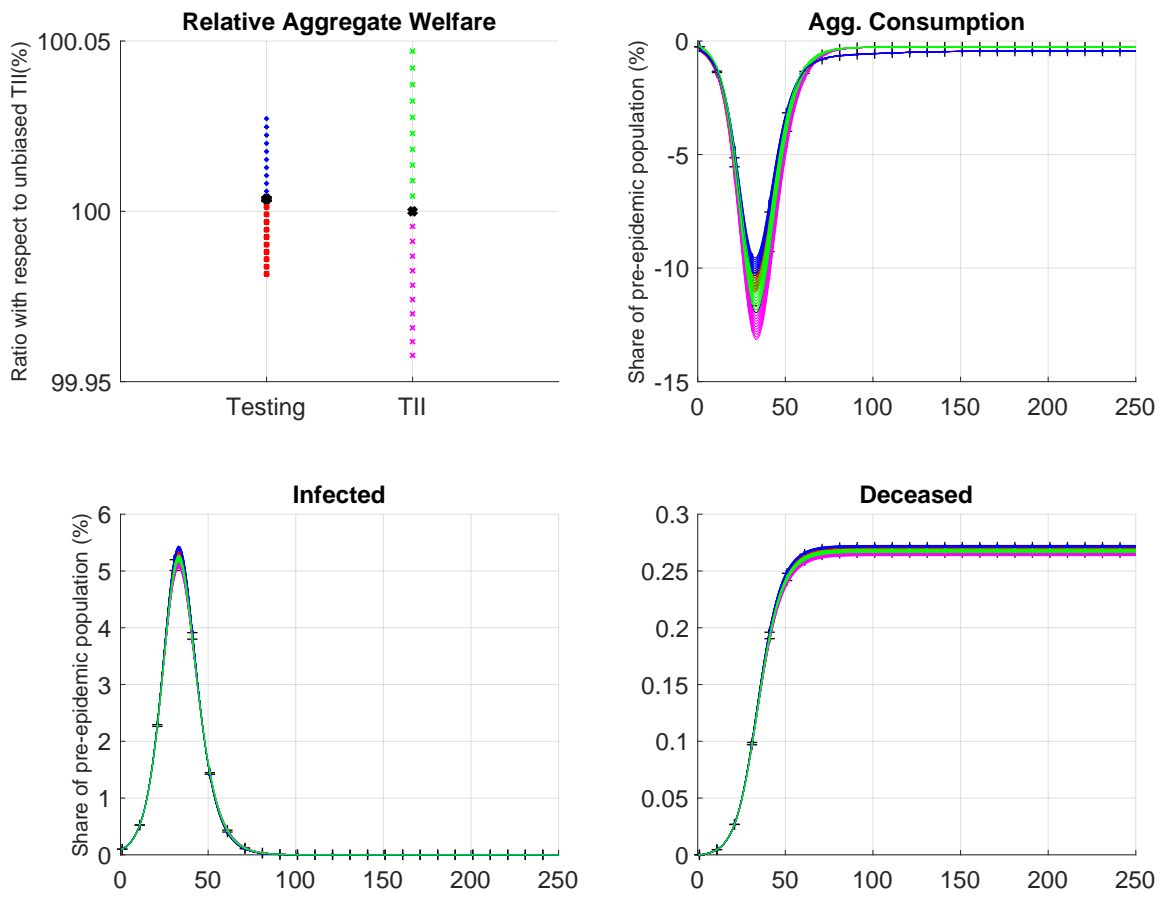


Figure 21: Beliefs Biases: TII vs Testing



## 8 Tables

Table 1: **Welfare, economic and epidemiological results for Sections 2 and 3 - Information Scenarios and Containment Measures**

*This table summarizes the implications of the three information set-ups explained in Section 2 and the effects of the different containments considered in Section 3. These eight scenarios are simulated for 250 periods and analyzed through some indicators of welfare, economic activity, epidemiological dynamics and policy paths. The relative loss of aggregate welfare measures, in percentage points, the deviation of aggregate welfare under a certain specification with respect to the welfare of the Complete Information case. The maximum falls in aggregate consumption and aggregate hours are calculated relative to their pre-epidemic values and expressed in percentage points. The cumulative fall in aggregate consumption is the accumulation of all the foregone consumption during the simulation horizon, relative to a world where consumption remains all the time in its pre-epidemic value. The peak infection variable accounts for the total number of active infection cases at the height of the epidemic, as a percentage of the initial population. The final deaths and recoveries accumulate all the people that either died or recovered during the simulation horizon and express them as shares of the initial population. The containment measures show the maximum value of the consumption tax levied by the government for each type of containment.*

	Complete Information (CI)	Partial Incomplete Information (PII)	Total Incomplete Information (TII)	CI General Containment	CI Conditional Containment	PII General Containment	PII Conditional Containment	TII General Containment
Relative loss of Aggregate Welfare	0	-0.2005	-0.2314	0	0.0003	-0.1741	-0.164	-0.1955
Max Fall in Aggregate Consumption %	-0.33	-9.94	-11.96	-0.33	-0.33	-28.53	-7.02	-30.78
Cumulative Fall in Aggregate Consumption %	-0.17	-1.24	-1.54	-0.17	-0.17	-4.97	-1.14	-5.88
Max Fall in Aggregate Hours %	-0.33	-9.94	-11.96	-0.33	-0.33	-28.53	-7.02	-30.78
Peak Infection %	0.32	5.53	5.15	0.32	0.32	3.37	3.58	3.11
Final Deaths %	0.06	0.27	0.26	0.06	0.06	0.22	0.23	0.21
Final Recoveries %	13.74	54.49	53.05	13.74	13.69	43.89	46.58	42.12
Peak of General Containment %	-	-	-	0	-	73.05	-	82.34
Peak of Symptomatic Containment %	-	-	-	-	7.89	-	199.98	-
Peak of Asymptomatic Containment %	-	-	-	-	6.28	-	194.58	-

Table 2: **Welfare, economic and epidemiological results for Section 4- Information Policy Tools**

*This table presents the simulation results for the modified model under different information and containment policies. These eight scenarios are simulated for 250 periods and analyzed through some indicators of welfare, economic activity, epidemiological dynamics and policy paths. The relative loss of aggregate welfare measures, in percentage points, the deviation of aggregate welfare under a certain specification with respect to the welfare of the Complete Information case. The maximum falls in aggregate consumption and aggregate hours are calculated relative to their pre-epidemic values and expressed in percentage points. The cumulative fall in aggregate consumption is the accumulation of all the foregone consumption during the simulation horizon, relative to a world where consumption remains all the time in its pre-epidemic value. The peak infection variable accounts for the total number of active infection cases at the height of the epidemic, as a percentage of the initial population. The final deaths and recoveries accumulate all the people that either died or recovered during the simulation horizon and express them as shares of the initial population. The containment measures show the maximum value of the consumption tax levied by the government for each type of containment. The information policies variables are expressed as a share of the initial population. The population that is informed is the people that acquire and incorporate the available private information of others health statuses. The averages are calculated for all of the 250 weeks simulated.*

	Complete Information	Partial Incomplete Information	Total Incomplete Information	Testing	Testing Free Divulagation	Testing Costly Divulagation	Optimal Mix Free Divulagation	Optimal Mix Costly Divulagation
Relative loss of Aggregate Welfare	0	-0.2005	-0.2314	-0.2079	-0.0334	-0.0705	-0.0287	-0.0655
Max Fall in Aggregate Consumption %	-0.33	-9.94	-11.96	-10.51	-1.44	-2.09	-1.31	-1.99
Cumulative Fall in Aggregate Consumption %	-0.17	-1.24	-1.54	-1.34	-0.64	-1.12	-0.62	-1.08
Max Fall in Aggregate Hours %	-0.33	-9.94	-11.96	-9.24	-0.87	-0.63	-0.72	-0.50
Peak Infection %	0.32	5.53	5.15	5.54	0.39	0.49	0.36	0.47
Final Deaths %	0.06	0.27	0.26	0.27	0.07	0.08	0.07	0.08
Final Recoveries%	13.74	54.49	53.05	54.43	14.92	17.19	14.39	16.85
Peak of General Conatinment%	-	-	-	-	-	-	0	0
Peak of Symptomatic Containment %	-	-	-	-	-	-	78.15	63.51
Peak of Asymptomatic Containment %	-	-	-	-	-	-	66.72	56.88
Average % of Population Tested per Week	-	-	-	11.03	13.93	24.88	16.78	25.45
Max % of Population Tested per Week	-	-	-	95.96	34.68	42.26	35.18	38.00
Average % of Population Informed per Week	-	-	-	-	89.49	76.02	89.92	74.97
Max % of Population Informed per Week	-	-	-	-	99.93	94.76	99.93	94.00

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## 10 Appendix A: Modified Model

### 10.1 Player $i$ is asymptomatic

Player  $j$  type is unknown ( $J = U$ )

$$\begin{aligned}
 U_t^{S,A^U} &= u(c_t^{A^U}, n_t^{A^U}) + \beta \left[ \left( 1 - p_t^{I^E} \tau_t^{I^E U} - p_t^{I^A} \tau_t^{I^A U} \right) U_{t+1}^{S,A^U} \right. \\
 &\quad \left. + \left( p_t^{I^E} \tau_t^{I^E U} + p_t^{I^A} \tau_t^{I^A U} \right) U_{t+1}^I \right] \\
 \tau_t^{I^E U} &= \pi_1 c_t^{A^U} c_t^{I^E} + \pi_2 n_t^{A^U} n_t^{I^E} + \pi_3 \\
 \tau_t^{I^A U} &= \pi_1 c_t^{A^U} c_t^{I^A} + \pi_2 n_t^{A^U} n_t^{I^A} + \pi_3
 \end{aligned}$$

Then, Player  $i$ 's optimal decisions are:

$$[c_t^{A^U}] : \frac{\partial u(c_t^{A^U \star}, n_t^{A^U \star})}{\partial c_t^{A^U \star}} + q^S \beta \pi_1 (p_t^{I^E} c_t^{I^E \star} + p_t^{I^A} c_t^{I^A \star}) (U_{t+1}^I - U_{t+1}^{S,A^U}) = \lambda_t^{A^U} (1 + \mu_t)$$

$$[n_t^{A^U}] : \frac{\partial u(c_t^{A^U \star}, n_t^{A^U \star})}{\partial n_t^{A^U \star}} + q^S \beta \pi_2 (p_t^{I^E} n_t^{I^E \star} + p_t^{I^A} n_t^{I^A \star}) (U_{t+1}^I - U_{t+1}^{S,A^U}) = -\lambda_t^{A^U} w_t$$

Player  $j$  type is known and there is no contagion risk ( $J = NI$ )

$$U_t^{S,A^{NI}} = u(c_t^{A^{NI}}, n_t^{A^{NI}}) + \beta U_{t+1}^{S,A}$$

Then, Player  $i$ 's optimal decisions are:

$$[c_t^{A^{NI}}] : \frac{\partial u(c_t^{A^{NI} \star}, n_t^{A^{NI} \star})}{\partial c_t^{A^{NI} \star}} = \lambda_t^{A^{NI}} (1 + \mu_t)$$

$$[n_t^{A^{NI}}] : \frac{\partial u(c_t^{A^{NI} \star}, n_t^{A^{NI} \star})}{\partial n_t^{A^{NI} \star}} = -\lambda_t^{A^{NI}} w_t$$



Player  $j$  type is known and there is risk of contagion from  $I^A$  ( $J = A$ )

$$\begin{aligned} U_t^{S,A^{IA}} &= u\left(c_t^{IA}, n_t^{IA}\right) + \beta \left[ \left(1 - \tau_t^{IA}\right) U_{t+1}^{S,A} + \tau_t^{IA} U_{t+1}^I \right] \\ \tau_t^{IA} &= \pi_1 c_t^{IA} c_t^{IA} + \pi_2 n_t^{IA} n_t^{IA} + \pi_3 \end{aligned}$$

Then, Player  $i$ 's optimal decisions are:

$$[c_t^{IA}] : \frac{\partial u\left(c_t^{IA*}, n_t^{IA*}\right)}{\partial c_t^{IA*}} + \beta \pi_1 c_t^{IA*} \left(U_{t+1}^I - U_{t+1}^{S,A}\right) = \lambda_t^{IA} (1 + \mu_t)$$

$$[n_t^{IA}] : \frac{\partial u\left(c_t^{IA*}, n_t^{IA*}\right)}{\partial n_t^{IA*}} + \beta \pi_2 n_t^{IA*} \left(U_{t+1}^I - U_{t+1}^{S,A}\right) = -\lambda_t^{IA} w_t$$

Player  $j$  type is known and there is contagion risk from  $I^E$  ( $J = E$ )

$$\begin{aligned} U_t^{S,A^{IE}} &= u\left(c_t^{IE}, n_t^{IE}\right) + \beta \left[ \left(1 - \tau_t^{IE}\right) U_{t+1}^{S,A} + \tau_t^{IE} U_{t+1}^I \right] \\ \tau_t^{IE} &= \pi_1 c_t^{IE} c_t^{IE} + \pi_2 n_t^{IE} n_t^{IE} + \pi_3 \end{aligned}$$

Then, Player  $i$ 's optimal decisions are:

$$[c_t^{IE}] : \frac{\partial u\left(c_t^{IE*}, n_t^{IE*}\right)}{\partial c_t^{IE*}} + \beta \pi_1 c_t^{IE*} \left(U_{t+1}^I - U_{t+1}^{S,A}\right) = \lambda_t^{IE} (1 + \mu_t)$$

$$[n_t^{IE}] : \frac{\partial u\left(c_t^{IE*}, n_t^{IE*}\right)}{\partial n_t^{IE*}} + \beta \pi_2 n_t^{IE*} \left(U_{t+1}^I - U_{t+1}^{S,A}\right) = -\lambda_t^{IE} w_t$$

## 10.2 Player $i$ is a tested susceptible

Player  $j$  type is unknown ( $J = U$ )

$$\begin{aligned} \max U_t^{S^{X,U}} &= u\left(c_t^{S^{X,U}}, n_t^{S^{X,U}}\right) + \beta \left[ \left(1 - p_t^{I^E} \tau_t^{I^E S^{X,U}} - p_t^{I^A} \tau_t^{I^A S^{X,U}}\right) U_{t+1}^{S^{X,U}} \right. \\ &\quad \left. + \left(p_t^{I^E} \tau_t^{I^E S^{X,U}} + p_t^{I^A} \tau_t^{I^A S^{X,U}}\right) U_{t+1}^I \right] \end{aligned}$$

$$\begin{aligned} s.a. (1 + \mu_t) c_t^{S^{X,U}} &= w_t n_t^{S^{X,U}} + \Gamma_t + \Gamma_t^{Inf} \\ \wedge \tau_t^{I^E S^{X,U}} &= \pi_1 c_t^{S^{X,U}} c_t^{I^E} + \pi_2 n_t^{S^{X,U}} n_t^{I^E} + \pi_3 \\ \wedge \tau_t^{I^A S^{X,U}} &= \pi_1 c_t^{S^{X,U}} c_t^{I^A} + \pi_2 n_t^{S^{X,U}} n_t^{I^A} + \pi_3 \end{aligned}$$

Then, Player  $i$ 's optimal decisions are:

$$[c_t^{S^{X,U}}]: \frac{\partial u\left(c_t^{S^{X,U}*}, n_t^{S^{X,U}*}\right)}{\partial c_t^{S^{X,U}*}} + \beta \pi_1 \left(p_t^{I^E} c_t^{I^E*} + p_t^{I^A} c_t^{I^A*}\right) \left(U_{t+1}^I - U_{t+1}^{S^{X,U}}\right) = \lambda_t^{S^{X,U}} (1 + \mu_t)$$

$$[n_t^{S^{X,U}}]: \frac{\partial u\left(c_t^{S^{X,U}*}, n_t^{S^{X,U}*}\right)}{\partial n_t^{S^{X,U}*}} + \beta \pi_2 \left(p_t^{I^E} n_t^{I^E*} + p_t^{I^A} n_t^{I^A*}\right) \left(U_{t+1}^I - U_{t+1}^{S^{X,U}}\right) = -\lambda_t^{S^{X,U}} w_t$$

Player  $j$  type is known and there is no contagion risk ( $J = NI$ )

$$\begin{aligned} \max U_t^{S^{X,NI}} &= u\left(c_t^{S^{X,NI}}, n_t^{S^{X,NI}}\right) + \beta U_{t+1}^{S^{X,A}} \\ s.a. (1 + \mu_t) c_t^{S^{X,NI}} &= w_t n_t^{S^{X,NI}} + \Gamma_t + \Gamma_t^{Inf} \end{aligned}$$

Then, Player  $i$ 's optimal decisions are:

$$[c_t^{S^{X,NI}}]: \frac{\partial u\left(c_t^{S^{X,NI}*}, n_t^{S^{X,NI}*}\right)}{\partial c_t^{S^{X,NI}*}} = \lambda_t^{S^{X,NI}} (1 + \mu_t)$$

$$[n_t^{S^X, NI}] : \frac{\partial u(c_t^{S^X, NI^*}, n_t^{S^X, NI^*})}{\partial n_t^{S^X, NI^*}} = -\lambda_t^{S^X, NI} w_t$$

Player  $j$  is known and there is contagion risk from  $I^A$  ( $J = A$ )

$$\begin{aligned} \max U_t^{S^X, I^A} &= u\left(c_t^{S^X, I^A}, n_t^{S^X, I^A}\right) + \beta \left[ \left(1 - \tau_t^{IAS^X, I^A}\right) U_{t+1}^{S^X, A} + \tau_t^{IAS^X, I^A} U_{t+1}^I \right] \\ \text{s.a.} (1 + \mu_t) c_t^{S^X, I^A} &= w_t n_t^{S^X, I^A} + \Gamma_t + \Gamma_t^{Inf} \\ \wedge \tau_t^{IAS^X, I^A} &= \pi_1 c_t^{S^X, I^A} c_t^{I^A} + \pi_2 n_t^{S^X, I^A} n_t^{I^A} + \pi_3 \end{aligned}$$

Optimal consumption and hours worked given by:

$$[c_t^{S^X, I^A}] : \frac{\partial u(c_t^{S^X, I^A^*}, n_t^{S^X, I^A^*})}{\partial c_t^{S^X, I^A^*}} + \beta \pi_1 c_t^{I^A^*} (U_{t+1}^I - U_{t+1}^{S^X, A}) = \lambda_t^{S^X, I^A} (1 + \mu_t)$$

$$[n_t^{S^X, I^A}] : \frac{\partial u(c_t^{S^X, I^A^*}, n_t^{S^X, I^A^*})}{\partial n_t^{S^X, I^A^*}} + \beta \pi_2 n_t^{I^A^*} (U_{t+1}^I - U_{t+1}^{S^X, A}) = -\lambda_t^{S^X, I^A} w_t$$

Player  $j$  type is known and there is contagion risk from  $I^E$  ( $J = E$ )

$$\begin{aligned} \max U_t^{S^X, I^E} &= u\left(c_t^{S^X, I^E}, n_t^{S^X, I^E}\right) + \beta \left[ \left(1 - \tau_t^{IES^X, I^E}\right) U_{t+1}^{S^X, A} + \tau_t^{IES^X, I^E} U_{t+1}^I \right] \\ \text{s.a.} (1 + \mu_t) c_t^{S^X, I^E} &= w_t n_t^{S^X, I^E} + \Gamma_t + \Gamma_t^{Inf} \\ \wedge \tau_t^{IES^X, I^E} &= \pi_1 c_t^{S^X, I^E} c_t^{I^E} + \pi_2 n_t^{S^X, I^E} n_t^{I^E} + \pi_3 \end{aligned}$$

Optimal consumption and hours worked are given by:

$$[c_t^{S^X, I^E}] : \frac{\partial u(c_t^{S^X, I^E^*}, n_t^{S^X, I^E^*})}{\partial c_t^{S^X, I^E^*}} + \beta \pi_1 c_t^{I^E^*} (U_{t+1}^I - U_{t+1}^{S^X, A}) = \lambda_t^{S^X, I^E} (1 + \mu_t)$$

$$[n_t^{S^X, I^E}] : \frac{\partial u(c_t^{S^X, I^E^*}, n_t^{S^X, I^E^*})}{\partial n_t^{S^X, I^E^*}} + \beta \pi_2 n_t^{I^E^*} (U_{t+1}^I - U_{t+1}^{S^X, A}) = -\lambda_t^{S^X, I^E} w_t$$