Optimal sustainable monetary policy for commodity-exporting economies

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March 21, 2024
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Abstract

Optimal monetary policy analysis is conducted for a small open- and -commodity-exporting economy, subject to a constraint on foreign borrowing. Results show that the operational optimal quasi-sustainable policy, designed to be implementable, aligns with the optimal policy under commitment. This suggests that, under certain conditions, reputation can effectively replicate commitment. Furthermore, the equilibrium under the operational policy mirrors the competitive outcome, highlighting its practical feasibility. These findings offer valuable insights for managing sustainable policies in commodity-dependent economies.

Key words: optimal sustainable monetary policy, financial friction, sustainability constraint, sustainable equilibrium, commodity price.

JEL classification: E52, E58, E61, F41, F44, Q02.

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1 Introduction

The paper by Clarida, Galí and Gertler (1999) underscores the prevalence of the optimal monetary policy under the full commitment approach, initially proposed by Currie and Levine, 1993 and Woodford, 1998, as the primary framework for welfare-based monetary policy evaluation in the literature. Within this context, Drechsel, McLeay and Tenreyro (2019, hereafter DMT) employs this approach to conduct a welfare-based optimal monetary policy analysis for a small open- and -commodity-exporting economy (SOCEE).

In response to a positive commodity price shock (CPS) under the optimal commitment policy, DMT observes that the policymaker must tighten monetary conditions and allow for an appreciation of the nominal exchange rate to address the inefficiencies introduced by the shock. Additionally, in this scenario, domestic inflation and the domestic output gap exhibit lower responses and volatilities compared to those obtained under simple and implementable policy rules, while the nominal exchange rate experiences higher measures.

Considering Frankel's (2003) observation that small open- and -commodity-exporting economies also fear to float (i.e., policymakers in such countries prefer policy rules that smooth out fluctuations in the nominal exchange rate as much as possible), the present paper explores whether the results obtained by DMT would change if the optimal sustainability policy framework à la Kurozumi (2008) is used instead of the standard optimal commitment policy à la Clarida et al. (1999). Specifically, this paper asks whether the optimal sustainable policy framework would achieve a lower response and volatility for the nominal exchange rate, while at the same time if it would still obtain the lowest responses and volatilities for the domestic inflation rate and the domestic output gap.

The optimal sustainable policy framework, inspired by Chari and Kehoe's (1990) optimal sustainable plans concept, focuses on achieving an optimal sustainable equilibrium based on the policymaker's reputation rather than commitment. This framework is derived from policy games between competitive private agents and their government in infinite-horizon economies. In such games, these authors characterize the entire set of sustainable equilibrium outcomes, find the worst sustainable equilibrium and conclude that the optimal government's strategy requires the continuation as an outcome –or policy–, provided it has been adopted in the past; otherwise, the strategy requires to opt for the worst sustainable equilibrium outcome.

Kurozumi (2008) extrapolates this concept to the optimal monetary policy welfare-based design framework, showing that the optimal sustainable monetary policy relies on the policymaker's reputation, dispensing with the commitment assumption. Furthermore, the optimal sustainable policy can be implemented through an operational optimal quasi-sustainable policy scheme, derived from a welfare-based approach with a sustainability constraint. Where, the highest welfare comes from the optimal commitment policy, while the worst sustainable equilibrium policy, from the optimal discretionary policy.

This paper evaluates the optimal monetary policy by contrasting the optimal quasi-sustainable policy, optimal commitment policy, and discretionary policy regimes within the SOCEE model proposed by DMT. The model incorporates a representative commodity sector with ad hoc borrowing limits to nominal foreign loans denominated in foreign currency. To obtain a numerical solution, the paper follows the strategy proposed by Sunakawa (2015), implementing a variant of the policy function iteration method suggested by Kehoe and Perri (2002).

This paper analyzes the impact of commodity price shocks on selected macroeconomic variables and assesses the slackness condition of the sustainability constraint under both optimal quasi-sustainable and commitment policies. It is demonstrated that, under the specified assumptions of the DMT model, these policies coincide – leading to three key results. First, the competitive equilibrium achieved by the optimal commitment policy is consistent with the sustainable equilibrium obtained under the optimal quasi-sustainable policy. Second, the reputation technology (assumed under the optimal quasi-sustainable policy) corresponds to the commitment technology (assumed under the optimal commitment policy). Finally, business cycle fluctuations are more volatile under the optimal discretionary policy, as expected.

The coincident equilibrium between the optimal quasi-sustainable policy and the optimal commitment policy is a result in line with the findings documented by Sunakawa (2015). In concrete, such a result comes from the standard calibration value assigned in the literature to the subjective discount factor of the economy, which implies realistic real interest rates for the economies. Consequently, such high values of the subjective discount factor (or low real interest rate values) prevent the sustainability constraint to bind and observe any possible difference between the optimal quasi-sustainable policy and the optimal commitment policy. With all of that, this paper also finds that in the face of a CPS, the optimal sustainable policy framework delivers the same responses and volatilities (for the nominal exchange rate, the domestic inflation, the domestic output gap and other selected macroeconomic variables) that are accounted under the optimal commitment policy.

Two additional findings related to a CPS are noteworthy. First, as the inelasticity of foreign nominal borrowings with respect to international commodity prices increases, the business cycle becomes more volatile. Second, an increase in the commodity inputs share in the economy leads to a more volatile business cycle.

The rest of the document proceeds with Section 2 where theoretical aspects of the SOCEE model and those of the optimal monetary policy approaches are presented and discussed. Section 3 describes the main quantitative findings and details about the optimal monetary policy evaluation. Section 4 concludes.

2 The model

This section discusses the crucial aspects of a commodity-exporting economy that are relevant for our welfare-based monetary policy assessment under three different policy regimes: optimal commitment, optimal discretion, and optimal sustainability. Additionally, it also details the chosen solution method and calibration used within the model.

2.1 The commodity-exporting economy model

Drechsel, McLeay and Tenreyro's (2019) model builds upon Ferrero and Seneca (2019) and Galí and Monacelli's (2005) frameworks. The basic structure of DMT's model is given by a two-country dynamic general equilibrium model for a small open economy with sticky prices, competitive markets and a non-distorted steady state where zero inflation rate and labor subsidy are assumed.

The economy incorporates a representative household sector, a sector of non-traded final goods firms, a representative commodity-producing firm, a foreign sector, and a monetary authority agent.[1]

The representative household consumes a basket of foreign and domestic consumption goods, provides hours of work, keeps a portfolio of (foreign and domestic) state-contingent securities, and receives rebated dividend profits from the firms. The macroeconomic environment is characterized by a cash-less economy with complete asset markets where the international perfect risk-sharing condition holds, the law of one price holds, there is a fully passed-through effect from changes in the nominal exchange rate to imported goods prices and the uncovered interest rate parity (UIP) condition holds.

The productive sector of the economy has two main sectors: the non-traded final goods firms and the representative commodity-producing firm. The first sector is integrated by wholesale and retail firms.

Wholesale firms are made up of a continuum of firms that producing intermediate goods. They operate in monopolistic competitive markets employing labor and the homogeneous technology of the industry to manufacture differentiated goods, using a linear production function technology (with constant returns to scale, CRS), and pricing their products à la Calvo (1983).

Retail firms operate in perfectly competitive markets, selling aggregate final goods that are produced as intermediate goods. Final goods from this sector are destined for domestic consumption (demanded by households), while intermediate goods/inputs (demanded by the commodity-producing sector) are also saleable as final goods in other sectors. "Final goods" and "intermediate goods" both refer to goods that are, respectively, either ready for direct consumption or used in part to produce other goods (i.e., inputs). The optimality condition for this sector delivers the New Keynesian Phillips Curve (NKPC) for this commodity-exporting economy,

$$\pi_{h,t} = \beta E_t \pi_{h,t+1} + \xi x_{h,t} + \xi (y^e_{h,t} - y^n_{h,t}), \tag{1}$$

where $\pi_{h,t}$, $x_{h,t}$, $y^e_{h,t}$ and $y^n_{h,t}$ represent the domestic inflation rate, the relevant output gap of the economy and the efficient and natural outputs, respectively. All these four variables are expressed in log-linearized terms and in deviation with respect to its respective steady state level. For its part, ξ measures the slope of the NKPC which, in this case, it is also weighting the gap between the efficient and natural outputs ($y^e_{h,t}$ and $y^n_{h,t}$).

The representative commodity-producing firm operates in perfectly competitive markets producing (commodity) goods by using intermediate goods (inputs) under a decreasing returns to scale production function. All production is exported, and international creditors are assumed to meet all the firm's credit demands fully.

The firm faces an ad hoc working capital constraint on foreign credits used to finance intermediate goods purchases. As a result, international borrowings/loans ($L_{c,t}$) fully cover the demand of intermediate goods purchases ($M_{h,t}$), valued in domestic prices ($P_{h,t}$),

$$L_{c,t} = P_{h,t} M_{h,t}.$$

[1] The primary objective of this paper is to assess a welfare-based monetary policy employing the DMT model within the framework of optimal commitment, discretionary, and sustainable policies. For a comprehensive understanding of the model, readers are encouraged to consult Drechsel et al.'s (2019) paper, which offers detailed information not covered within this manuscript.

Borrowings are constrained by a time-varying proportion of the working capital $(P_{c,t}Y_{c,t})$,

$$L_{c,t} \leq \Big[\bar{\chi}\Big(\frac{P^*_{c,t}}{P^*_t}\Big)^{\chi}\Big]P_{c,t}Y_{c,t},$$

where χ is an elasticity that measures borrowing conditions with respect to international commodity prices $(P^*_{c,t})$ relative to international prices (P^*_t), and $Y_{c,t}$ is the constant returns to scale production function of the sector. The firm's production technology exhibits homogeneity of degree one, implying $Y_{c,t} = A_{c,t}M_{h,t}$, where $A_{c,t}$ represents the total factor productivity of the sector.

Optimality conditions determine the demand of intermediate goods and borrowings/loans, maximizing the firm's discounted value.

International commodity prices, denoted by $P^*_{c,t}$, are determined exogenously in international markets and denominated in foreign currency. The real commodity price shock $p^*_{c,t}$ follows a first-order autoregressive exogenous process[2]

$$p^*_{c,t} = \rho_{p^*_c}p^*_{c,t-1} + \epsilon_{p^*_c,t}, \tag{2}$$

where $\epsilon_{p^*_c,t} \sim (0, \sigma^2_{p^*_c})$ is bounded (i.e., there is a $B > 0$ such that $|p^*_{c,t}| < B$), independent and identically distributed with zero mean and $\sigma^2_{p^*_c} > 0$ standard deviation. Observe that the persistence of the process is given by $\rho_{p^*_c} \in (-1, 1)$ and an initial condition $p^*_{c,t-1|t=0} = 0$.

Regarding the rest of the exogenous processes that characterize the foreign sector as well as the total factor productivity processes of the two producing sector of this economy, unlike DMT, here, it is assumed that they are nil. Namely, the foreign output process, the foreign interest rate process, the foreign inflation rate process and the productivity processes in the commodity and non-traded final domestic goods sectors.

Given the last set of assumptions, it is possible then to re-express the term $\xi(y^e_{h,t} - y^n_{h,t})$ in the NKPC (1) only in terms of the commodity price shock (CPS) as

$$\xi(y^e_{h,t} - y^n_{h,t}) = \xi(\frac{-\frac{s_{c,ss}s_{m,ss}\nu}{\mathcal{W}_{ss}(1-v)^2}}{\frac{\lambda_\tau}{\mathcal{W}_{ss}} + \phi\mathcal{W}_{ss}} - \frac{\frac{s_{m,ss}}{1-\nu}}{1+\phi\mathcal{W}_{ss}})(1-\chi)p^*_{c,t} \equiv \omega p^*_{c,t}, \tag{3}$$

where ω can be positive if $\chi > 1$; zero if $\chi = 1$; or negative if $\chi < 1$. Note that ω encompasses all the structural parameters of the model written in the middle of the last equality. By inspection, one verifies that the weighted gap between the efficiency and natural outputs in this economy is being determined by the borrowing elasticity of the commodity sector (χ), the constant returns to scale in the commodity-producing sector (ν), the slope of the NKPC (ξ), the share of domestic consumption goods produced in the economy $(s_{c,ss})$, the share of domestic intermediate goods produced in the economy for the commodity sector $(s_{m,ss})$, and the wedge at the steady state $(\mathcal{W}_{ss})$.[3]

The new equivalent expression for the term $\xi(y^e_{h,t} - y^n_{h,t})$ in (3) allows to rewrite the same

[2]Note that $p^*_{c,t} \equiv \ln(P^*_{c,t}/P^*_{c,ss})/\ln(P^*_t/P^*_{ss})$ where $P_{c,ss}$ and P^*_{ss} are the foreign commodity price and the foreign consumer price index at their steady state values.

[3]Note that $\xi \equiv \kappa(1+\phi\mathcal{W}_{ss})/\mathcal{W}_{ss}$, $\kappa = (1-\theta)(1-\beta\theta)/\theta$, $\mathcal{W}_{ss} \equiv s^e_{c,ss} + 1/(1-\nu)s^e_{m,ss} > 1$, $\lambda_\tau = s_{c,ss} + s_{m,ss}/(1-\nu)^2$, where β is the subjective discount factor, ϕ is the inverse Frisch elasticity labor supply and $1-\theta$ is the Calvo price re-set probability.

short-run aggregate supply curve (or NKPC) as a weighted function of the CPS,

$$\pi_{h,t} = \beta E_t \pi_{h,t+1} + \xi x_{h,t} + \omega p^*_{c,t}. \tag{4}$$

The weighting parameter ω in equation (4) plays a crucial role in determining the impact of the CPS on the NKPC. Notably, the sign of ω influences the direction of the CPS shock effect, which is indirectly modulated by the value of the financial channel parameter (χ), as it can be seen in equation (3).

Specifically, under unfavorable borrowing conditions in the commodity sector ($\chi < 1$, $\omega < 0$), a positive CPS leads to a downward displacement of the NKPC. Conversely, when borrowing conditions are favorable ($\chi > 1$, $\omega > 0$), the model predicts an upward displacement of the NKPC in response to the same positive CPS.

The policymaker's block finally closes the equilibrium conditions of DMT's model. The optimal monetary policy under full commitment in the spirit of Clarida et al. (1999) is obtained by choosing the state contingent sequences of domestic inflation rate and output gap $\{\pi_{h,t}, x_{h,t}\}_{t=0}^{\infty}$ that maximize social welfare under full commitment, subject to the NKPC (1). In particular, the problem of the policymaker is solved here using the reformulated NKPC (4).

The welfare function is derived through a second-order approximation to the representative household's expected utility (based on the consumption basket of domestic and foreign goods and the disutility from labor or hours at work: $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{\ln C_t - N_t^{1+\phi}/[1+\phi]\}$) in period zero, given by

$$-E_0 \sum_{t=0}^{\infty} \beta^t \frac{\Omega}{2} (\pi_{h,t}^2 + \lambda_x x_{h,t}^2). \tag{5}$$

in this second-order approximation (or welfare loss function), Ω weights for the entire squared sum of the deviations of the domestic inflation and domestic output gap sequences from their steady state values, while λ_x focuses solely on stabilizing the domestic output gap relative to domestic inflation.[4]

2.2 Optimal monetary policy approaches

In this subsection, three approaches for optimal commitment, discretionary and sustainable policies are presented and derived to analyze the economic predictions according to DMT's model.

2.2.1 Optimal commitment policy

Designing the optimal monetary policy under full commitment involves the policymaker publicly announcing a state-contingent rule and adhering to it indefinitely in pursuit of a predetermined set of policy goals. This commitment presumes perfect credibility for the policymaker, who acts as the head of the central monetary institution.

Under DMT's model framework, whenever a CPS hits the economy, its monetary authority faces a policy trade-off that challenges the fulfillment of its commitment made to economic

[4] See that $\Omega = (1-\alpha)\epsilon/(\kappa \mathcal{W}_{ss})$ and that $\lambda_x = \kappa/\epsilon(\lambda_\tau/\mathcal{W}_{ss}^2 + \phi)$, where α is the foreign consumption bias and ϵ is the elasticity of substitution between home and foreign consumption goods.

agents.

In fact, a CPS changes the efficient allocation of the economy by boosting the demand for inputs in the commodity sector, increasing the supply of domestic goods, causing a domestic inflation fall, but an expansion of the output gap due to the commodity boom. Thus, with domestic inflation dropping and the output gap rising, the monetary authority faces a trade-off and must stabilize the economy. This objective needs to be achieved considering its state-contingent policy rule, the commitment to its ultimate goals and the expectations (or beliefs) of forward-looking private agents, which turns the policymaker's monetary policy design into a time-consistency problem. This issue is usually known as 'stabilization bias'.[5]

On the other hand, as it was stated before, DMT's model rules out the possibility of 'inflation bias', given that subsidies are assumed to offset the monopolistic distortion to obtain an efficient steady state (with a zero output gap and zero domestic inflation at the steady state). Therefore, no inflation bias is present and the stabilization bias is the only source of time-inconsistency.

The usual fashion in which the optimal monetary policy problem under full commitment is solved, as mentioned in the previous section, is by choosing the state contingent sequences of domestic inflation rate and output gap $\{\pi_{h,t}, x_{h,t}\}_{t=0}^{\infty}$ that maximize the welfare gains stated in (5), subject to the NKPC presented in (4). Under full commitment, the resulting optimality condition for domestic inflation policy rule in terms of the domestic output gap is:

$$\pi_{h,0} = -\frac{\lambda_x}{\xi} x_{h,0}, \quad \text{and} \quad \pi_{h,t} = -\frac{\lambda_x}{\xi}(x_{h,t} - x_{h,t-1}), \, \forall t > 0. \tag{6}$$

As it can be appreciated in (6), the optimality condition for the policy rule prescribes that the monetary authority must stabilize the domestic inflation whenever the economy experiences a boom. Notably, the optimality condition for the policy rule in (6) differs between the initial period ($t = 0$) and subsequent periods ($t > 0$). This discrepancy across time periods renders the policy time-inconsistent, leading to a stabilization bias. Note that the timing subscript t refers to the period since the policy begins its implementation. That is to say, no previous commitment to period zero are made (which implies that no commitment previous to $t < 0$ is made about the output gap, $x_{h,-1} = 0$; though this not implies a zero output gap in period $t = -1$ or $t < 0$). Subsequently, the time by which the policy is being implemented is independent from (and may even be parallel to) the period in which the economy is going through. Accordingly, note that the optimality condition in (6) also requires an initial condition for the output gap ($x_{h,0}$) that may be distinct from zero (Kurozumi, 2008).

The time-inconsistency policy problem addressed here is also referred to as the 'time-0 perspective' commitment policy (see Sunakawa, 2015) and it differs from the 'timeless perspective' commitment policy approach formulated by Woodford (2003). The key distinction lies in the assumption. The timeless perspective assumes the optimal policy was already implemented in the distant past. While this assumption guarantees time consistency, the resulting 'optimal' policy might not always be the best course of action in the present. In some cases, discretionary policy may even prove superior (Dennis, 2010; Sauer, 2010). For this reason, the time-0 perspective approach is the only policy design under full commitment that is being addressed here.

[5]Technically, private rational expectations (or beliefs) on future domestic inflation behavior ($\beta E_t \pi_{h,t+1}$) are consigned in the NKPC (4). Note, however, that the CPS term ('$\omega p^*_{c,t}$') of this SOCEE setting is also present there.

The system for the optimal monetary policy under full commitment for the commodity-exporting economy is determined by the welfare gains (5), the NKPC (4), the CPS process (2) and the initial conditions for the output gap and the shock. Once the system is solved a unique rational expectation equilibrium is attained.

The optimal commitment policy rule for the output gap ($x^c_{h,t}$) as a function of the commodity price shock is given by

$$x^c_{h,t} = b^- x^c_{h,t-1} + \omega \frac{-\frac{\xi}{\lambda_x}}{\beta(b^+ - \rho_{p^*_c})} p^*_{c,t} \equiv \omega a_x p^*_{c,t} + b_x x^c_{h,t-1}, \quad (7)$$

where $a_x < 0$ and $b^- = b_x \in (0,1)$.[6] While the optimal commitment policy rule for the domestic inflation is given by,

$$\pi^c_{h,t} = \frac{\lambda_x}{\xi}(1-b^-)x^c_{h,t-1} + \omega \frac{1}{\beta(b^+ - \rho_{p^*_c})} p^*_{c,t} \equiv \omega a_\pi p^*_{c,t} + b_\pi x^c_{h,t-1}, \quad (8)$$

where $b_\pi > 0$ and $a_\pi > 0$.

As it can be noted in (7) and (8), the sign of the parameter ω (and indirectly the borrowing conditions to the commodity sector, given by χ in [3]) exerts an influence over the responses of the domestic inflation and the output gap to a CPS. Correspondingly, under unfavorable circumstances to borrowings emanating from the commodity sector ($\chi < 1$, $\omega < 0$), a positive CPS would lead domestic inflation to show a negative response (a negative deviation with respect to its steady state level) that would reverse over time as the CPS dissipates. However, in the case of the output gap, a positive response (a positive deviation with respect to its steady state level) would be observed in response to the same CPS and calibrations ($\chi < 1$, $\omega < 0$).

DMT's model predicts contrasting responses for domestic inflation and the output gap under favorable financial circumstances ($\chi > 1$, $\omega > 0$) despite a positive CPS. According to equations (7) and (8), inflation increases (positive response) while the output gap contracts (negative response). This positive inflation response resembles a 'cost-push shock' (as introduced by Clarida et al., 1999) or an inefficient supply shock' (as referred to by Woodford, 2003). However, this predicted positive inflation response challenges empirical evidence. For instance, Bergholt et al. (2019) finds a negative domestic inflation response to a positive CPS.

The validity of DMT's predictions hinges on the elasticity of the borrowing parameter χ. If data confirms χ is elastic (greater than 1), the model's predictions hold true. Conversely, inelastic χ (less than 1) would require reevaluation of the model's predictions in this scenario.

2.2.2 Optimal discretionary policy

A policymaker adopts an optimal discretionary policy by implementing the measure that is most beneficial at the current time and under the prevailing circumstances. This approach disregards both future consequences and the expectations of private agents regarding future policies. In such a scenario, commitment to any future action becomes impossible

[6]Note that the system yields $f(b) \equiv \beta b^2 - (1+\beta+\xi^2/\lambda_x)b + 1 = 0$, where $b^{\pm} = \{(1+\beta+\xi^2/\lambda_x) \pm [(1+\beta+\xi^2/\lambda_x)^2 - 4\beta]^{\frac{1}{2}}\}/(2\beta)$ are the two (different) real roots $0 < b^- < 1 < b^+$.

for the policymaker.

The optimal discretionary policy rule is found by selecting the state-contingent sequences of domestic inflation rate and output gap, denoted as $\{\pi_{h,t}, x_{h,t}\}_{t=0}^{\infty}$, that maximize the welfare gains expressed in (5). This is done subject to the NKPC in (4), where private agent's expectations (the forward-looking variables) are taken as given.

Under discretion, the optimality condition for the domestic inflation policy rule in terms of the domestic output is defined as

$$\pi_{h,t} = -\frac{\lambda_x}{\xi} x_{h,t}, \; \forall t \geq 0. \tag{9}$$

This optimality condition, being unique across all time periods, ensures that the optimal discretionary policy avoids the time-inconsistency problem. In simpler terms, whenever the economy deviates from its steady state level at the present time, the policymaker reacts by attempting to stabilize inflation concurrently.

The system for the optimal monetary policy for the commodity-exporting economy under full commitment is determined by the welfare gains (5), the NKPC (4), the CPS process (2) and the initial condition for the shock. Solving this system leads to a unique rational expectations equilibrium.

The optimal policy rule under discretion for the domestic inflation rate ($\pi^d_{h,t}$) is defined as:

$$\pi^d_{h,t} = \frac{1}{1 - \beta\rho_{p^*_c} + \frac{\xi^2}{\lambda_x}} \omega p^*_{c,t} \equiv \omega c_\pi p^*_{c,t}, \tag{10}$$

where $c_\pi > 0$. Similarly, the optimal discretionary policy rule for the output gap ($x^d_{h,t}$) is:

$$x^d_{h,t} = \frac{-\frac{\xi}{\lambda_x}}{1 - \beta\rho_{p^*_c} + \frac{\xi^2}{\lambda_x}} \omega p^*_{c,t} \equiv \omega c_x p^*_{c,t}, \tag{11}$$

where $c_x < 0$.

Comparing the optimal discretionary policy rules ([10] & [11]) with those under optimal commitment ([7] & [8]), one observes a weaker response to the CPS under commitment. This is because the response to shocks is dampened by the lagged term of the domestic output gap.

Furthermore, the influence of the parameter ω (and χ) on the optimal discretionary policy rules (Equations 10 & 11) mirrors the observations made for the optimal commitment policy rules (Equations 7 & 8).

2.2.3 Optimal sustainable policy

So far, two contrasting policy approaches have been presented: the optimal commitment and the optimal discretionary policies. These policy regimes, respectively, assume the existence of full commitment technology or the complete absence of such technology. However, in the real world, monetary authorities neither fully commit to a policy forever nor completely abandon them.

The concept of the optimal sustainable policy emerges as a viable approach that allows the monetary authority to navigate between these two extremes (or even implement elements of both) in the absence of a commitment technology. The optimal sustainable policy outcome can still achieve an optimal welfare-based policy that produces a sustainable equilibrium and a higher welfare level than the one delivered by the discretionary policy. In this context, the worst welfare level is the one resulting from the optimal discretionary policy, while the highest one occurs under the optimal commitment policy.

As Sunakawa (2015) points out, the optimal sustainable policy is the solution to a "policy game between an infinite number of private agents and the policymakers" that delivers a sustainable equilibrium that overcomes the time-inconsistency issue present in the optimal commitment policy.

The sustainable equilibrium

In the policy game of this infinite-horizon economy, the policymaker acts first, followed by the private agents.[7] The history of commodity price shocks and output gaps up to period t is recursively defined as $h_t = (h_{t-1}, x_{h,t-1}, p^*_{c,t})$, $\forall t > 0$ and $h_0 = p^*_{c,0}$.[8]

Given the history h_t, the policymaker formulates its strategy by setting the current domestic output $x_{h,t}$ (as a function of the history, $x_{h,t} = \sigma(h_t)$) along with a contingent plan $(\sigma_s)_{s\geq t+1}$ for future domestic output gaps and possible future histories. On the other hand, given the current history $(h_t, x_{h,t})$ and the policymaker's contingent plan $(\sigma_s)_{s\geq t+1}$, private agents formulate their strategy specifying current domestic inflation $\pi_{h,t}$ as a function of the history and the domestic output gap $\pi_{h,t} = f_t(h_t, x_{h,t})$ and a contingent plan $(f_s)_{s\geq t+1}$ for any possible future histories. Then, given the aforementioned setting, the sustainable equilibrium for the economy can be defined as follows.

Definition 1. The pair of strategies (σ, f) formulated by the monetary authority and private agents, respectively, is a sustainable equilibrium that
(i) satisfies the continuation condition of private agents' reaction function f given by the optimality condition

$$f_t(h_t, x_{h,t}) = \beta E_t[f_{t+1}(h_{t+1}, \sigma_{t+1}(h_{t+1}))] + \xi x_{h,t} + \omega p^*_{c,t}, \quad t \geq 0,$$
$$f_s(h_s, \sigma_s(h_s)) = \beta E_s[f_{s+1}(h_{s+1}, \sigma_{s+1}(h_{s+1}))] + \xi \sigma_s(h_s) + \omega p^*_{c,s}, \quad s \geq t+1,$$

given the policy strategy (σ) and the current history $(h_t, x_{h,t})$ for all possible future histories induced by σ;
(ii) solves the policymaker's problem (by choosing the current and future policy strategy $(\sigma_s)_{s\geq t}$ subject to private agents' reaction function f and current history h_t) defined as

$$\max_{(\tilde{\sigma}_s)_{s\geq t}} \quad -E_t \sum_{s=t}^{\infty} \beta^{s-t} \{[f_s(h_s, \tilde{\sigma}_s(h_s))]^2 + \lambda_x[\tilde{\sigma}_s(h_s)]^2\}$$
$$s.t. \quad f_s(h_s, \tilde{\sigma}_s(h_s)) = \beta E_s[f_{s+1}(h_{s+1}, \tilde{\sigma}_{s+1}(h_{s+1}))] + \xi \tilde{\sigma}_s(h_s) + \omega p^*_{c,t},$$

for all possible future histories induced by $(\tilde{\sigma}_s)_{s\geq t}$.

The sustainability constraint

Within a policy game setting, policymakers consider the sustainability constraint:

$$W^c(p^*_{c,t}, x_{h,t-1|t=0}) \geq W^d(p^*_{c,t}). \tag{12}$$

This constraint reflects the goal of achieving the highest possible welfare level. Ideally, policymakers aim for the level attained under the optimal commitment policy, denoted by

[7] The definition of the sustainable equilibrium closely follows Sunakawa (2015) and Kurozumi (2008).

[8] As in Chari and Kehoe (1990), Kurozumi (2008) and Sunakawa (2015) private agents are policy takers. Then, the history of domestic inflation $(\pi_{h,t})$ is excluded from public history and only the policymaker can deviate from its current policy.

$W^c(p^*_{c,t}, x_{h,t-1|t=0})$. However, at a minimum, they seek to achieve the level associated with the worst sustainable equilibrium, which arises under the optimal discretionary policy, denoted by $W^d(p^*_{c,t})$. The associated welfare measures under the optimal commitment and the optimal discretionary policies are detailed in equations (A.1) and (A.2) of the Appendix A.1.

Kurozumi (2008) demonstrates that the optimal discretionary policy rule delivers the worst sustainable equilibrium. This is established through two propositions.

Proposition 1. The rational expectations equilibrium (REE) prescribed by the optimal discretionary policy is the worst sustainable equilibrium of the model.

This proposition characterizes the complete set of outcomes generated by the sequences of functions representing sustainable equilibria outcomes, denoted by $(\sigma, f) = \{(\sigma_t), (f_t)\}_{t\geq 0}$. Notably, the sustainable equilibrium yields the pair of contingent sequences for domestic inflation rates and domestic output gaps, denoted by $(\pi_{h,t}, x_{h,t}) = \{(\pi_{h,t}), (x_{h,t})\}_{t\geq 0}$ and, known as the outcome of the equilibrium.

Proposition 2. Any arbitrary pair of contingent sequences of domestic inflation rates and domestic output gaps, denoted by (π_h, x_h), is considered an outcome of a sustainable equilibrium if and only if:
(i) the pair (π_h, x_h) satisfies the NKPC in (4) every period $t \geq 0$; and,
(ii) the sustainability constraint in (12) holds in every period $t \geq 0$.

This proposition establishes necessary and sufficient conditions for the existence of a sustainable equilibrium with a specific outcome (π_h, x_h). Furthermore, the constraints defined in this proposition encompass the entire set of sustainable equilibrium outcomes.

The aforementioned propositions transform the optimal sustainable policy into a strategy for achieving the best sustainable equilibrium without relying on commitment technologies. As stated by Kurozumi (2008), "the optimal sustainable policy becomes a policy strategy which specifies to continue the optimal quasi-sustainable policy as long as it has been adopted in the past; otherwise, the strategy specifies to switch to the optimal discretionary policy forever".

In summary, the optimal sustainable policy does not necessitate a commitment technology assumption but rather a reputation assumption (similar to Chari and Kehoe, 1990). Second, the optimal sustainable policy represents a regime that falls between the optimal commitment and discretionary policies, potentially even coinciding with either one. Finally, the optimal sustainable policy leverages the optimal quasi-sustainable policy as a tool to achieve the best sustainable equilibrium outcome.

Characterization of the optimal sustainable policy

The optimal quasi-sustainable policy is obtained by maximizing the social welfare function (5) within the constraints (4) and (12), starting from period zero onwards. This set of constraints defines the entire set of sustainable equilibrium outcomes, as stated in Proposition 2.

The Lagrangian associated with the optimal quasi-sustainable policy is given by:

$$\mathcal{L} \equiv E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ -\frac{\Omega}{2}(\pi_{h,t}^2 + \lambda_x x_{h,t}^2) + \varphi_{1,t}(\pi_{h,t} - \beta E_t \pi_{h,t+1} - \xi x_{h,t} - \omega p_{c,t}^*) - \varphi_{2,t}[E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{\Omega}{2}(\pi_{h,s}^2 + \lambda_x x_{h,s}^2) + W^d(p_{c,t}^*)] \Big\}, \tag{13}$$

where $\varphi_{1,t}$ and $\varphi_{2,t}$ are the Lagrangian multipliers on the constraints (4) and (12), respectively, for $t \geq 0$.

Building on prior work by Kurozumi (2008) and Sunakawa (2015), the same problem can be rewritten by applying the recursive formulation as proposed by Marcet and Marimon (2019) and implementing Abel's summation formula. The recursive formulation can be written as

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ -\Psi_t \frac{\Omega}{2}(\pi_{h,t}^2 + \lambda_x x_{h,t}^2) + (\varphi_{1,t} - \varphi_{1,t-1})\pi_{h,t} - \varphi_{1,t}(\xi x_{h,t} + \omega p_{c,t}^*) + \varphi_{2,t} W^d(p_{c,t}^*) \Big\}, \tag{14}$$

where Ψ_t is the multiplier that recursively measures the tightness of the sustainability constraint ($\varphi_{2,t}$) and the fulfillment of past commitments (Ψ_{t-1}): $\Psi_t \equiv 1 + \sum_{i=0}^{t} \varphi_{2,i} = \Psi_{t-1} + \varphi_{2,t}$, $\forall t \geq 0$, with initial conditions for $\Psi_{t-1|t=0} = 1$ and $\varphi_{2,t|t=0} = 0$.

The first-order conditions for $\pi_{h,t}$ and $x_{h,t}$ yield the optimality condition for the optimal quasi-sustainable policy

$$\pi_{h,t} = -\frac{\lambda_x}{\xi}(x_{h,t} - \frac{\Psi_{t-1}}{\Psi_t} x_{h,t-1}), \ \forall t \geq 0, \tag{15}$$

where the ratio $\Psi_{t-1}/\Psi_t = \Psi_{t-1}/(\Psi_{t-1} + \varphi_{2,t}) \in (0,1]$ reflects the degree of commitment steadfastness.

When the sustainability constraint is binding ($\varphi_{2,t} > 0$), $0 < \Psi_{t-1}/\Psi_t < 1$, and the optimal quasi-sustainable policy deviates from the optimal commitment policy (a reputational mechanism replaces commitment technology). Conversely, when the constraint is slack ($\varphi_{2,t} = 0$), $\Psi_{t-1}/\Psi_t = 1$, and the optimal quasi-sustainable policy coincides with the optimal commitment policy (implying that commitment and reputation are equivalent under such conditions).

In conclusion, the recursive formulation demonstrates that the optimal quasi- sustainable policy offers a solution free from time-inconsistency issues.

2.3 Solution method

This section describes the solution strategy, numerical method, and calibration employed to solve for the optimal monetary policies.

2.3.1 Solving for the optimal policies

The optimal quasi-sustainable policy solution depends on the solution of the optimal commitment and the optimal discretionary policies. Consequently, the system to be solved for the optimal quasi-sustainable policy incorporates:

its relevant welfare level (which is given by the welfare under the optimal commitment policy),

$$W_t^c = -\frac{\Omega}{2}(\pi_{h,t}^2 + \lambda_x x_{h,t}^2) + \beta E_t W_{t+1}^c;$$

its optimality condition,

$$x_{h,t} = -\frac{\xi}{\lambda_x}\pi_{h,t} + z_t x_{h,t-1};$$

the NKPC,

$$\pi_{h,t} = \beta\pi_{h,t+1} + \xi x_{h,t} + \omega p_{c,t}^*;$$

and the sustainability constraint,

$$W^c(p_{c,t}^*, x_{h,t-1|t=0}) \geq W_t^d(p_{c,t}^*).$$

Here, $z_t = \Psi_{t-1}/\Psi_t \in (0, 1]$ is the commitment steadfastness measure (as discussed previously), and the initial conditions are given by $x_{h,-1}$ and $\Psi_{-1} = 1$.

The system comprises four endogenous variables $\{x_{h,t}, \pi_{h,t}, \Psi_t, \varphi_t\}_{t=0}^{\infty}$ and one exogenous variable $\{p_{c,t}^*\}_{t=0}^{\infty}$. Given the potential for the sustainability constraint to be binding, a variant of the policy function iteration method is employed, as implemented by Kehoe and Perri (2002) and Sunakawa (2015).[9]

The system can be rewritten in a state-space representation by defining $u = (p_{c,t}^*, x_{h,-1}) \in P \times X$ where P and X are closed sets. The relevant welfare level measure for the optimal quasi-sustainable policy then becomes:

$$W^u(u) = -\frac{\Omega}{2}([\pi_h^u(u)]^2 + \lambda_x[x_h^u(u)]^2) + \beta\sum_{p_{c,t}^{*}{}'} p(p_{c,t}^{*}{}'|p_{c,t}^*)W^u(p_{c,t}^{*}{}', x_h^u(u)).$$

The optimality condition,

$$x_h^u(u) = -\frac{\xi}{\lambda_x}\pi_h^u(u) + z(u)x_{h,-1}^u;$$

the NKPC,

$$\pi_h^u(u) = \xi x_h^u(u) + \beta\sum_{p_{c,t}^{*}{}'} p(p_{c,t}^{*}{}'|p_{c,t}^*)\pi_h^u(p_{c,t}^{*}{}', x_h^u(u)) + \omega p_{c,t}^*;$$

and the sustainability constraint,

$$W^u(u) \geq \hat{W}^d(p_{c,t}^*);$$

are written similarly, using $W^u(u)$, $\pi_h^u(u)$, $x_h^u(u)$ and $z(u)$ as the policy functions, where $\hat{W}^d(p_{c,t}^*)$ is the numerically computed worst sustainable equilibrium value under the dis-

[9] To solve for the optimal commitment policy and the optimal discretionary policy, note that one only needs to change the optimality condition for each regime and assume that the commitment steadfastness is unitary under commitment and zero under discretion.

cretionary policy. The matrix $p(p_{c,t}^{*}{}'|p_{c,t}^{*})$ represents the transition probability, which is approximated by a first-order auto-regressive process, as specified in equation (2).

The recursive structure allows implementation of the policy function iteration method and addresses the occasionally binding constraint for the entire space $u \in P \times X$. Appendix A.2 provides further details on the numerical algorithm implementation.

2.3.2 Calibration

The baseline model adopts the parameter values established by DMT. These authors, in turn, utilize calibrated parameters that sometimes coincide with those set by Ferrero and Seneca (2019) and Galí and Monacelli (2005).

Specifically, the quarterly subjective domestic and foreign discount factors are calibrated to $\beta = \beta^{*} = 0.9963$, corresponding to annual real domestic and foreign interest rates of $r = r^{*} \approx 1.5\%$ and a zero-inflation steady state. The home consumption bias of the economy, $1 - \alpha$, is set to 60%, while the elasticity of substitution is set to 6, consistent with a desired markup of 20%. The Calvo price re-setting probability is set to 25%, and the inverse Frisch elasticity of labor supply is set to 3.

The share of intermediate goods used in the production of commodity goods, s_m, is set to 15%, while the share of the commodity-producing sector relative to the gross domestic product of the economy, s_{yc}, is set to 20%. The returns to scale in the commodity production function, ν, are calibrated to 0.38, while the elasticity of borrowing conditions relative to international commodity prices, χ, is set to 0.5. The exogenous process parameters are defined by the persistence (autoregressive) parameter, $\rho_{p_c^*}$, and the volatility of the shock, $\sigma_{p_c^*}$, which are set to 0.9 and 0.1, respectively.

Based on these parameters, the slope of the New Keynesian Phillips Curve (NKPC), ξ, and the weight on the output gap, λ_x, are equal to 0.3299 and 0.0567, respectively. Under the baseline calibration, the weight on the commodity price shock, ω, in the NKPC yields a negative value of -0.0137.

The upper bound of the disturbance, $m\sigma_{p_c^*}$, for computational purposes is set to $m = 6$. Likewise, the maximum and minimum values of the grid for the domestic output gap grid, X, are chosen consistently using the same upper bound value for optimal volatility. The number of grids for the commodity price shock, $n_{p_c^*}$, is set to 31, while the number of grids for the output gap, n_{x_h}, is set to 15. The exogenous process for the commodity price shock is approximated and bounded as a first-order autoregressive process according to Tauchen's (1986) method.

3 Results

Results under the baseline and alternative calibrations of the model are presented in this section.

3.1 Results under the baseline calibration

Assuming a commodity price shock (CPS) hits the small open- and -commodity-exporting economy, the responses of selected macroeconomic variables are analyzed under each one of the monetary policy regimes discussed in the previous section.

Figure 1 shows that commitment steadfastness over the horizon the CPS hits the economy remains full (equal to one). As explained earlier, a non-binding sustainability constraint implies that the optimal quasi-sustainable policy coincides with the optimal commitment policy. Furthermore, the equilibria attained under both regimes are identical. In other words, the sustainable equilibrium under the optimal quasi-sustainable policy aligns with the competitive equilibrium delivered by the optimal commitment policy.

The high value of the subjective discount factor ($\beta = 0.9963$) prevents the sustainability constraint from binding, leading to full commitment steadfastness. This finding aligns with Sunakawa (2015), who obtained the same conclusion for a small open economy (without a commodity-producing sector) using standard calibration values.[10]

Observing the responses of selected macroeconomic variables to a 10% CPS shock under each optimal policy regime reveals that domestic inflation, the nominal exchange rate, and the domestic output gap exhibit the most significant differences (Figure 1). As expected, the responses of these variables are lower under the optimal commitment and quasi-sustainable policies and higher under the optimal discretionary policy. These differences stem from the technology assumptions underlying each policy regime. Evidently, the commitment and reputational technologies, reflected in their respective policy rules for domestic inflation and the output gap, contribute to smoother macroeconomic variable responses to the shock. Notably, domestic inflation and the output gap directly affect the representative's welfare function. Therefore, the responses of these variables highlight the welfare differences associated with each monetary policy regime.

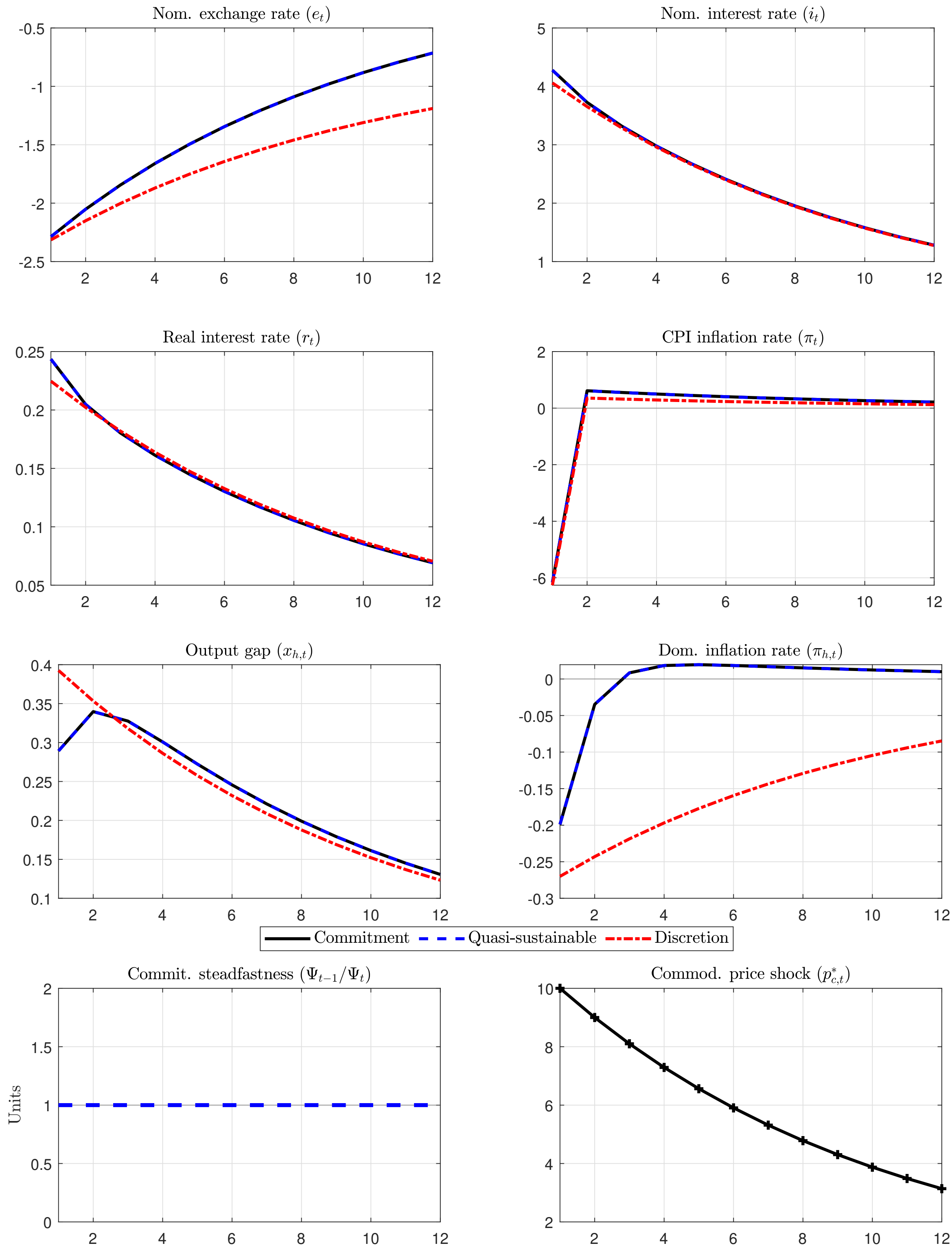
In terms of second moments of the macroeconomic variables, only domestic inflation and the domestic output gap display statistically significant differences in volatilities. These volatility measures are assessed under the optimal discretionary policy and the two identical regimes (the optimal commitment and the optimal quasi-sustainable policies). Table A.1 presents simulated second moments of selected macroeconomic variables and F-tests for equal variances results, indicating statistically significant differences at $p < 1\%$ (***), $p < 5\%$ (**) and $p < 10\%$ (*), respectively. As mentioned, the statistically significant differences in this baseline scenario are observed for domestic inflation and the domestic output gap under the aforementioned policy regimes. Volatility difference tests for the remaining selected macroeconomic variables are not statistically significant.

Regarding the monetary conditions of the economy in response to a positive CPS, Figure 1 shows that all optimal policy regimes agree to tighten monetary conditions. The tightest initial conditions are prescribed under the optimal commitment and the optimal quasi-sustainable policies, although they become less tightened over time compared to the optimal discretionary policy.

In the baseline scenario, when the representative commodity-producing sector observes a rise in international commodity prices, it decides to increase production to generate more profits. To achieve this goal, the firm contracts international borrowings denominated in nominal foreign currency to finance input purchases for increased commodity production.

[10] Read more about in subsection 3.2.3.

Figure 1: The baseline model under different optimal monetary policy regimes (in %)



Note. The impulse–response function (IRF) plots display a 10% positive commodity price shock. The shock is equivalent to a one standard deviation percentage from its respective steady state. Unless it is exclusively specified, horizontal and vertical axes indicate quarters and percentage deviation from the steady state, respectively.

With a higher demand for inputs, domestic prices rise, relative prices drop, the terms of trade deteriorate (fall), and this depreciation appreciates the nominal and real exchange rates of the economy.

The change in relative prices also leads to changes in optimal allocations (e.g., total and domestic consumption goods decrease). The non-traded final goods sector produces more, contracts more hours at work, and the commodity boom leads to a positive output gap. With domestic inflation dropping (due to the fall in total and domestic consumption demands) and the domestic output gap increasing (due to the commodity boom), the policymaker faces a trade-off to return the economy to its optimal equilibrium.

The analyzed optimal monetary policy regimes (optimal commitment, optimal quasi-sustainable, and optimal discretionary) all prescribe an increase in the nominal interest rate and an appreciation of the nominal and real exchange rates, which are subsequently reversed over time.

3.2 Results under alternative calibrations

Three main variations to the baseline model are introduced. First, the analysis assumes that the financial channel of the DMT model varies. Second, the model's predictions are examined when the commodity input share in the economy changes. Third, alternative values for the subjective discount factor are evaluated to determine if commitment steadfastness decreases. Finally, some alternative modifications to the baseline model are analyzed in Appendix A.1 and Supplementary Material B.

3.2.1 Varying the financial channel

Small open- and -commodity-exporting economies are more sensitive to CPS as the inelasticity of borrowings with respect to international commodity prices condition increases ($\chi \to 0$). This implies that higher inelasticity leads to higher responses and volatilities of domestic inflation and the output gap.[11]

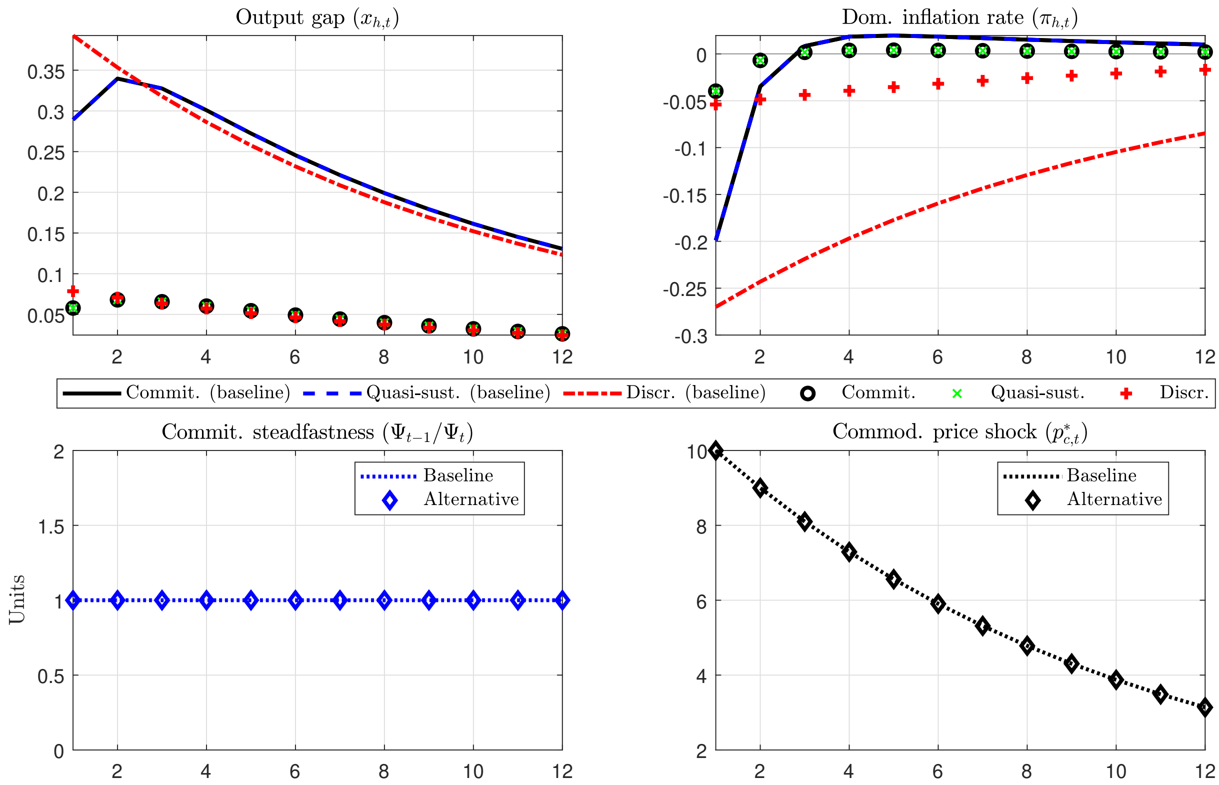
Comparing the baseline calibration ($\chi = 0.5$) with the alternative calibration ($\chi = 0.9$), one observes that as the elasticity of borrowings with respect to the international commodity prices condition tends towards unity ($\chi \to 1$), the responses and volatilities of domestic inflation and the output gap decrease (see Figure 2 and Table A.1). This holds true under all policy regimes, where it is still verified that the sustainability constraint does not bind in any case; meaning that the optimal commitment policy remains consistent with the optimal sustainable policy. Notice that the optimal discretionary policy continues to offer the highest responses and volatilities for macroeconomic variables to the CPS.

The current analysis confirms that the elasticity of borrowings with respect to the international commodity prices condition plays a key role as a financial amplification channel for monetary policy. In simpler terms, when favorable factors positively affect commodity prices in international markets, the representative commodity firm can borrow more, produce more, and the commodity boom becomes a stronger driver of the business cycle.

Conversely, in this alternative calibration, the monetary policy condition is expected to be less restrictive due to the lower variations in domestic inflation and the output gap in response to the same CPS. The same assertion holds for the nominal interest rate, the CPI inflation rate, and the real and nominal exchange rates.

[11]This prediction of the DMT model aligns with the one made by Báez's (2022) model.

Figure 2: Varying the financial channel: $\chi = \{0.5, 0.9\}$, $\omega = \{-0.01, -0.003\}$ (in %)



Note. The impulse–response function (IRF) plots display a 10% positive commodity price shock. The shock is equivalent to a one standard deviation percentage from its respective steady state. Unless it is exclusively specified, horizontal and vertical axes indicate quarters and percentage deviation from the steady state, respectively. The solid, dashed and dash-dotted lines are variable responses under the baseline calibration values (or baseline scenario). While the circle, cross and plus signs are variable responses under the alternative calibration values (or alternative scenario).

3.2.2 Varying the commodity inputs share

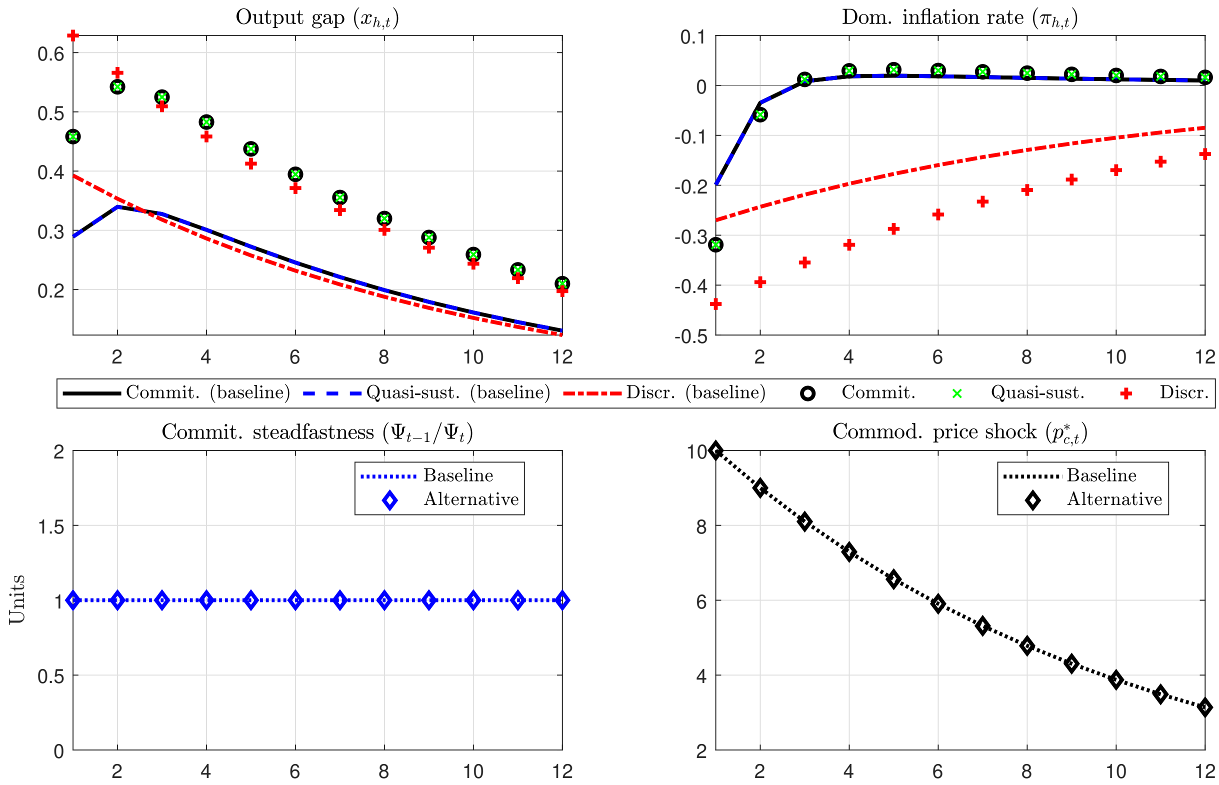
Small open- and commodity-exporting economies exhibit greater sensitivity to CPSs as the share of commodity inputs increases. In other words, a higher ratio amplifies the responsiveness and volatility of domestic inflation and output gaps.

An increase in the commodity input share (s_m) signifies a larger proportion of intermediate goods (demanded by the representative commodity-producing firm) relative to the output produced by the non-traded final goods firms sector. Consequently, the proportion of final domestic consumption goods in the economy (s_c) declines, as both variety goods are produced by firms within the same sector.

When a CPS shock hits the economy, a higher proportion of domestic intermediate goods destined for commodity production exacerbates the deflationary process and economic boom in the SOCEE (Figure 3). The alternative calibration results in a more pronounced decline in domestic inflation and a larger increase in the domestic output gap. Furthermore, the second moments for these two variables (as shown in Table A.1) also exhibit higher magnitudes under the current alternative calibrations. As expected, the optimal discretionary policy regime leads to the highest volatilities for these two variables.

This analysis reaffirms that the sustainability constraint remains non-binding. Consequently, the optimal commitment policy aligns with the optimal sustainable policy (Fig-

Figure 3: Higher commodity inputs share: $s_m = \{0.15, 0.30\}$ (in %)



Note. The impulse–response function (IRF) plots display a 10% positive commodity price shock. The shock is equivalent to a one standard deviation percentage from its respective steady state. Unless it is exclusively specified, horizontal and vertical axes indicate quarters and percentage deviation from the steady state, respectively. The solid, dashed and dash-dotted lines are variable responses under the baseline calibration values (or baseline scenario). While the circle, cross and plus signs are variable responses under the alternative calibration values (or alternative scenario).

ure 3), despite the significant role played by the commodity input share in amplifying the CPS shock.

3.2.3 Varying the discount factor

A lower discount factor (which corresponds to a higher nominal interest rate) leads to larger responses in the output gap and domestic inflation (Figure 4).

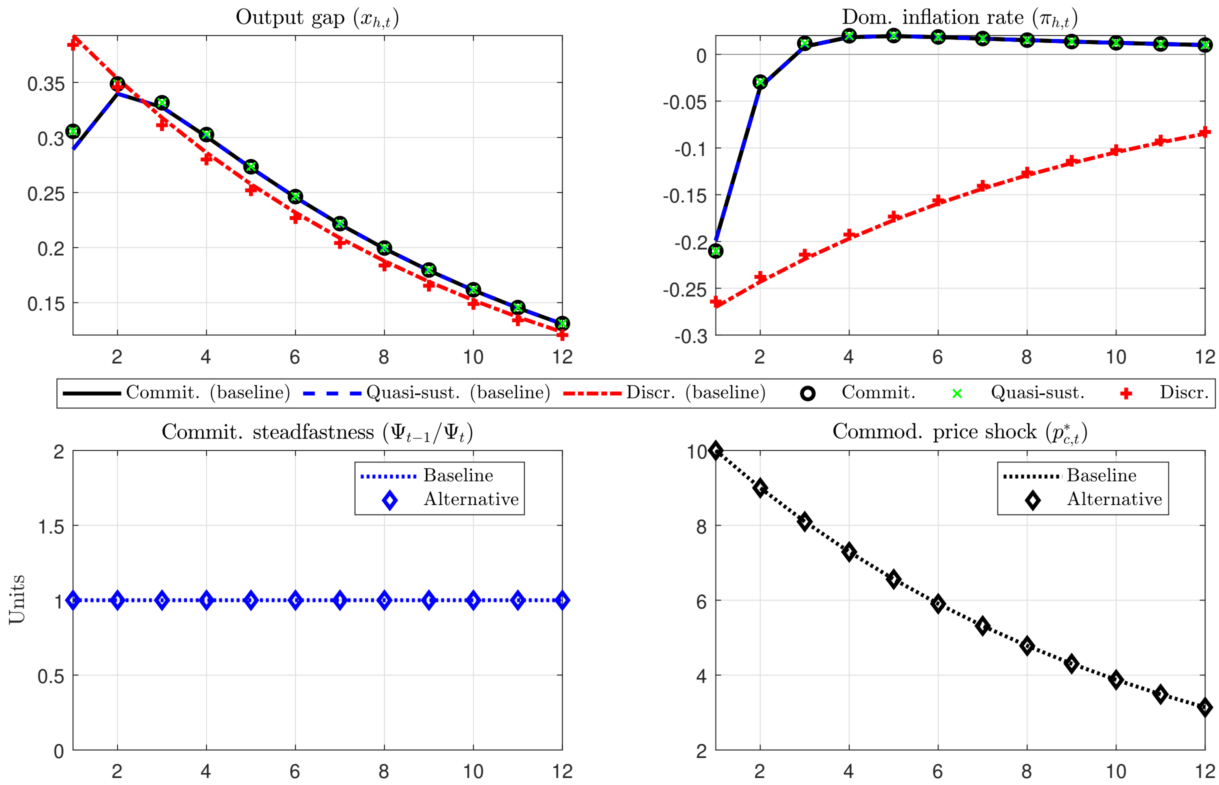
Standard calibrations for $\beta \geq 0.9$ (as those in the literature) imply that the optimal quasi-sustainable policy is consistent with the optimal commitment policy.[12] Specifically, the discount factor parameter (β) varies from 0.9936 to 0.900, corresponding to annual real interest rates of approximately 1.5% and 52.4%, respectively (Figure 4).

The discount factor (and its relationship with the nominal interest rate) is a key parameter for all economies, including commodity exporters. This is because it plays a crucial role in determining welfare under the sustainability constraint (optimal commitment policy welfare $\geq$ optimal discretionary policy welfare).

A lower discount factor ($\beta = \{0.9963, 0.6\}$; $i_{ss} \approx \{1.5\%, 671.6\%\}$) implies an even higher

[12] As mentioned in subsection 3.1, this results aligns with the findings of Sunakawa (2015) for small open economies without a commodity sector.

Figure 4: Varying the discount factor: $\beta = \{0.9963, 0.9\}$ (in %)



Note. The impulse–response function (IRF) plots display a 10% positive commodity price shock. The shock is equivalent to a one standard deviation percentage from its respective steady state. Unless it is exclusively specified, horizontal and vertical axes indicate quarters and percentage deviation from the steady state, respectively. The solid, dashed and dash-dotted lines are variable responses under the baseline calibration values (or baseline scenario). While the circle, cross and plus signs are variable responses under the alternative calibration values (or alternative scenario).

steady-state nominal interest rate. While unrealistic, this scenario highlights how commitment deviates from unity (the sustainability constraint binds). In other words, the optimal sustainable policy is no longer consistent with the optimal commitment policy, and their respective equilibria diverge (Figure A.1 in Appendix A.1).

The difference between the optimal quasi-sustainable and optimal commitment equilibria arises not only from the discount factor change but also from the optimal sustainable policy design itself. The assumption of an infinite punishment period (Section 2.2) with a lower welfare level (for moving from the optimal commitment policy to the optimal discretionary policy) explains this outcome.

This outcome suggests that a finite punishment period could lead to less frequent consistency between the optimal quasi-sustainable and optimal commitment equilibria. Therefore, reputation technology would become more relevant in monetary policy design.

In this context, it is important to remember that the concept of reputation and/or credibility as a substitute for commitment in monetary policy originates from the work of Barro and Gordon (1983). Their analysis models optimal monetary policy within a one-punishment period. When the policymaker attempts to benefit from unanticipated inflation shocks, they consequently face inflation-related costs that negatively impact their reputation (or credibility). The authors also explore the conditions under which the optimal enforceable rule is binding, seeking to identify a sustainable outcome.

Accordingly, such a rule minimizes expected inflation costs while satisfying an 'enforceability constraint'. This constraint ensures the present value of punishment for breaking the rule (enforcement costs) is at least as high as the benefits of reneging (temptation costs). The spirit of this constraint and the resulting optimal sustainable policy build upon similar concepts explored in their previous work.

However, this paper leaves the design of an optimal sustainable policy with a finite number of punishment periods as an open question for future research.

Other predictions of the DMT model

Additional results and discussions about them can be found in the Supplementary Material B that accompanies this paper. In particular, this material analyzes specific results of the DMT model when the borrowing elasticity is elastic, when the share of the commodity sector to the GDP increases, among other alternative parameterizations from the model (like variations in the price rigidity, the desired markup price, the size of the CPS, and the inverse Frisch elasticity rate).

4 Conclusion

A welfare-based optimal monetary policy for a small open- and -commodity-exporting economy is conducted under the concept of the optimal sustainable policy, as proposed by Kurozumi (2008).

Relying on Drechsel et al.'s (2019) model and assuming that a commodity price shock (CPS) hits the economy, the optimal sustainable policy is evaluated following Sunakawa's (2015) quantitative approach.

The optimal quasi-sustainable policy serves as an operational platform for the optimal sustainable policy framework. This platform is compared with the optimal commitment and optimal discretionary monetary policies under baseline and alternative calibrations. The results demonstrate that the optimal quasi-sustainable policy always aligns with the optimal commitment policy. This implies that the competitive equilibrium achieved under the optimal commitment policy coincides with the sustainable equilibrium attained under the optimal quasi-sustainable policy.

In response to the commodity price shock, the monetary authority tightens monetary policy by raising the nominal interest rate and allowing for an appreciation of the nominal and real exchange rates. This mitigates the inefficiencies caused by the commodity boom. As expected, the responses and volatilities of domestic inflation, the nominal exchange rate, and the output gap are larger under the optimal discretionary policy compared to the optimal commitment and optimal quasi-sustainable policies.

This paper identifies two additional key findings. First, small, open, commodity-exporting economies with a more inelastic borrowing elasticity in their commodity sector experience greater responses and volatilities in domestic inflation and the output gap. Second, these economies also observe more responsive and volatile domestic inflation and output gaps when the share of commodity inputs is higher.

Finally, there are two key assumptions of the DMT model that can be further explored to assess the robustness of these conclusions. The first concerns the linear-quadratic approximation approach, and the second, the exogeneity of the financial channel. On the one hand, one could drop the assumptions of the labor subsidy and the zero inflation at the steady state, and on the other hand, one could consider an endogenous financial channel.

In this regard, the non-linear model proposed by Báez (2022) provides a framework that overcomes the two aforementioned limitations and could be used for further research. Specifically, under the approach of the optimal sustainable policy à la Kurozumi (2008) one could examine the results from the optimal commitment, sustainable y discretionary policies, following a similar approach to Leith and Liu (2016) and Sunakawa (2015).

Acknowledgments and disclaimers

The author expresses sincere gratitude to Dr. Paulo Santos Monteiro for his invaluable guidance and support during the development of this paper. Special thanks are also extended to the members of the Thesis Advisory Panel, Dr. Neil Ranking and Dr. João Madeira, for their insightful comments and feedback. Naoki Yago and Andrea Ferrero are likewise thanked for their valuable suggestions.

Financial support provided by the Central Bank of Paraguay and the National Postgraduate Scholarship Program 'Don Carlos Antonio López' of the Republic of Paraguay is gratefully acknowledged.

The author assumes full responsibility for any errors or omissions in this paper. The views expressed herein are those of the author and do not necessarily reflect the position of the aforementioned institutions or compromise any of their interests.

A Appendix

A.1 Policy rules: additional details

Welfare measures

The associated welfare measure under the optimal commitment policy is given by:

$$W^c(p^*_{c,t}, x^c_{t-1}) = -\frac{\Omega}{2}\omega^2\Big\{(a_\pi^2 + \lambda_x a_x^2)[\frac{1}{1-\beta\rho^2}(p^*_{c,t})^2 + \beta\sigma^2_\epsilon\frac{1}{1-\beta}\frac{1}{1-\rho^2}]$$

$$+(b_\pi^2 + \lambda_x b_x^2)\frac{1}{1-\beta b_x^2}[(\omega a_x)^2\rho^2\frac{1}{1-b_x^{-2}\rho^2}(p^*_{c,t})^2 + (\omega a_x)^2\sigma^2_\epsilon\frac{1}{1-b_x^2}\frac{1}{1-\rho^2}]\Big\}$$

$$-\frac{\Omega}{2}\omega^2\frac{(b_\pi^2 + \lambda_x b_x^2)}{1-\beta b_x^2}b_x^2(x^c_{h,t-1})^2. \tag{A.1}$$

The welfare measure related to the optimal discretionary policy is defined as:

$$W^d(p^*_{c,t}) = -\frac{\frac{\Omega}{2}\omega^2(c_\pi^2 + \lambda_x c_x^2)}{1-\beta\rho^2}\big[(p^*_{c,t})^2 + \frac{\beta\sigma^2_\epsilon}{1-\beta}\frac{1-\beta\rho^2}{1-\rho^2}\big]. \tag{A.2}$$

Unconditional variances

Commodity price shock,

$$\sigma^2_{p^*_c} = \frac{\sigma^2_\epsilon}{1-\rho^2_{p^*_{c,t}}}. \tag{A.3}$$

Domestic output gap under the optimal commitment policy,

$$(\sigma^c_x)^2 = \frac{(\omega a_x)^2\sigma^2_{p^*_c}}{1-b_x^2}. \tag{A.4}$$

Domestic inflation under the optimal commitment policy,

$$(\sigma^c_\pi)^2 = \frac{(\omega a_\pi)^2\sigma^2_{p^*_c}}{1-b_\pi^2}. \tag{A.5}$$

Domestic output gap under the optimal discretionary policy,

$$(\sigma^d_x)^2 = (\omega c_x)^2\sigma^2_{p^*_c}. \tag{A.6}$$

Domestic inflation under the optimal discretionary policy,

$$(\sigma^d_\pi)^2 = (\omega c_\pi)^2\sigma^2_{p^*_c}. \tag{A.7}$$

A.2 Numerical algorithm: details

Given the state-space representation indicated in subsubsection 2.3.1, one can calculate $\hat{W}^d(p^*_{c,t})$ as follows.

As of equations (4) and (9), one may write the state-space representation,

$$(1+\frac{\xi^2}{\lambda_x})\pi_h(p^*_{c,i}) = \beta\sum_j p(p^*_{c,j}|p^*_{c,i})\pi_h(p^*_{c,j}) + \omega p^*_{c,i}.$$

Equivalently, in matrix notation,

$$\begin{bmatrix} \frac{1}{\omega}[1+\frac{\xi^2}{\lambda_x}-\beta p(p^*_{c,1}|p^*_{c,1})] & -\frac{\beta}{\omega}p(p^*_{c,2}|p^*_{c,1}) & \cdots & -\frac{\beta}{\omega}p(p^*_{c,n_{p^*_c}}|p^*_{c,1}) \\ -\frac{\beta}{\omega}p(p^*_{c,1}|p^*_{c,2}) & \frac{1}{\omega}[1+\frac{\xi^2}{\lambda_x}-\beta p(p^*_{c,2}|p^*_{c,2})] & \cdots & -\frac{\beta}{\omega}p(p^*_{c,n_{p^*_c}}|p^*_{c,2}) \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\beta}{\omega}p(p^*_{c,1}|p^*_{c,n_{p^*_c}}) & -\frac{\beta}{\omega}p(p^*_{c,2}|p^*_{c,n_{p^*_c}}) & \cdots & \frac{1}{\omega}[1+\frac{\xi^2}{\lambda_x}-\beta p(p^*_{c,n_{p^*_c}}|p^*_{c,n_{p^*_c}})] \end{bmatrix} \begin{bmatrix} \pi_h(p^*_{c,1}) \\ \pi_h(p^*_{c,2}) \\ \vdots \\ \pi_h(p^*_{c,n_{p^*_c}}) \end{bmatrix} = \begin{bmatrix} p^*_{c,2} \\ p^*_{c,2} \\ \vdots \\ p^*_{c,n_{p^*_c}} \end{bmatrix},$$

where each grid for the shock makes possible to solve for $\pi_h(p^*_{c,i})$. Subsequently, once $\pi_h(p^*_{c,i})$ is known, substituting (9) into (5) one can solve for $\hat{W}^d(p^*_{c,i})$. In recursive terms,

$$\hat{W}^d(p^*_{c,i}) = -\frac{\Omega}{2}(1+\frac{\xi^2}{\lambda_x})[\pi_h(p^*_{c,i})]^2 + \beta\sum_j p(p^*_{c,j}|p^*_{c,i})\hat{W}^d(p^*_{c,j}).$$

The numerical algorithm follows the same steps as Sunakawa (2015). Next points describe the algorithm steps.

1. Initial guess values for functions $\hat{W}^{(0)}(u)$ and $\pi_h^{(0)}(u)$ are set according to each grid point on $P \times X$ of the space $u = (p^*_c, x_{h,-1})$.
2. For every grid point u, the equations for $(W^{i,u}, \pi_h^{i,u}, x_h^{i,u}, z^{i,u})$ are solved, given the functions $W^{(i-1)}(p^{*\prime}_c, x_h)$ and $\pi_h^{i-1}(p^{*\prime}_c, x_h)$.
3. Set the new functions as $W^{(i)}(u) = \{W^{i,u}\}_{u\in P\times X}$ and $\pi_h^{(i)}(u) = \{\pi_h^{i,u}\}_{u\in P\times X}$.
4. Iterate steps 2 and 3 until the functions $W^{(i)}(u)$ and $\pi_h^{(i)}(u)$ converge at each grid point.

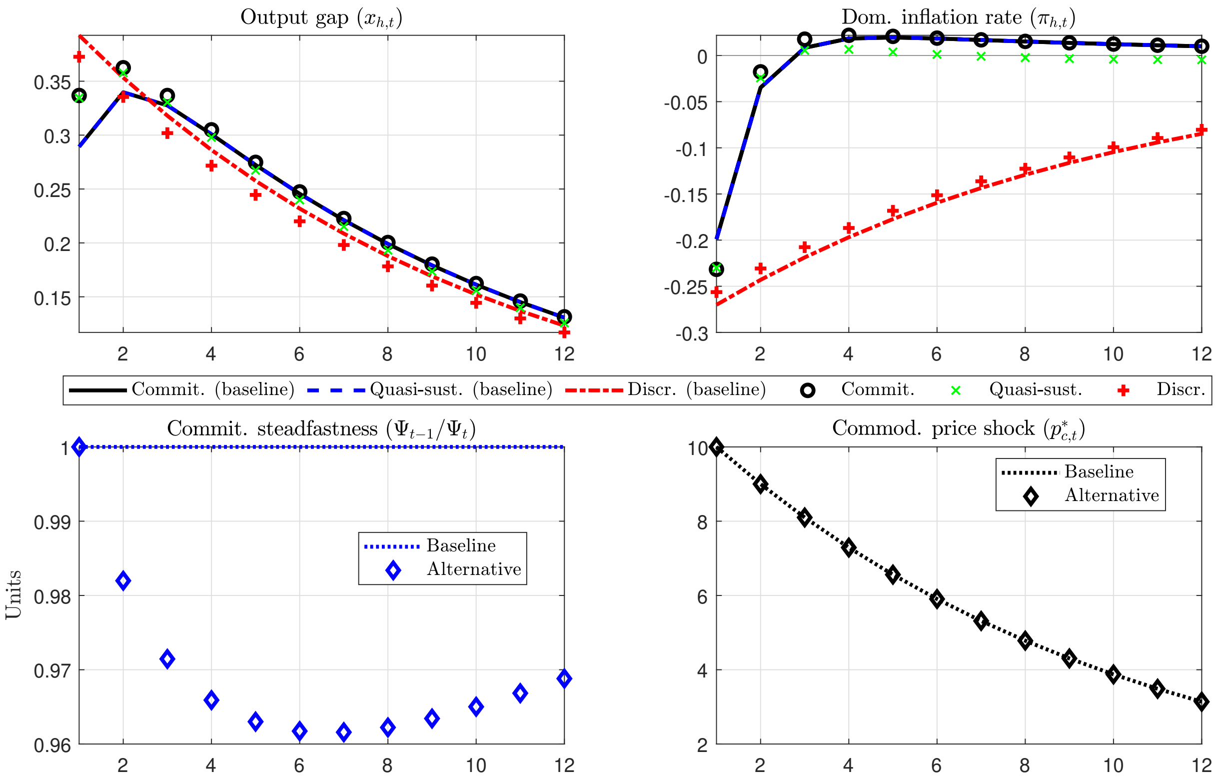
The two relevant cases for the sustainability constraint are that it may bind or not.

In case the sustainability constraint binds, $z^{i,u} = 1$. Solve
$W^{i,u} = -\frac{\Omega}{2}([\pi_h^{i,u}]^2 + \lambda_x[x_h^{i,u}]^2) + \beta\sum_{u'} p(u'|u)W^{(i-1)}(u', x_h^{i,u})$,
$x_h^{i,u} = -\frac{\xi}{\lambda_x}\pi_h^{i,u} + x_{h,-1}$,
$\pi_h^{i,u} = \xi x_h^{i,u} + \beta\sum_{p^{*\prime}_c} p(p^{*\prime}_c|p^*_c)\pi_h^{(i-1)} + \omega p^*_c$,
for the values of $(x_h^{i,u}, \pi_h^{i,u}, W^{i,u})$.

In case the sustainability constraint does not bind, $z^{i,u} \in (0,1)$ and $W^{i,u} = \tilde{W}^d(p^*_c)$. Solve
$\tilde{W}^d(p^*_c) = -\frac{\Omega}{2}([\pi_h^{i,u}]^2 + \lambda_x[x_h^{i,u}]^2) + \beta\sum_{u'} p(u'|u)W^{(i-1)}(u', x_h^{i,u})$,
$x_h^{i,u} = -\frac{\xi}{\lambda_x}\pi_h^{i,u} + z^{i,u}x_{h,-1}$,
$\pi_h^{i,u} = \xi x_h^{i,u} + \beta\sum_{p^{*\prime}_c} p(p^{*\prime}_c|p^*_c)\pi_h^{(i-1)} + \omega p^*_c$,
for the values of $(x_h^{i,u}, \pi_h^{i,u}, z^{i,u})$.

Then, as $x_h^{i,u}$ may not be on the grid point, these functions are approximated using a spline

Figure A.1: Varying the discount factor: $\beta = \{0.9963, 0.6\}$ (in %)



Note. The impulse–response function (IRF) plots display a 10% positive commodity price shock. The shock is equivalent to a one standard deviation percentage from its respective steady state. Unless it is exclusively specified, horizontal and vertical axes indicate quarters and percentage deviation from the steady state, respectively. The solid, dashed and dash-dotted lines are variable responses under the baseline calibration values (or baseline scenario). While the circle, cross and plus signs are variable responses under the alternative calibration values (or alternative scenario).

interpolation for those points of X, while an outerpolation, for those outside of X. For conditional expectations, cubic splines are used. So that, $h_{w,i}(x_h) = \sum_{p^*_{c,j}} p(p^*_{c,j}|p^*_{c,i})W(p^*_{c,j}, x_h)$ and $h_{\pi_h,i}(x_h) = \sum_{p^*_{c,j}} p(p^*_{c,j}|p^*_{c,i})\pi_h(p^*_{c,j}, x_h)$, for each point of the grid $(i = 1, ..., n_{p^*_c})$.

Error approximation

Error computations are effectuated using the residual function specification,

$$R(\tilde{u}) = -W(\tilde{u}) - ([\pi_h(\tilde{u})]^2 + \lambda_x[x_h(\tilde{u})]^2) + \beta h_{w,i}(x_h(\tilde{u})),$$

where $\tilde{u} \in \tilde{X} \times P$ is a grid with a larger number of points, in comparison to the ones used for result computations.

As in Sunakawa (2015), the total number of grids are 201. They are used to compare $\tilde{X}$ against X, which is obtained using 15 grids. To evaluate $W(\tilde{u})$, $\pi_h(\tilde{u})$ and $x_h(\tilde{u})$, a linear interpolation is used. Moreover, absolute and relative errors are also computed as $e^{abs} = \|R(\tilde{u})\|_\infty$ and $e^{abs} = \|R(\tilde{u})/W(\tilde{u})\|_\infty$. Note that $R(u) = 0$ holds for $u \in X \times P$. Details for the calculated errors are displayed in Table A.3.

A.3 Results: additional details

Table A.1: Simulated standard deviations (in %)

Order	Model scenario	$\pi_{h,t}^{\{s,c\}}$	$\pi_{h,t}^{d}$	$x_{h,t}^{\{s,c\}}$	$x_{h,t}^{d}$	$\Delta e_t^{\{s,c\}}$	Δe_t^{d}	$\pi_t^{\{s,c\}}$	π_t^{d}	$i_t^{\{s,c\}}$	i_t^{d}	$r_t^{\{s,c\}}$	r_t^{d}	$s_t^{\{s,c\}}$	s_t^{d}	$p_{c,t}^{*}$
1	Baseline ($\chi = 0.5$)	***0.22	0.63	*0.91	0.92	2.34	2.31	6.25	6.14	5.39	5.48	2.35	2.35	5.19	5.26	23.46
2	$\chi = \{0.5, 0.9\}$	***0.04	0.13	*0.18	0.18	2.97	2.96	7.92	7.90	6.60	6.62	2.98	2.98	6.57	6.58	23.47
3	$\chi = \{0.5, 1.5\}$	***0.22	0.63	*0.91	0.92	3.91	3.95	10.44	10.59	8.48	8.41	3.92	3.92	8.63	8.56	23.45
4	$\chi = \{0.5, 2\}$	***0.43	1.27	*1.81	1.84	4.70	4.78	12.55	12.87	10.04	9.94	4.70	4.70	10.32	10.18	23.39
5	$s_{y_c} = \{0.2, 0.3\}$	***0.22	0.63	*0.91	0.92	2.34	2.31	6.24	6.14	5.38	5.47	2.35	2.35	5.18	5.25	23.42
6	$s_m = \{0.15, 0.30\}$	***0.35	1.03	*1.45	1.47	3.72	3.67	9.91	9.74	8.63	8.78	3.73	3.73	8.24	8.33	23.44
7	$1 - \theta = \{0.25, 0.40\}$	***0.26	0.66	0.94	0.95	2.34	2.31	6.24	6.13	5.41	5.49	2.35	2.35	5.19	5.26	23.44
8	$DM = \{1.2, 1.3\}$	***0.28	0.86	**0.89	0.90	2.34	2.30	6.24	6.11	5.38	5.49	2.35	2.35	5.19	5.28	23.42
9	$\rho_{p_c^*} = \{0.9, 0.0\}$	***0.19	0.19	***0.23	0.27	3.10	3.09	8.24	8.21	2.64	2.76	3.13	3.13	2.22	2.22	10.06
10	$\sigma_{p_c^*} = \{0.1, 0.5\}$	***1.08	3.17	*4.54	4.61	11.71	11.56	31.22	30.69	26.94	27.39	11.75	11.74	25.95	26.31	117.27
11	$\beta = \{0.9963, 0.9\}$	***0.23	0.62	***0.92	0.90	2.34	2.31	6.25	6.14	5.39	5.46	2.35	2.35	5.18	5.25	23.42
12	$\phi = \{3, 1\}$	***0.38	1.25	***1.75	1.76	2.56	2.52	6.84	6.68	6.08	6.29	2.57	2.57	5.68	5.82	23.40
13	$\beta = \{0.9963, 0.6\}$	***0.21	0.60	***0.93	0.87	2.34	2.31	6.23	6.14	5.40	5.45	2.35	2.35	5.18	5.24	23.39

Note. Simulation results for each policy rule are obtained using the same pseudo-random numbers to perform 2000 replications for 1100 initial periods (10% of the initial periods are discarded to avoid initial value effects). The probability of binding the sustainability constraint is nil for each one of the first twelve scenarios, $Pr(z < 1) = 0.00$; although positive and equal to 79.6% for the last scenario. Superscript s = optimal quasi-sustainable policy; Superscript c = optimal commitment policy; Superscript d = optimal discretionary policy. β = subjective discount factor; χ = elasticity borrowing limit to commodity price (financial channel parameter); DM = desired markup; $\rho_{p_c^*}$ = auto-correlation parameter of the foreign commodity price shock; $\sigma_{p_c^*}$ = standard deviation parameter of the foreign commodity price shock; s_m = share of inputs used for commodity production goods; $1 - \theta$ = Calvo price re-set probability; s_{y_c} = Share of the comm. producing sector to GDP; ϕ = Inverse Frisch elasticity rate. Statistical significance for a Two-sample F-test of equal variances denoted as: * $p < 0.1$,** $p < 0.05$,*** $p < 0.01$.

Table A.2: Simulated correlation matrices (in %)

Variables	$\pi_{h,t}^{\{s,c\}}$	$x_{h,t}^{\{s,c\}}$	$\pi_{h,t}^{d}$	$x_{h,t}^{d}$	$p_{c,t}^{*}$
$\chi = 0.5$, $\omega = -0.01$ (baseline)					
$\pi_{h,t}^{\{s,c\}}$	100.00	-18.48	31.40	-31.40	-30.83
$x_{h,t}^{\{s,c\}}$	-18.48	100.00	-99.11	99.11	87.47
$\pi_{h,t}^{d}$	31.40	-99.11	100.00	-100.00	-88.67
$x_{h,t}^{d}$	-31.40	99.11	-100.00	100.00	88.67
$p_{c,t}^{*}$	-30.83	87.47	-88.67	88.67	100.00
$\chi = 0.9$, $\omega = -0.003$					
$\pi_{h,t}^{\{s,c\}}$	100.00	-17.16	29.27	-29.27	-27.00
$x_{h,t}^{\{s,c\}}$	-17.16	100.00	-99.23	99.23	89.20
$\pi_{h,t}^{d}$	29.27	-99.23	100.00	-100.00	-89.98
$x_{h,t}^{d}$	-29.27	99.23	-100.00	100.00	89.98
$p_{c,t}^{*}$	-27.00	89.20	-89.98	89.98	100.00
$\chi = 1.5$, $\omega = 0.01$					
$\pi_{h,t}^{\{s,c\}}$	100.00	-17.56	30.37	-30.37	26.24
$x_{h,t}^{\{s,c\}}$	-17.56	100.00	-99.13	99.13	-87.80
$\pi_{h,t}^{d}$	30.37	-99.13	100.00	-100.00	88.48
$x_{h,t}^{d}$	-30.37	99.13	-100.00	100.00	-88.48
$p_{c,t}^{*}$	26.24	-87.80	88.48	-88.48	100.00
$\chi = 2$, $\omega = 0.0273$					
$\pi_{h,t}^{\{s,c\}}$	100.00	-17.12	29.06	-29.06	27.23
$x_{h,t}^{\{s,c\}}$	-17.12	100.00	-99.34	99.34	-89.39
$\pi_{h,t}^{d}$	29.06	-99.25	100.00	-100.00	90.20
$x_{h,t}^{d}$	-29.06	99.25	-100.00	100.00	-90.20
$p_{c,t}^{*}$	27.23	-89.39	90.20	-90.20	100.00

Note. Simulation results for each policy rule are obtained using the same pseudo-random numbers to perform 2000 replications for 1100 initial periods (10% of the initial periods are discarded to avoid initial value effects). Superscript s = optimal quasi-sustainable policy; Superscript c = optimal commitment policy; Superscript d = optimal discretionary policy.

Table A.3: Errors summary for the optimal quasi-sustainable monetary policy (in %)

Order	Model scenario	Absolute			Relative		
		$W(\tilde{u})$	$\pi_{h,t}(\tilde{u})$	$x_{h,t}(\tilde{u})$	$W(\tilde{u})$	$\pi_{h,t}(\tilde{u})$	$x_{h,t}(\tilde{u})$
1	Baseline ($\chi = 0.5$)	9.94E-04	7.59E-16	2.78E-15	4.21E-02	7.55E-16	2.71E-15
2	$\chi = \{0.5, 0.9\}$	8.19E-04	1.39E-16	3.52E-16	5.62E+00	1.39E-16	3.50E-16
3	$\chi = \{0.5, 1.5\}$	9.94E-04	7.37E-16	2.08E-15	4.21E-02	7.33E-16	2.04E-15
4	$\chi = \{0.5, 2\}$	9.93E-04	1.47E-15	4.16E-15	9.68E-03	1.46E-15	3.99E-15
5	$s_{y_c} = \{0.2, 0.3\}$	9.94E-04	7.59E-16	2.78E-15	4.21E-02	7.55E-16	2.71E-15
6	$s_m = \{0.15, 0.30\}$	9.93E-04	1.21E-15	3.47E-15	1.95E-02	1.20E-15	3.35E-15
7	$1 - \theta = \{0.25, 0.40\}$	9.96E-04	1.83E-15	2.78E-15	4.00E-02	1.82E-15	2.69E-15
8	$DM = \{1.2, 1.3\}$	9.93E-04	6.94E-16	2.08E-15	4.30E-02	6.89E-16	2.08E-15
9	$\rho_{p_c^*} = \{0.9, 0.0\}$	9.82E-04	1.88E-11	6.94E-16	1.25E+01	1.88E-11	6.88E-16
10	$\sigma_{p_c^*} = \{0.1, 0.5\}$	1.02E-02	3.90E-15	1.11E-14	1.25E-02	3.76E-15	1.02E-14
11	$\beta = \{0.9963, 0.9\}$	2.62E-04	2.06E-05	3.07E-02	3.41E-01	2.06E-05	3.05E-02
12	$\phi = \{3, 1\}$	9.95E-04	7.81E-16	4.16E-15	2.04E-02	7.68E-16	4.04E-15
13	$\beta = \{0.9963, 0.6\}$	2.71E-04	6.41E-04	8.20E-02	2.68E+00	6.38E-04	8.18E-02

Note. $\tilde{u}$ = state space defined as of the simulated commodity price shock ($\tilde{P_c^*}$) and the lagged output gap ($\tilde{X}$), $\tilde{u} \in \tilde{X} \times P_c^*$. β = subjective discount factor; χ = elasticity borrowing limit to commodity price (financial channel parameter); DM = desired markup; $\rho_{p_c^*}$ = auto-correlation parameter of the foreign commodity price shock; $\sigma_{p_c^*}$ = standard deviation parameter of the foreign commodity price shock; s_m = share of inputs used for commodity production goods; $1 - \theta$ = Calvo price re-set probability; s_{y_c} = Share of the comm. producing sector to GDP; ϕ = Inverse Frisch elasticity rate.

B Supplementary material

B.1 Other predictions of the DMT model

B.1.1 Varying the financial channel: elastic elasticities

The financial channel, represented by the parameter χ in the NKPC (4), influences the model's predictions.

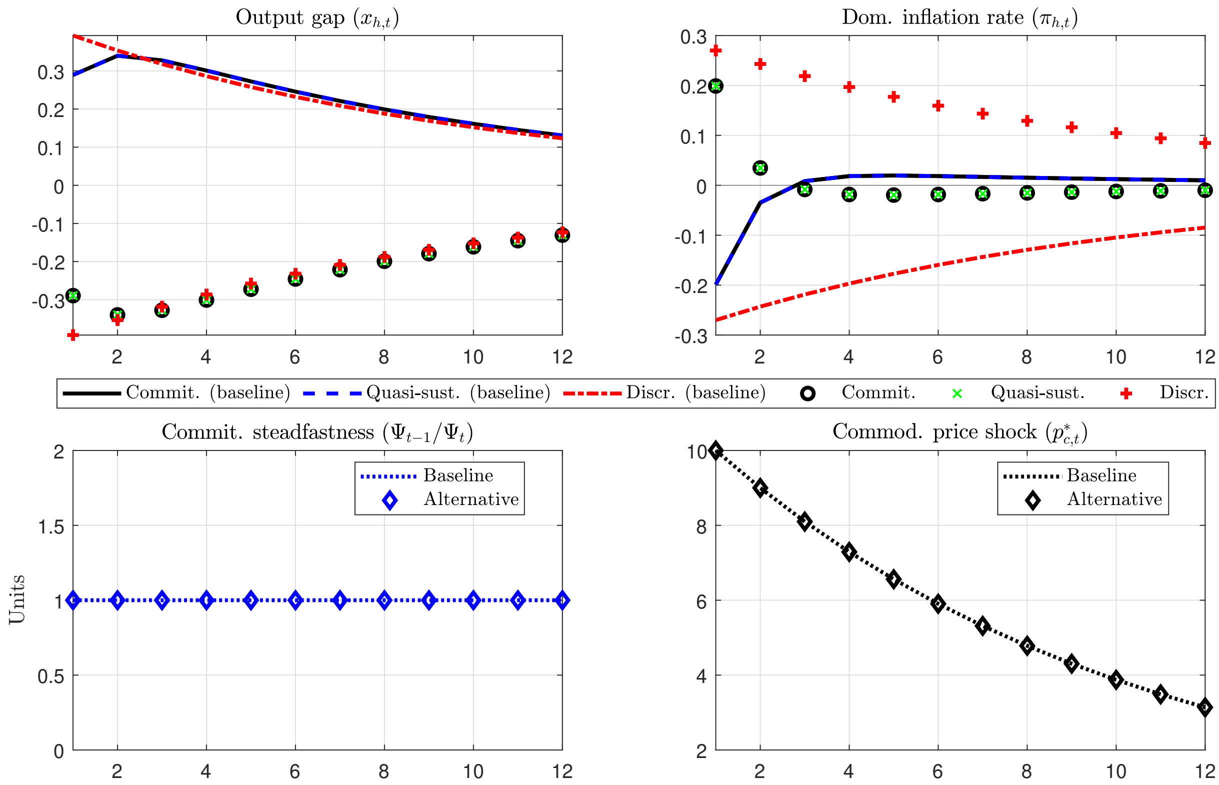
Assuming an elasticity of $\chi = 1.5 > 1$, which reflects favorable borrowing conditions for commodity producers relative to price fluctuations, the NKPC parameter ω (3) becomes positive ($\omega = 0.01 > 0$). This scenario leads to a symmetric variation in the domestic inflation rate and output gap when a 10% CPS and optimal policies are applied (Figure B.2). These results suggest that, under this specific model configuration, a CPS can induce inflationary pressures. This behavior aligns with standard cost-push or markup shocks documented in the literature. As a result, the model predicts a trade-off for policymakers, with rising inflation accompanied by a declining output gap.

In contrast, when borrowing conditions are unfavorable for commodity producers ($\chi = 0.5 < 1$, $\omega = -0.01 < 0$), a CPS triggers deflationary pressures and an economic boom, as shown in the model estimates by Bergholt et al., 2019.[13]

Furthermore, increasing the elasticity of borrowing conditions for commodity producers (e.g., $\chi = \{0.5, 2\}$) leads to higher volatilities in domestic inflation and the output gap

[13]This aligns with the predictions of the model by Báez (2022), even when the financial channel parameter in his model is positive but remains below unity. In essence, a CPS is associated with a positive output gap and a negative correlation with domestic inflation (Table A.2).

Figure B.2: Varying the financial channel: $\chi = \{0.5, 1.5\}$, $\omega = \{-0.01, 0.01\}$ (in %)



Note. The impulse–response function (IRF) plots display a 10% positive commodity price shock. The shock is equivalent to a one standard deviation percentage from its respective steady state. Unless it is exclusively specified, horizontal and vertical axes indicate quarters and percentage deviation from the steady state, respectively. The solid, dashed and dash-dotted lines are variable responses under the baseline calibration values (or baseline scenario). While the circle, cross and plus signs are variable responses under the alternative calibration values (or alternative scenario).

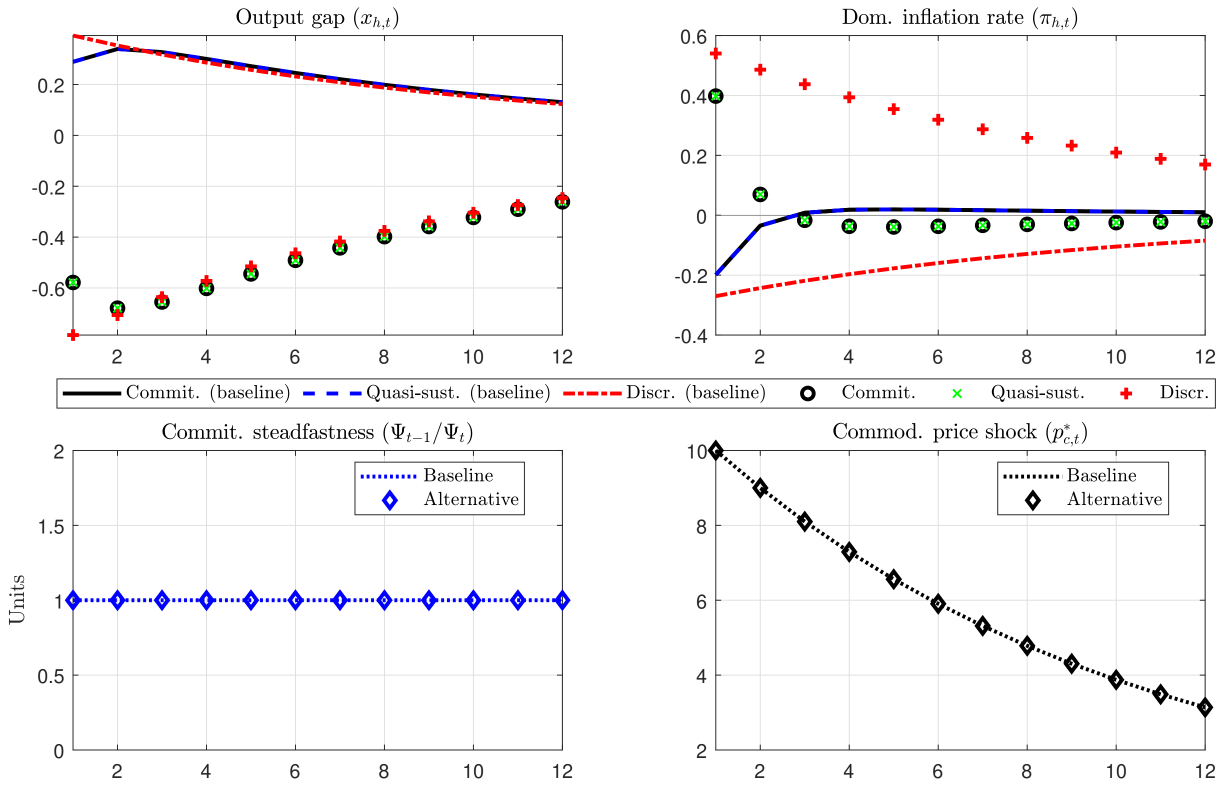
(Figure B.3). The same trend holds for real and nominal exchange rate volatilities. When the parameter χ is symmetrical ($\chi = \{0.5, 1.5\}$), the volatilities of macroeconomic variables become equal (Table A.1).

B.1.2 Varying the commodity sector share to GDP

The results from the DMT model predict that variations in the commodity sector's share relative to the economy's gross domestic product (GDP) are not statistically significant. This finding contrasts with the key monetary policy implications identified when examining variations in the economy's commodity inputs share (as detailed in subsubsection 3.2.2).

Specifically, varying the commodity sector share relative to GDP (s_m) between 20% and 30% does not alter the economy's dynamics or its steady state Figure B.4). As the figure demonstrates, the crucial factor within the DMT model framework is the share of commodity inputs produced domestically (s_m).

Figure B.3: Varying the financial channel: $\chi = \{0.5, 2\}$, $\omega = \{-0.01, 0.03\}$ (in %)



Note. The impulse–response function (IRF) plots display a 10% positive commodity price shock. The shock is equivalent to a one standard deviation percentage from its respective steady state. Unless it is exclusively specified, horizontal and vertical axes indicate quarters and percentage deviation from the steady state, respectively. The solid, dashed and dash-dotted lines are variable responses under the baseline calibration values (or baseline scenario). While the circle, cross and plus signs are variable responses under the alternative calibration values (or alternative scenario).

B.1.3 Varying other parameters of DMT's model

The following simulations show variations of some selected parameters of the DMT model.

The simulations consistently show that the sustainability constraint remains inactive under the alternative calibrations. Consequently, the optimal commitment policy aligns with the optimal quasi-sustainable policy, leaving the optimal discretionary policy as the least welfare-favorable outcome. Additionally, all results exhibit symmetry; the opposite scenario holds true for each simulation presented, delivering the corresponding opposite outcome.

Varying the price rigidity

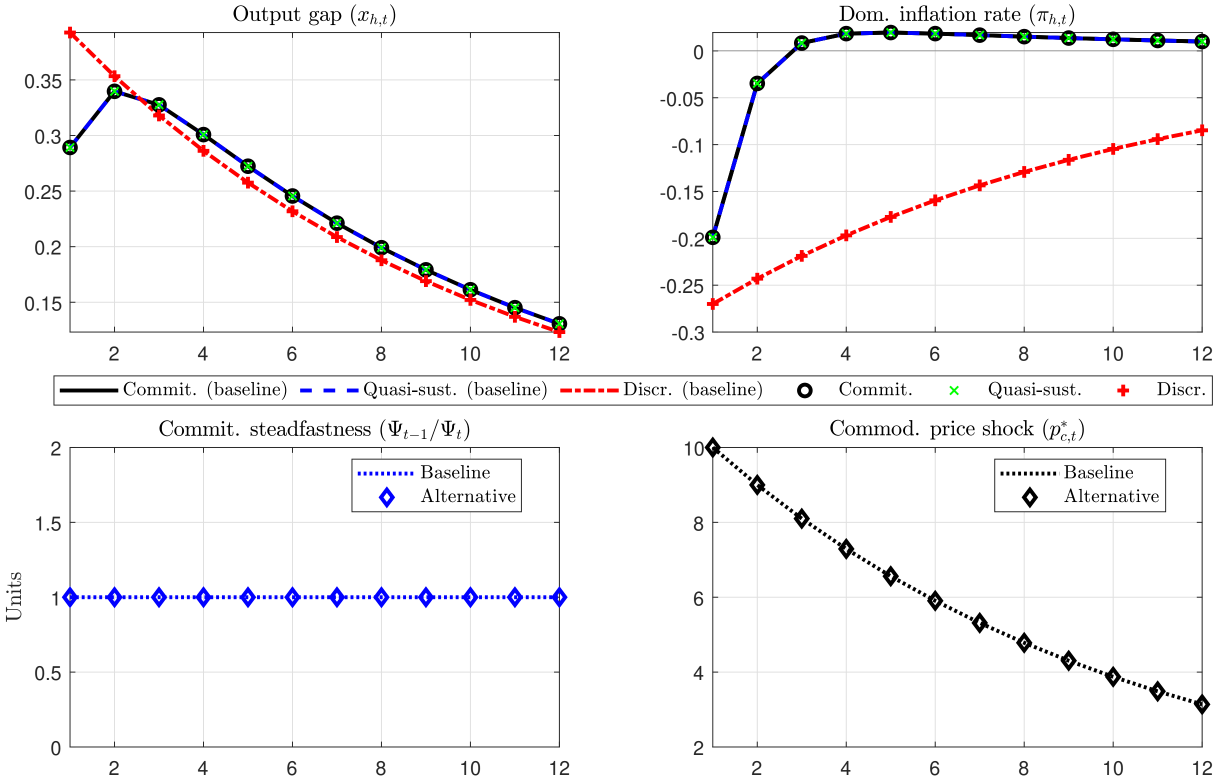
Greater price flexibility in the economy leads to larger responses in the domestic inflation rate and the output gap to the CPS (Figure B.5).

The Calvo parameter $1 - \theta$ varies from 25% (baseline calibration) to 40% (alternative calibration) representing the probability of firms adjusting prices per quarter.

Varying the desired markup price

A higher desired markup price translates to stronger effects on domestic inflation but weaker effects on the output gap (Figure B.6).

Figure B.4: Higher commodity sector share to GDP: $s_{y_c} = \{0.2, 0.3\}$ (in %)



Note. The impulse–response function (IRF) plots display a 10% positive commodity price shock. The shock is equivalent to a one standard deviation percentage from its respective steady state. Unless it is exclusively specified, horizontal and vertical axes indicate quarters and percentage deviation from the steady state, respectively. The solid, dashed and dash-dotted lines are variable responses under the baseline calibration values (or baseline scenario). While the circle, cross and plus signs are variable responses under the alternative calibration values (or alternative scenario).

The desired markup price parameter is assumed to vary from 20% (baseline calibration) to 30% (alternative calibration).

Amplifying the shock

The optimal quasi-sustainable policy always coincides with the optimal sustainable policy, regardless of the CPS size. Consequently, a larger shock translates to larger responses in the output gap and the domestic inflation rate (Figure B.7).

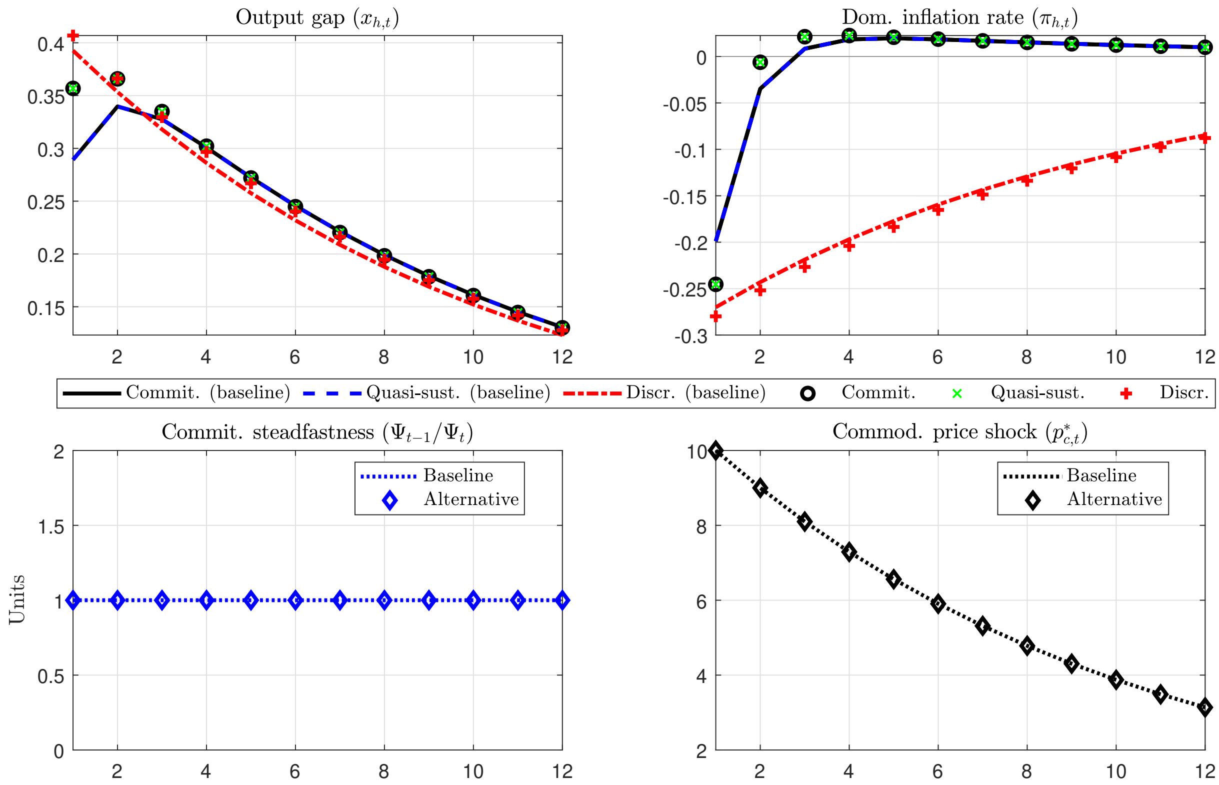
It is noteworthy that even with a significant variation in the shock size (a fivefold increase, from 10% to 50%, in the parameter $\sigma_{p^*_{c,t}}$), the sustainability constraint remains inactive. This implies full commitment steadfastness.

Varying the inverse Frisch elasticity rate

A lower elasticity to wages leads to a stronger response in domestic output but a weaker response in domestic inflation (Figure B.8).

The simulation assumes the inverse Frisch elasticity rate (ϕ) drops from 3 to 1. As a result, the lower sensitivity of working hours to real wages under the alternative scenario leads to an increase in the amount of work during economic booms (as measured by the domestic output gap). The domestic inflation rate, however, exhibits a more significant decline because marginal costs are lower under the alternative scenario.

Figure B.5: Lower price rigidity: $1 - \theta = \{0.25, 0.40\}$ (in %)



Note. The impulse–response function (IRF) plots display a 10% positive commodity price shock. The shock is equivalent to a one standard deviation percentage from its respective steady state. Unless it is exclusively specified, horizontal and vertical axes indicate quarters and percentage deviation from the steady state, respectively. The solid, dashed and dash-dotted lines are variable responses under the baseline calibration values (or baseline scenario). While the circle, cross and plus signs are variable responses under the alternative calibration values (or alternative scenario).

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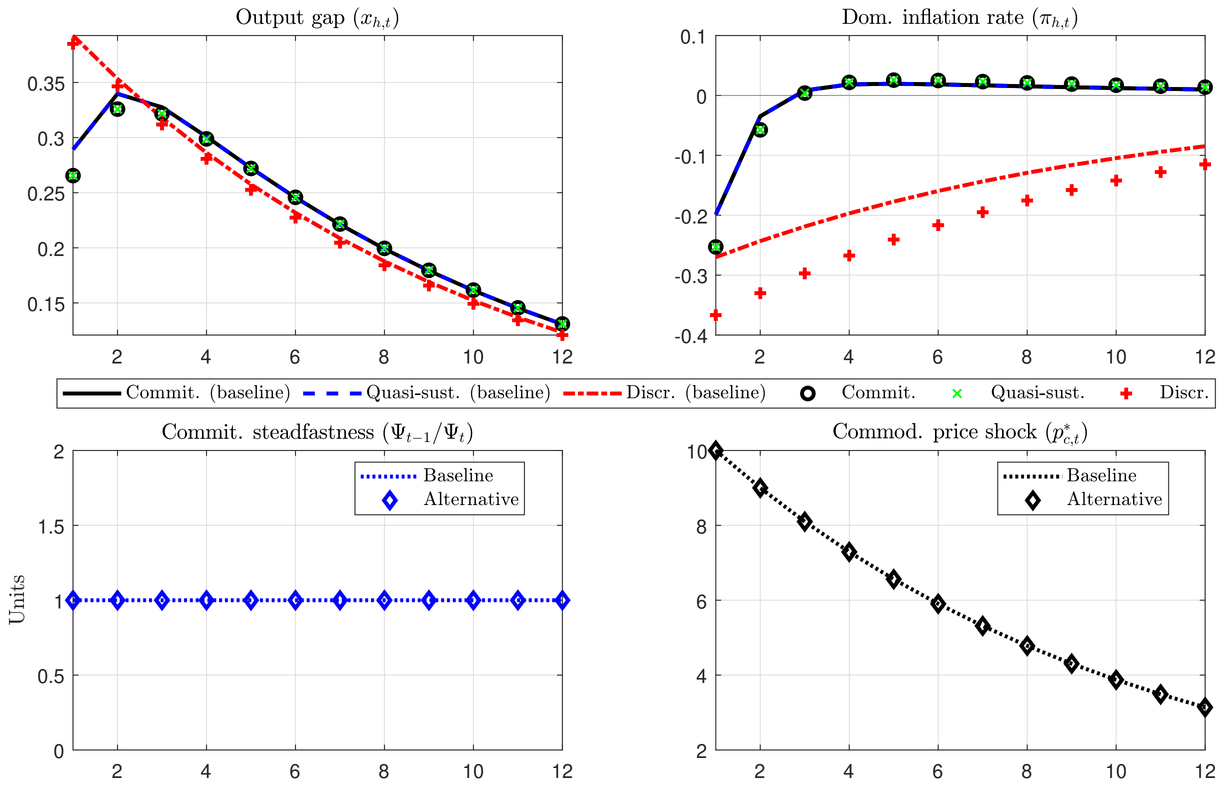
Figure B.6: Varying the desired markup price: $DM = \{1.2, 1.3\}$ (in %)



Note. The impulse–response function (IRF) plots display a 10% positive commodity price shock. The shock is equivalent to a one standard deviation percentage from its respective steady state. Unless it is exclusively specified, horizontal and vertical axes indicate quarters and percentage deviation from the steady state, respectively. The solid, dashed and dash-dotted lines are variable responses under the baseline calibration values (or baseline scenario). While the circle, cross and plus signs are variable responses under the alternative calibration values (or alternative scenario).

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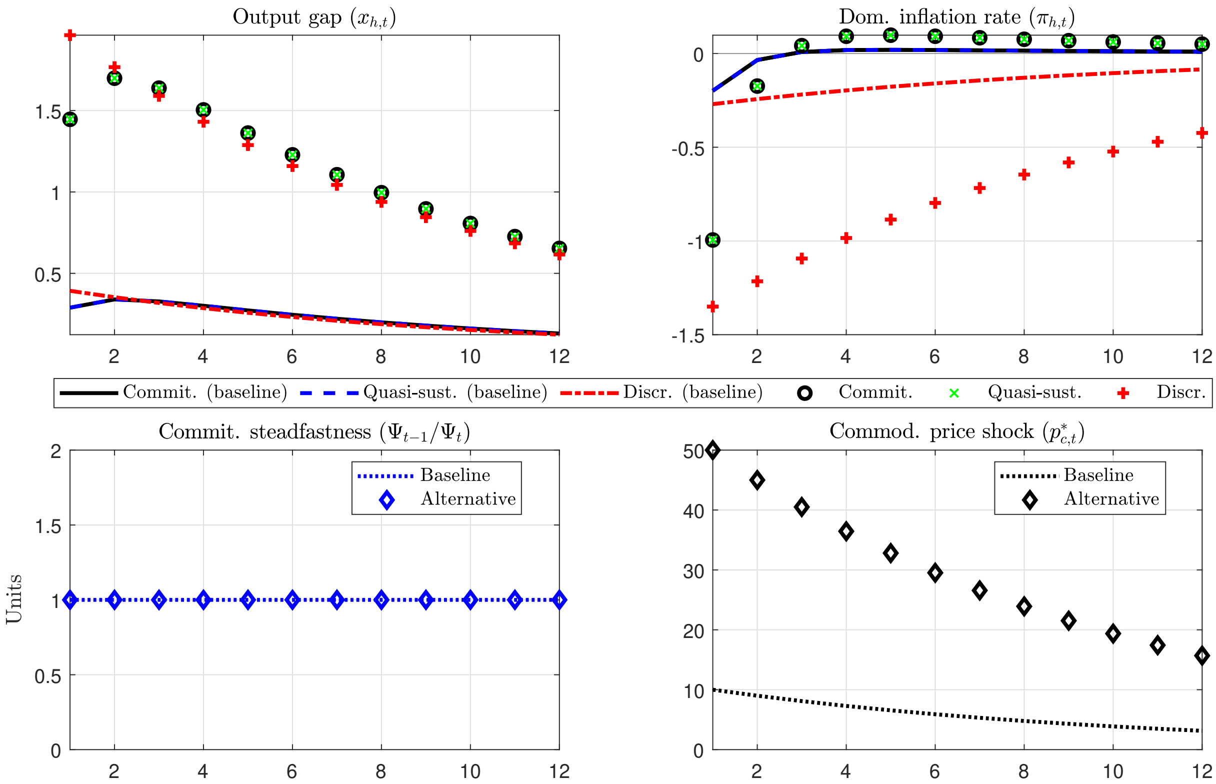
Figure B.7: Amplifying the shock: $\sigma_{p^*_{c,t}} = \{0.1, 0.5\}$ (in %)



Note. The impulse–response function (IRF) plots display a 10% positive commodity price shock. The shock is equivalent to a one standard deviation percentage from its respective steady state. Unless it is exclusively specified, horizontal and vertical axes indicate quarters and percentage deviation from the steady state, respectively. The solid, dashed and dash-dotted lines are variable responses under the baseline calibration values (or baseline scenario). While the circle, cross and plus signs are variable responses under the alternative calibration values (or alternative scenario).

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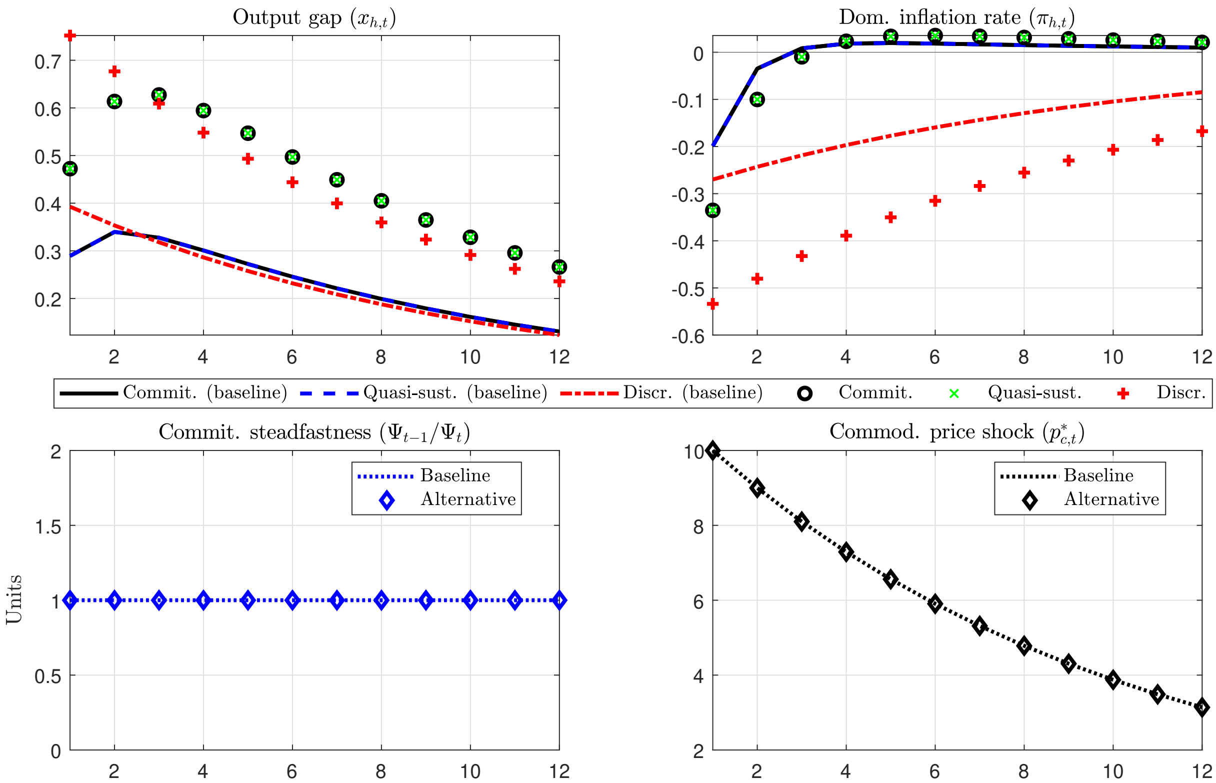
Figure B.8: Varying the inverse Frisch elasticity rate: $\phi = \{3, 1\}$ (in %)



Note. The impulse–response function (IRF) plots display a 10% positive commodity price shock. The shock is equivalent to a one standard deviation percentage from its respective steady state. Unless it is exclusively specified, horizontal and vertical axes indicate quarters and percentage deviation from the steady state, respectively. The solid, dashed and dash-dotted lines are variable responses under the baseline calibration values (or baseline scenario). While the circle, cross and plus signs are variable responses under the alternative calibration values (or alternative scenario).

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