CBDC: Banking and Anonymity *

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Abstract

We examine the optimal user anonymity in Central Bank Digital Currency (CBDC) in the context of bank lending. Anonymity, defined as the lender's inability to discern entrepreneurs' actions that enable fund diversion, influences payment instrument choice due to its impact on bank lending decisions. We show that moderate anonymity in CBDC leads to an inefficient pooling equilibrium. To avoid this, CBDC anonymity should be either low, reducing attractiveness, or high, discouraging bank lending. Specifically, the anonymity should be high when CBDC significantly benefits sales, and low otherwise. However, competition between deposits and CBDC may hinder the implementation of low anonymity.

Key words: CBDC; Anonymity; Bank lending
JEL Codes: E42, E58, G28

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1 Introduction

The possibility of issuing a central bank digital currency (CBDC) hinges on the currency's design. Central banks have a wide range of choices in designing public money in the form of a CBDC. This flexibility has sparked active discussions that have evaluated the benefits and costs from various perspectives. A major policy concern is user privacy: Governor Bowman of the Federal Reserve Board emphasized in her speech that "safeguarding privacy is a top concern ... we need to think about how to protect the privacy of consumers and businesses" (Bowman, 2023). While much of the existing literature focuses on consumer privacy, the privacy of business, which we call "anonymity," has been less thoroughly explored. Entrepreneurs might indeed favor anonymity, but this preference may introduce challenges to the banking sector. This lack of information emerges as a key friction in bank lending, and historical records of transactions and cash flows can provide valuable data for banks when screening entrepreneurs. To address such a conflict of desires, this paper asks: To what extent should user anonymity be incorporated in the design of a CBDC?

Bank lending decisions indeed utilize the transaction histories of borrowing entrepreneurs. For example, the fintech innovator Square relies on such data in screening firms to make loans. Square was originally founded to provide point-of-sale (PoS) terminal devices. However, as its business model evolved, Square expanded into lending under the name Square Capital and has since re-branded itself as Square Loans, which has facilitated more than $9 billion in small business loans by 2021. Square makes refinancing decisions based on sales data reported by its PoS devices. Consider, for example, a small business entrepreneur, like a convenience store owner, borrowing from Square. They could accept cash from their customers, which offers more anonymity, but such transactions would not be recorded on the PoS machine, potentially impacting their refinancing prospects. With the issuance of a CBDC, the possibility arises for central banks to collaborate with entities like Square to develop terminal devices, forming *public-private partnerships*.[1] However, it is not trivial whether these entities should be allowed to utilize transaction information for their lending business. Our question is thus narrowed down to: to what extent should lending banks be allowed to monitor CBDC transactions of borrowers?

To answer this question, motivated by the example of Square, we build a signaling model wherein an entrepreneur in need of rollover financing chooses a payment instrument in selling their product.[2] Each instrument has a different degree of anonymity, potentially serving as a signaling tool to the lending bank. In our framework, "anonymity" represents the lender's inability to discern the entrepreneur's actions, thereby allowing the entrepreneur to divert more funds. Thus, depending on what payment methods an entrepreneur uses, the lending bank has a different degree of *control* over funds generated by the entrepreneur. We first consider two payment methods: a debit card (akin to a bank deposit), which offers limited anonymity; and cash, which provides greater anonymity.[3] There are also two types of entrepreneurs, distinguished by the productivity of their projects. Entrepreneurs, upon privately learning their types after borrowing from the bank, choose what payment method they accept. Based on the chosen method, the bank makes inferences about the quality of the project in deciding whether to refinance the loan in an intermediate stage. Because higher anonymity reduces the bank's payoff, entrepreneurs with high-quality projects may

[1] See, for example, U.S. Department of the Treasury (2022) and Balz (2022).

[2] Our model is presented in the context of firm borrowing. However, the model can be relabeled in the consumer-finance context and used to study consumers' privacy and payment choices. For instance, consumers may borrow from the bank for current consumption and repay from future income. These consumers, having different productivity levels, may choose various methods to receive their income, such as cash or deposit. In fact, Prescott and Tatar (1999) documented that 15% of the US households were unbanked and suggested that one potential reason is the desire for privacy from their lenders.

[3] The idea that banks can better monitor borrowers through their bank accounts is not new. See, for example, Black (1975), Fama (1985), and Mester, Nakamura, and Renault (2006).

strategically choose less anonymous instruments to induce bank lending. On the other hand, entrepreneurs with low-quality projects may choose more anonymous instruments to divert profits as much as they can. We structure the benchmark model in a way that there exists a unique, efficient separating equilibrium: high-type entrepreneurs choose the debit card, while low-type entrepreneurs choose cash. In this equilibrium, the bank refinances exclusively the high-type entrepreneur. This model is our lens for studying a CBDC, and we study equilibrium credit allocation by adding a CBDC into this framework, where its degree of anonymity is a policy choice.

When a CBDC is introduced as the third payment method, a pooling equilibrium may arise at a modest level of anonymity, where both types choose a CBDC and the bank continues with both. We assume low-quality projects have negative net present values (NPV); thus, this pooling equilibrium is inefficient and features credit misallocation. This equilibrium exists when the low-type entrepreneur can borrow and still divert a substantial fraction of profits. One way to eliminate this inefficient equilibrium is to reduce the anonymity of the CBDC, making it akin to a deposit, so that the CBDC becomes less attractive to the low type, leading the low-type entrepreneur to switch to cash. Alternatively, increasing the CBDC's anonymity to resemble cash could also be effective, as high anonymity would lead the bank to refrain from lending to any entrepreneurs using the CBDC, thus discouraging low-type entrepreneurs from choosing it. Therefore, to prevent credit misallocation, the anonymity of the CBDC should be sufficiently low or sufficiently high.

While a pooling equilibrium may arise, there also exists a unique separating equilibrium for each degree of CBDC anonymity. Although the equilibrium strategies that constitute a separating equilibrium change depending on the degree of anonymity, every equilibrium yields an outcome as least as *good* as the benchmark case. We evaluate the outcomes by the total net output of the economy, and hence, any separating equilibria where only high-type entrepreneurs can get refinanced are efficient. In such a case, the optimal degree of CBDC anonymity is either sufficiently high or sufficiently low.

We then study how (exogenous) benefits of a CBDC for sales change the equilibrium outcomes. Such benefits change the attractiveness of a CBDC for merchants. For example, a CBDC may incur lower payment fees compared to debit and cash, thereby enhancing the entrepreneur's profit in every sale.[4] We find that, when CBDC has small benefits for sales, it should be significantly less anonymous, resembling a deposit. In such a case, the low-type entrepreneur chooses cash while the high-type entrepreneur chooses CBDC. Since the high type operates over a longer term in equilibrium, they benefit more from choosing CBDC, and hence their use of CBDC increases their total net output. However, when a CBDC has substantially larger benefits for sales, it becomes too attractive, and the low type also chooses CBDC, even though it offers less anonymity than cash. To avoid such an inefficient pooling equilibrium, CBDC should be designed as highly anonymous, so that, anticipating that the bank will not refinance any types, the low type refrains from mimicking the high type. The resulting outcome features the high type choosing debit while the low type chooses cash. Since the low type operates for a shorter period, their use of CBDC is less desirable than for the high type.

These equilibria, however, may not be desirable for the bank because CBDC may allow entrepreneurs to divert more than the benchmark. We finally consider banks' strategic responses to the introduction of CBDC as competition for debit. Specifically, we allow the bank to change the degree of anonymity in debit. For example, banks might intensify their

[4] Verdier (2023) and Liu, Reshidi, and Rivadeneyra (2023) discuss how a CBDC may lower payment fees. However, our focus is on how the benefits of a CBDC change the equilibrium outcomes, and not on the form of the benefits. In fact, there are some other ways to interpret those benefits, which do not undermine our results or implications. See our discussions in Section 4.3.1.

loan scrutiny, tighten credit standards, or even re-evaluate their risk assessment models to decrease debit anonymity. Such proactive measures change the payoffs in the game, potentially preventing outcomes that are not in the bank's favor. For example, the bank may prefer that high-type entrepreneurs use debit instead of CBDC so that the high type diverts less. We find that, if the bank can strategically change debit anonymity before a CBDC is introduced, the bank can hinder the implementation of the most efficient outcome where the high type uses CBDC. In such a case, while low CBDC anonymity is the most desirable in the absence of the bank's strategic response, the central bank now has to choose high CBDC anonymity. On the other hand, if the central bank acts before the banks, it can consistently implement the equilibrium it desires. Our analysis suggests that whether or not, and when, the bank responds to the introduction of CBDC affect the optimal design of CBDC.

Related literature

Our paper adds to the growing literature on the value of information in payments, which has largely focused on consumer privacy in digital transactions (see Norberg, Horne, and Horne (2007), Athey, Catalini, and Tucker (2017) for privacy paradox; Kahn, McAndrews, and Roberds (2005) and Garratt and van Oordt (2021) for privacy externality; Parlour, Rajan, and Zhu (2022) and He, Huang, and Zhou (2022) for fintech competition and information spillover; Agur, Ari, and Dell'Ariccia (2023) for household privacy and data monopoly; Xiao (2021) for risk-sharing; also see Acquisti, Taylor, and Wagman (2016) for a recent survey). In the context of firms, Ahnert, Hoffmann, and Monnet (2022) study anonymity, developing a model that demonstrates that while online platforms facilitate sales for the merchant, they also lead to a loss of privacy in the form of information rent. They show that a CBDC can improve welfare by maintaining both digital services and privacy. Similar to Ahnert et al. (2022), we also examine anonymity in the context of firms, but we model payment methods as a signaling tool to induce bank lending, characterizing the optimal design of CBDC anonymity.

Our study sheds new light on the role of bank lending as a key aspect in designing the anonymity of a CBDC. The literature on the value of information in payments has often defined anonymity in terms of users' ability to keep their information private. For instance, Agur, Ari, and Dell'Ariccia (2022) build a model in which introducing a more anonymous CBDC may crowd out cash, while a less anonymous CBDC may compete with deposits, thereby trading off the benefit of a variety of payment instruments against the size of financial intermediation. Garratt and van Oordt (2021) and Garratt and Lee (2022) examine an environment in which firms can acquire information from consumers' payments, allowing them to price discriminate. They discuss how introducing an anonymous CBDC can improve welfare by preventing firms from acquiring this information. Our paper is the first to consider the design of anonymity in the context of bank lending.

Our study contributes to the classic literature on signaling the quality of investments in credit markets by introducing a new dimension: payment methods. Previous research has suggested various methods of signaling quality, such as the amount of own funds invested (Ross (1977) and Leland and Pyle (1977)), the use of collateral (Bester (1985)), and the size of the loan (Milde and Riley (1988)). Our study expands on this literature by exploring the use of payment methods as signaling tools in bank lending, and we examine the strategic interactions between firms and banks as firms choose payment methods to signal the quality of their investments.

The remainder of the paper is organized as follows. Section 2 outlines the baseline environment. Section 3 analyzes equilibrium without a CBDC. Section 4 introduces the CBDC and examines the optimal design. Section 5 studies the strategic interplay between banks and the central bank. Section 6 concludes.

2 Model

In this section, we describe our model of bank lending and payment instruments. Payment instruments are used by an entrepreneur to receive investment returns, and we consider payment instruments tools for the entrepreneur to use, to signal profitability to the bank. We analyze equilibrium with and without the CBDC in the next two sections.

2.1 The environment

There are three periods, $t \in \{0, 1, 2\}$, and two risk-neutral agents: a monopolistic bank and an entrepreneur. The entrepreneur has access to a single, constant-returns-to-scale investment technology, called *project*, which requires one amount of funding in $t = 0$ and k amount of funding in $t = 1$. The bank has deep pockets, but the entrepreneur has no endowment. To invest in the project, the entrepreneur needs to borrow from the bank in periods 0 and 1.

The project can be of two types: a high type (h) or a low type (ℓ), with an ex-ante probability of $\theta \in (0, 1)$ that the project will be of the high type. The project yields returns in periods 1 and 2: both projects yield r in period 1, and in period 2, the high type yields R_h and the low type yields $R_\ell < R_h$. The entrepreneur will have either a high-type project only or a low-type project only ex-post.[5] The high type project has a positive net present

[5] In what follows, we refer to an entrepreneur with a low- or high-type project as L-type or H-type, respectively.

value (NPV) in period 1, whereas the low type has a negative NPV: $R_L < k < R_h$. We let M denote the expected return in period 2 such that $M \equiv \theta R_h + (1-\theta)R_l$.

After making an investment in period 0, the entrepreneur privately learns the project type and subsequently chooses a payment technology to receive future investment returns. We first consider that the entrepreneur has the option of two payment methods: *Debit* and *Cash*. The distinction between these methods is the degree of *anonymity*, which we define as the inability to monitor firms' actions, allowing for a greater proportion of fund diversion. Specifically, Debit, a product of the lending bank, allows the entrepreneur to divert only a fraction of the returns, denoted as $\lambda < 1$. In contrast, using Cash enables the entrepreneur to divert the entirety of the returns.[6] The bank collects the non-divertible portion of the returns as repayments in periods 1 and 2. This premise underscores the notion that Debit provides the bank with enhanced monitoring or *control* over the funds, making diversion by the entrepreneur more challenging. Conversely, with Cash, the bank's capacity to deter fund diversion diminishes significantly.[7]

The bank may or may not continue the investment in period 1. To continue the investment, the bank has to provide additional funding of $k > 0$ to the entrepreneur.[8] This additional funding can be interpreted as a liquidity shock to the project, such as expenditures for device maintenance or the entrepreneur's plan to expand their business into new markets or locations. If the bank does not lend it, the bank cannot recover any amount of

[6] It is also feasible to assume that with Cash, only a fraction of the revenue can be diverted. As long as Debit permits a greater diversion than Cash, our findings remain consistent.

[7] When Debit is used, the funds reside in the bank's vault, which may allow the bank to seize them or reject transfer requests. Mester et al. (2006) provide the empirical evidence that transaction histories in deposit accounts help banks monitor borrowers. Square Capital is a recent example of how the lender keeps some funds in the borrower's account for future repayments, which Udell (2004) calls "inside" collateral. It is worth noting that Cash can be interpreted as Debit provided by a different bank that is not lending to the entrepreneur.

[8] We assume that the entrepreneur cannot finance k using the period-1 proceeds from the investment. This restriction may be seen as a timing issue or due to the insufficiency of the proceeds.

the original investment. Note that, given an L-type's negative NPV, the bank will always discontinue the investment if the entrepreneur is believed to be a L-type.

Finally, the bank cannot pre-commit to a continuation decision. The loan contract between the bank and the entrepreneur is thus negotiable after the bank chooses a payment technology. Specifically, the entrepreneur makes a take-it-or-leave-it offer in period 1, and as a result, the bank just extracts as much as it can.

2.2 Timeline

The sequence of events is summarized in Figure 1.

$t = 0$	$t = 1$	$t = 2$ (if continued)
• Loan is made • Firm's type is revealed • Firm chooses a payment technology	• Repayments • Bank's continuation decision	• Repayments

Figure 1: Timeline of the events

In period 0, the entrepreneur borrows from the bank and invests in their project. After the investment is made, they learn the type of their project and then choose a payment technology. This choice becomes known to the bank immediately. In period 1, the project yields returns, and the entrepreneur makes a repayment to the bank, the amount of which depends on the chosen payment instrument. The bank then makes a decision about whether to continue the investment. The period ends after these decisions are made. In period 2, the project yields returns again, and the entrepreneur makes a repayment.

2.3 Discussion

We have assumed that the entrepreneur can divert a fraction λ of the returns by using Debit. An alternative way to specify λ is as the probability that diversion is successful. If diversion is successful, the entrepreneur can divert the entire amount. If diversion is unsuccessful, the entrepreneur obtains nothing. This alternative specification will yield equivalent results.

3 Equilibrium without CBDC

We first characterize equilibrium in the absence of CBDC. This analysis will serve as our lens to study how the introduction of CBDC will change equilibrium outcomes. Our focus is on how the entrepreneur's incentives in choosing a payment technology shape the outcome of bank lending. Specifically, we will study a signaling game in which the entrepreneur chooses a payment technology, and the bank updates its beliefs about the type based on the entrepreneur's choice. The entrepreneur's payoffs are determined by the bank's decisions regarding the continuation of the investment. We define an equilibrium as:

Definition 1. The equilibrium of the signaling game consists of the entrepreneur's choice of payment technology in maximizing their payoff and the bank's corresponding belief. This belief is updated using Bayes' rule whenever possible and is consistent with the entrepreneur's choice. The bank decides whether or not to lend in $t = 0, 1$ to maximize its payoff, given the belief.

For the off-equilibrium beliefs, we assume that the bank believes that the entrepreneur is a L-type upon observing any deviations from the path of the play. We let $\mu(h)$ denote the bank's belief that the entrepreneur is a H-type, where $h \in \{\ell, h\}$.[9]

[9] As will become clear, this approach yields some equilibria that are not robust. We will refine these

In proceeding with the analysis, we assume that the the L-type is substantially unproductive, while the H-type is substantially productive as follows.[10]

Assumption 1. $\lambda(r + R_l) < r < \lambda(r + R_h)$.

3.1 Equilibrium analysis

There are four possible combinations of choices, given that there are two payment technologies and two types of entrepreneurs. Assumption 1 ensures that an L-type will never choose Debit, even if doing so would allow refinancing in period 1. Consequently, we only need to consider two combinations of choices: (Debit, Cash) and (Cash, Cash), where the first element denotes the H-type's choice and the second indicates the L-type's choice.

We first show that (Cash, Cash) can constitute an equilibrium, but it does not survive the intuitive criterion. In this strategy profile, the bank will never continue with any entrepreneurs in period 1, as they would divert all returns in period 2. When the bank's belief is $\mu(h \mid Cash) = \theta$ and $\mu(h \mid Debit) = 0$, an H-type chooses Cash over Debit to divert a larger portion of r. Thus, both types rationally choose Debit with this belief. However, although (Cash, Cash) constitutes an equilibrium, it will not survive the Intuitive Criterion. Since only the H-type could potentially benefit from a deviation to Debit, the bank would infer that such a deviation must come from an H-type. If the bank continues with the player who deviates to Debit, the H-type becomes better off.

We now establish that the separating equilibrium (Debit, Cash) always exists with the belief $\mu(h \mid Debit) = 1$. The H-type chooses Debit to induce additional funding, although

equilibria using the intuitive criterion in the sense of Cho and Kreps (1987).

[10] This assumption is for exposition purposes and not pivotal to our main result. If we remove this assumption, (Debit, Debit) will be a unique equilibrium in the model without CBDC. In the following section, we aim for showing that (CBDC, CBDC) may constitute an equilibrium, but the existence of such an equilibrium itself does not depend on this assumption.

they have to give up a fraction $(1 - \lambda)$ of returns. The L-type chooses Cash to divert as much as possible, giving up the opportunity to continue the investment until period 2. The bank continues only with the H-type. This equilibrium is the only equilibrium that survive the intuitive criterion in this benchmark model:

Proposition 1. ***(Equilibrium).*** *There exists a unique equilibrium surviving the Intuitive Criterion in which the H-type chooses Debit and the L-type chooses Cash, and the bank continues with H-type. The H-type repays* $(1 - \lambda)r$ *in period 1 and* $(1 - \lambda)R_h$ *in period 2, while the L-type repays nothing.*

This equilibrium outcome is the same as it would be in the full information allocation, where the type is publicly known. In both cases, the bank desires to continue only with a H-type.

Anticipating this equilibrium outcome, the bank forms a loan contract in period 0. Due to the bank's lack of commitment, the bank cannot influence the entrepreneur's future action through lending terms, and hence any lending terms lead to the shown outcomes above.[11] However, our result can be interpreted as the entrepreneur promising to repay $(1 - \lambda)r$ in period 1 and $(1 - \lambda)R_h$ in period 2. The H-type will adhere to the repayment term, while the L-type will default on the loan contract. As the equilibrium outcome, the L-type indeed does not make any repayments and cannot be refinanced in period 1. In the analysis above, we assumed that the bank already lent to the entrepreneur in period 0. Our focus is on the ex-post signaling game, and the bank's $t = 0$ funding decision does not change our proposed mechanism. Having said that, it is straightforward to make a parametric assumption to guarantee the bank's $t = 0$ lending without undermining our mechanism. For example, the following assumption guarantees that $t = 0$ lending is rational in this section:

[11] It is also possible that the bank could bundle lending and payment services in writing the contract, which is not a credible threat either. See Section 4.3.2 for a further discussion.

Assumption 2. $(1-\lambda)\theta(r+R_h) \geq 1+\theta k$,

where the left-hand side represents the non-diversifiable part of the H-type's return, and the left-hand side is the total cost of funding.

Finally, we assess equilibrium in this framework by the expected output, netting out funding costs, as welfare. The net output under full information is $\theta(r+R_h-k)+(1-\theta)r-1$, which is characterized by (Debit, Cash) and the bank continuing only with the H-type. The separating equilibrium thus achieves the same welfare level as the full information allocation. We will study how introducing a CBDC changes the expected net output in the following section.

4 Equilibrium with CBDC

We now introduce a CBDC into our model as the third payment technology. Our analysis explores two distinct aspects of the CBDC: its benefits for sales and its level of anonymity. Regarding its benefits for sales, we consider the possibility that a CBDC augments business operations, leading to an enhancement in the entrepreneur's revenue by $\Delta \geq 0$ for every type in each period.[12] This exogenous revenue augmentation, Δ, can be attributed to the reduced transaction costs inherent to CBDCs, such as lower surcharge fees, as suggested by Verdier (2023) and Liu et al. (2023).[13]

[12] Theoretically, Δ can potentially be negative, implying that CBDC is more costly than Debit or Cash. However, in the context of our model where CBDC is being issued, this model implicitly assumes that the central bank has evaluated the benefits and costs and determined that the overall utility of CBDC, perhaps in terms of enhanced transaction efficiency or other non-monetary advantages, justifies its launch. However, how we interpret Δ itself is not crucial in delivering our message as our focus is on how a benefit of CBDC changes the equilibrium outcomes.

[13] The instant payment platform introduced by the Central Banco do Brazil, called *Pix*, provides a consistent real-world observation. Sarkisyan (2023) reports that the transaction cost of Pix for merchants is only 0.2% while debit cards cost about 1% in Brazil. Pix is viewed as a synthetic CBDC (Araujo (2022)), suggesting a similar benefit of a CBDC for merchants. See Section 4.3.1 for other interpretations of Δ.

Regarding the anonymity design, we assume that, when an entrepreneur chooses the CBDC as the payment method, they have the capability to divert a fraction $\eta \in [0, 1]$ of the returns. This fraction symbolizes the degree to which the bank can monitor funds within the entrepreneur's CBDC account.[14] When η approaches 1, the bank has no authority over the funds in the CBDC account, rendering the CBDC as anonymous as Cash. On the other hand, when η approaches λ, the CBDC has a debit-like nature, allowing the bank monitor and control over the entire funds as if they reside in the firm's primary deposit account. Intermediate values of η between λ and 1 suggest a scenario where the bank possesses partial access to the CBDC account, exerting control over a specific amount of funds. When η falls below λ, it can be interpreted as the central bank's augmented support to the banking institution in loan monitoring, potentially through measures like assigning a FinTech team to the entrepreneur for real-time loan tracking. Given η as a parameter, we study equilibria in the signaling game by adding this CBDC into the entrepreneur's strategy set. We later discuss the optimal level of η as the optimal CBDC design in our welfare analysis.

Before entering the formal analysis, we can narrow down potential equilibrium outcomes. Among the 3×3 payment choice combinations available to the two types of entrepreneurs, only four can emerge as potential equilibrium outcomes under Assumption 1. In particular, the H-type playing Cash will not constitute an equilibrium, or if it does, such an equilibrium would not survive the intuitive criterion. This is because, under the assumption $\lambda(r + R_h) > r$, only the H-type stands to gain potential profits by switching to Debit. Similarly, the L-type choosing Debit will not constitute an equilibrium that survives the intuitive criterion either.

[14] The degree of η implies the extent to which Privacy Enhancing Technologies (PETs) are embedded in CBDC transactions, such as zero-knowledge proofs, homomorphic encryption, differential privacy, and anonymization. See Darbha and Arora (2020) for the details.

Lemma 1. *There are 4 types of potential equilibrium outcomes: (Debit, Cash), (Debit, CBDC), (CBDC, Cash), and (CBDC, CBDC).*

4.1 A simple case of CBDC: $\Delta = 0$

We begin our analysis without the beneficial role of the CBDC, setting $\Delta = 0$. The distinction among the three payment technologies then lies solely in the diversion fraction. This approach simplifies the analysis while effectively conveying the insights that CBDC can lead to credit misallocation under a modest level of anonymity.

We first focus on separating equilibrium and establish that for each $\eta \in [0, 1)$, there uniquely exists an admissible separating equilibrium. Specifically, when η is less than λ, the CBDC is not attractive to both types due to its smaller diversion ratio compared to other payment methods. Therefore, the H-type chooses Debit and the L-type chooses Cash. However, as η exceeds λ, the CBDC starts to attract interest from the H-type, and consequently, (CBDC, Cash) constitute an equilibrium. Eventually, the CBDC becomes favorable to both types. We define $\underline{\eta}$ as the threshold above which the L-type starts to favor CBDC over Cash if doing so induces a rollover:

$$\underline{\eta}(r + R_l) = r.$$

When $\eta \geq \lambda$, (Debit, Cash) will constitute a unique separating equilibrium. This equilibrium is supported by the bank's belief that a player choosing a Debit must be the H-type, and any other plays, including choosing the CBDC, must come from the L-type, which survives the intuitive criterion. By Assumption 1, this $\underline{\eta}$ is always greater than λ, ensuring the existence of the (CBDC, Cash) equilibrium. Finally, when $\eta = 1$, CBDC is identical to Cash, therefore (Debit, CBDC) can constitute an equilibirum too. In all of these separating equilibria, the bank can infer the type of the entrepreneur and is hence able to make an efficient refinancing decision.

Proposition 2. ***(Separating equilibrium)***. *There exists a threshold $\underline{\eta}$ such that the separating equilibria that survive the intuitive criterion are (CBDC, Cash) when $\eta \in [\lambda, \underline{\eta}]$, (Debit, Cash) when $\eta \in [0, \lambda] \cup [\underline{\eta}, 1]$ and (Debit, CBDC) when $\eta = 1$. In all of the separating equilibria, the bank continues with the H-type only at $t = 1$.*

For $\eta \geq \underline{\eta}$, both types may rationally choose CBDC, giving rise to a pooling equilibrium, wherein both types choose CBDC. Given its inability to update belief in pooling equilibrium, the bank is willing to refinance if and only if doing so is still profitable, or $\eta \leq \bar{\eta}$, where $\bar{\eta}$ solves

$$(1 - \bar{\eta})M = k.$$

That is, the expected NPV must remain nonnegative at $t = 1$. Such a pooling equilibrium exists for each η within the range $[\underline{\eta}, \bar{\eta}]$:

Proposition 3. ***(Pooling equilibrium)***. *The pooling equilibria that survive the intuitive criterion are when $\eta \in [\underline{\eta}, \bar{\eta}]$. There are hence multiple equilibria in this range.*

We term this range a "modest level of anonymity". If the level of anonymity of CBDC is too low, the L-type may switch to Cash; conversely, if it is excessively high, the bank might resist rollover. Those pooling equilibria coexist with the (Debit, Cash) equilibria. To summarize our results, we illustrate the distribution of equilibrium in the following figure.

We finally evaluate the efficiency in these equilibria. Given that the L-type's negative NPV project receives funding in period 1, credit misallocation arises in the pooling equilib-

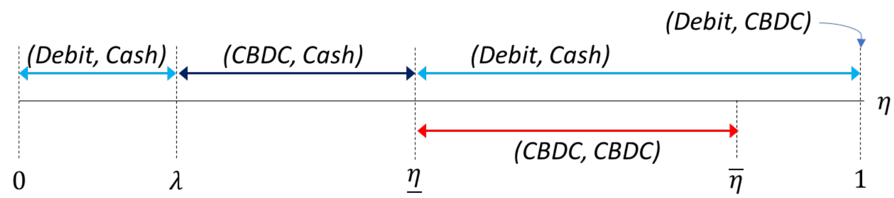

Figure 2: Distribution of equilibria.

rium. The net total output in those equilibria is

$$\Omega(\text{CBDC, CBDC}) = r + M - (1 + k),$$

which is strictly lower than in other separating equilibria, which is

$$\Omega(\text{Debit, Cash}) = r + \theta(R_h - k) - 1,$$

because $(1 - \theta)R_l < k$. Thus, maximum adoption of CBDC does not necessarily imply efficiency. The following example illustrates the difference of total net output with the distribution of equilibria. Notice that, when $\Delta = 0$, all separating equilibria yield the same total net output. If the central bank aims to optimize total net output while avoiding a potential credit misallocation introduced by pooling equilibria, the optimal anonymity level, denoted as η^*, should fall within the intervals $[0, \underline{\eta}]$ or $[\bar{\eta}, 1]$. This implies that the CBDC should closely resemble either Debit or Cash in its characteristics.

Proposition 4. ***(Optimal design of anonymity)****.* *When $\Delta = 0$, the optimal level of CBDC anonymity $\eta^* \in [0, \underline{\eta}] \cup [\bar{\eta}, 1]$.*

To conclude the section, we examine in detail the existence of pooling equilibria by

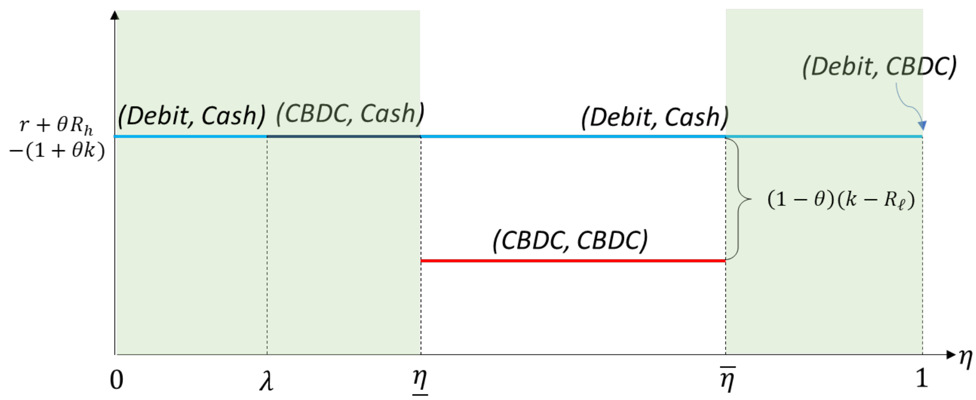

Figure 3: Comparing equilibria by total net output.

employing comparative statics to the parameters that guarantee $\underline{\eta} \leq \bar{\eta}$. Recalling that

$$\{\bar{\eta}, \underline{\eta}\} = \left\{1 - \frac{k}{\theta R_h + (1-\theta)R_l}, \frac{r}{r + R_l}\right\},$$

we establish the following comparative statics:

- The time-1 funding cost (k): An increase in k leads to a decline in $\bar{\eta}$, as the bank is less inclined to continue with both types. The range of pooling equilibria shrinks.
- The time-2 return to the L-type's project (R_l): An increase in R_l results in a decrease in $\bar{\eta}$ and a corresponding rise in $\underline{\eta}$. Consequently, the range of pooling equilibria expands.
- The time-2 return to the H-type's project (R_h): An increase in R_h raises $\bar{\eta}$, leaving $\underline{\eta}$ unaffected. This leads to an expansion in pooling equilibria.
- The time-1 project return (r): An increase in r raises $\underline{\eta}$, as it makes Cash more attractive to the L-type. Therefore, the parameter region of the pooling equilibria expands.

In the next section, we consider the benefits for sales in constructing equilibria and examine the optimal design of anonymity. The next proposition summarizes the observations above.

4.2 A general case of CBDC: $\Delta > 0$

We now turn our focus to a more general scenario where the CBDC has positive exogenous benefits for sales: $\Delta > 0$. This parameter uniquely differentiates CBDC from other payment technologies, making CBDC more attractive. To begin with, we slightly change Assumption 1 to

Assumption 3. $\lambda(r + R_l) < r < r + \Delta < \lambda(r + R_h)$,

which constrains the magnitude of Δ. This assumption implies that the H-type prefers Debit with rollover over CBDC without rollover. Additionally, we assume that even with the benefits associated with CBDC, the L-type's project still has a negative NPV:[15]

Assumption 4. $R_l + \Delta < k$.

It is perhaps worth emphasizing that these additional assumptions do not change any of the results we have presented so far. We introduce these assumptions to narrow down the range of Δ to study.

As in our prior analysis, we study equilibrium outcomes for each value of η given (λ, Δ), starting with separating equilibria. A major difference from the previous analysis is that the range of η that supports equilibria with a play of CBDC expands. In establishing a proposition, we introduce two new thresholds of η through exemplifying two notable changes. First, the separating equilibrium with (Debit, CBDC) is more likely to exist. We define

[15] If Δ makes the L-type's NPV positive, the problem becomes trivial: the pooling equilibrium will yield the highest total net output, and the information friction does not matter.

$\widehat{\eta} < 1$ such that (Debit, CBDC) exists when $\eta > \widehat{\eta}$. At this threshold level of η, the L-type is indifferent between Cash and CBDC without rollover:

$$\widehat{\eta}(r + \Delta) = r,$$

where $\widehat{\eta}$ is always less than 1 because $\Delta > 0$. Assumption 3 guarantees that the H-type then chooses Debit. Second, the equilibrium with (CBDC, Cash) can exist even when $\eta < \lambda$. The benefit of CBDC can offset its lesser degree of anonymity compared to Debit. The lowest level of η that supports the (CBDC, Cash) equilibrium, denoted by η^I, is given by

$$\eta^I(r + R_h + 2\Delta) = \lambda(r + R_h),$$

which is a strictly increasing function of λ.[16] However, note that when η exceeds $\underline{\eta}$, now characterized by

$$\underline{\eta}(r + R_l + 2\Delta) = r,$$

the L-type will start to mimic the H-type by choosing CBDC. Consequently, if $\eta^I(\lambda)$ exceeds $\underline{\eta}$, the (CBDC, Cash) equilibrium does not exist. We rewrite this condition as $\lambda \geq {\eta^I}^{-1}(\underline{\eta})$, where ${\eta^I}^{-1}(\cdot)$ is the inverse function of η^I.[17] The next proposition summarizes these results.

Proposition 5. ***(Separating equilibrium)***. *There exist three thresholds, $(\eta^I(\lambda), \underline{\eta}, \widehat{\eta})$, that govern what types of separating equilibria exist:*

1. *If $\lambda \leq {\eta^I}^{-1}(\underline{\eta})$, we have $\eta^I(\lambda) \leq \underline{\eta} < \widehat{\eta}$: The separating equilibria that survive the intuitive criterion consist of (CBDC, Cash) when $\eta \in [\eta^I(\lambda), \underline{\eta}]$, (Debit, Cash) when $\eta \in [0, \eta^I(\lambda)] \cup [\underline{\eta}, \widehat{\eta}]$ and (Debit, CBDC) when $\eta \in [\widehat{\eta}, 1]$;*

[16] See the proof of Proposition 5 in Appendix A for the detailed derivation.
[17] This exposition will later help us in endogenizing it.

2. *If* $\lambda \geq \eta^{I^{-1}}(\underline{\eta})$*, we have* $\underline{\eta} \leq \eta^{I}(\lambda) < \widehat{\eta}$*: The separating equilibria that survive the intuitive criterion consist of (Debit, Cash) when* $\eta \in [0, \widehat{\eta}]$ *and (Debit, CBDC) when* $\eta \in [\widehat{\eta}, 1]$*, there is no (CBDC, Cash) equilibrium.*

We now turn our focus to pooling equilibria. When $\Delta > 0$, the range of η that supports pooling equilibria expands by attracting both the L-type and the bank. It is clear that $\underline{\eta}$ which we just characterized is decreasing over Δ, implying that, for the L-type, a lower diversion ratio is compensated by a higher Δ. The other threshold, $\bar{\eta}$, is now determined by

$$(1-\bar{\eta})(\theta R_h + (1-\theta)R_l + \Delta) = k,$$

which shows that $\bar{\eta}$ is increasing over Δ, implying that, for the bank, a higher diversion ratio is compensated by a larger original pool of returns. Based on these thresholds, we establish the following proposition:

Proposition 6. ***(Pooling equilibrium)****.*

1. *If* $\lambda \leq \eta^{I^{-1}}(\underline{\eta})$*, the pooling equilibria that survive the intuitive criterion are when* $\eta \in [\underline{\eta}, \bar{\eta}]$

2. *If* $\lambda \geq \eta^{I^{-1}}(\underline{\eta})$*, the pooling equilibria that survive the intuitive criterion are* $\eta \in [\eta^{I}(\lambda), \bar{\eta}]$

As before, a pooling equilibrium arises when the CBDC's anonymity is at a moderate level. However, it is important to note that the existence of pooling equilibria now also depends on $\eta^{I}(\lambda)$ in addition to $\underline{\eta}$ and $\bar{\eta}$. In refining the pooling equilibria, the threshold $\eta^{I}(\lambda)$ plays a crucial role in shaping the bank's off-equilibrium beliefs, which in turn influences the existence of equilibrium. As λ increases, the range of η that can sustain a pooling equilibrium may shrink because H-type may be able to profit by deviating to Debit for some η. The distribution of these equilibria is illustrated in the following figures.

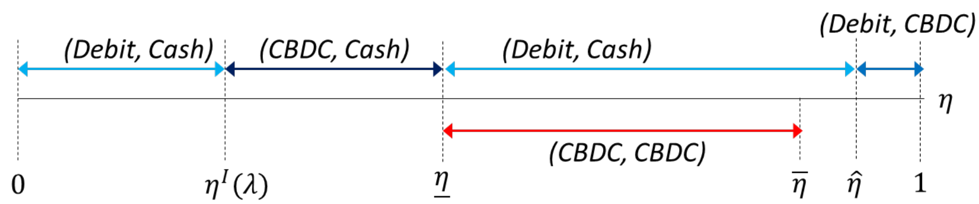

Figure 4: Distribution of equilibria: $\lambda \leq \eta^{I^{-1}}(\underline{\eta})$.

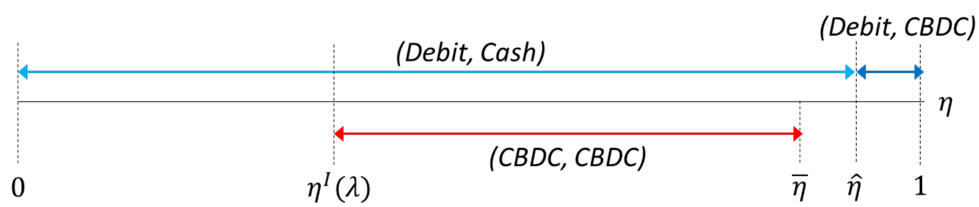

Figure 5: Distribution of equilibria: $\lambda \geq \eta^{I^{-1}}(\underline{\eta})$.

Regarding welfare implications, it is evident that maximum welfare is achieved under (CBDC, Cash). This outcome is attributed to the CBDC augmenting returns for the H-type's project across both periods. Following this, (Debit, CBDC) comes next, where the CBDC enhances returns for a single period. Among the separating equilibria, the welfare associated with (Debit, Cash) is the lowest. However, this welfare level still exceeds that of (CBDC, CBDC), guaranteed by Assumption 4. Formally, the welfare hierarchy is represented as:

$$\Omega(\text{CBDC, Cash}) > \Omega(\text{Debit, CBDC}) > \Omega(\text{Debit, Cash}) > \Omega(\text{CBDC, CBDC}).$$

If the central bank's aim is to maximize welfare while avoiding potential credit misallocation caused by pooling equilibria, the optimal anonymity level should adhere to the above hierarchy.

Proposition 7. ***(Optimal design of anonymity)****. When $\lambda \leq {\eta^I}^{-1}(\underline{\eta})$, the optimal level of CBDC anonymity $\eta^* \in [\eta^I(\lambda), \underline{\eta}]$; when $\lambda \geq {\eta^I}^{-1}(\underline{\eta})$, the optimal level of CBDC anonymity $\eta^* \in [\max\{\widehat{\eta}, \bar{\eta}\}, 1]$.*

The welfare comparison across different equilibria is illustrated across the distribution of these equilibria in the two following figures. Each corresponds to the figures presented above. The shaded areas denote the optimal ranges of anonymity.

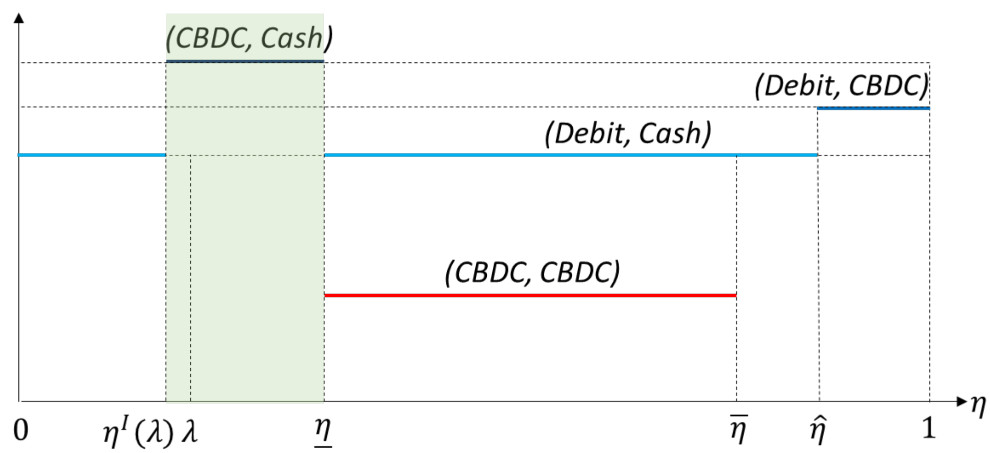

Figure 6: Total net output: $\lambda \leq {\eta^I}^{-1}(\underline{\eta})$.

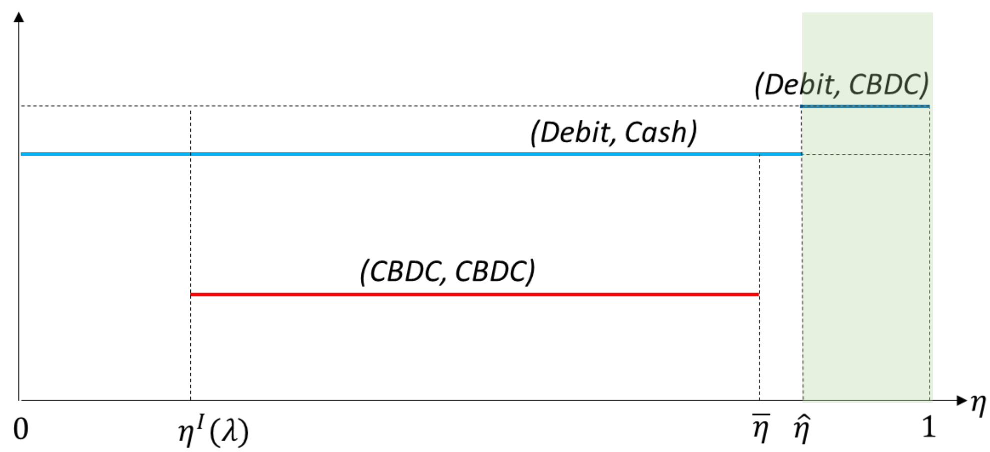

Figure 7: Total net output: $\lambda \geq {\eta^I}^{-1}(\underline{\eta})$.

Last but not least, we emphasize that determining which of the two cases is more probable depends crucially on λ and Δ. When Δ is substantially small, the CBDC is less attractive, and there exists (CBDC, Cash) equilibria, implying $\lambda \leq {\eta^I}^{-1}(\underline{\eta})$. In such a case, the optimal level of η is in the range of $[\eta^I(\lambda), \underline{\eta}]$. However, as Δ increases, the CBDC becomes more attractive. Specifically, an increase in Δ shrinks the parameter range where $\lambda \leq {\eta^I}^{-1}(\underline{\eta})$ holds, which eventually eliminate (CBDC, Cash) equilibria. The threshold of Δ that guarantees $\lambda \leq {\eta^I}^{-1}(\underline{\eta})$ solves $\eta^I(\lambda; \Delta) = \underline{\eta}(\Delta)$, which yields

$$\Delta \leq \widehat{\Delta} \equiv \frac{r - \lambda(r + R_\ell)}{2\lambda - \frac{r}{r+R_h}}.$$

Additionally, an increase in Δ shifts $\widehat{\eta}$ to the left, making $\widehat{\eta} < \bar{\eta}$ possible. Thus, when $\Delta \geq \widehat{\Delta}$, the optimal level of η lies in the range of $\eta^* \in [\bar{\eta}, 1]$. In other words, when Δ is substantially large, the range of η that supports the pooling equilibrium broadens in both the left and right directions. This effect first eliminates the (CBDC, Cash) as a potential equilibrium outcome and subsequently narrows the range of η that supports the (Debit, CBDC) equilibrium.

Corollary 1. *If $\Delta \leq \widehat{\Delta}$, the optimal level of CBDC anonymity is $\eta^* \in [\eta^I(\lambda), \underline{\eta}]$. If $\Delta \geq \widehat{\Delta}$, the optimal level of CBDC anonymity is $\eta^* \in [\max\{\widehat{\eta}, \bar{\eta}\}, \underline{\eta}]$.*

The intuition behind this result is straightforward: A higher Δ incentivizes the L-type to choose the CBDC, thereby limiting the scope for a "good" equilibrium to exist. While (CBDC, Cash) equilibria disappears as Δ rises, there will always exist at least a (Debit, CBDC) equilibrium because the bank does not continue when η is substantially close to 1. The next section explores the role of λ by allowing the bank to choose its level.

While our paper does not take any particular stance regarding the magnitude of Δ, our results suggest that any advantageous features of CBDC should be carefully examined. The most desirable outcomes that we have shown is the (Debit, Cash) equilibria, and the optimal design of CBDC is debit-like, which requires the small magnitude of Δ.[18]

4.3 Discussions

In the last part of this section, we provide some further extensions and discussions.

4.3.1 Interpretation of Δ

While we interpreted the merchant's benefit of Δ in using a CBDC as lower payment fees for merchants, it can be interpreted in some other ways. For example, such a benefit may come from in the form of interest (Barrdear and Kumhof (2022); Chiu, Davoodalhosseini, Jiang, and Zhu (2023); Keister and Sanches (2022)), programmable payments (Kahn and Van Oordt (2022)), personal loss recovery for digital cash (Kahn and Van Oordt (2021)), and privacy-sensitive consumers (Garratt and Lee (2022); Garratt and van Oordt (2021)). Since

[18] We assume that Δ is a parameter, and not the choice variable of the central bank. If the central bank can also choose Δ, the result in this section implies that central banks should choose a small magnitude of Δ and make a CBDC deposit-like.

our focus is how a merchant's benefits of choosing CBDC changes equilibrium outcomes and not the form of benefit, any of these interpretations work in our paper. In fact, these papers reinforce that Δ may be positive while the magnitude is still uncertain.

4.3.2 Bundling banking services

One of the important elements in our analysis is the lack of commitment: the bank cannot pre-commit to a loan contract. If the bank has commitment power, it can stipulate a loan contract such that it will not lend in period 1 if the entrepreneur chooses any payment instrument other than Debit. The H-type will therefore always choose Debit to continue the business until period 2. As a result, there will exist either (Debit, Cash) or (Debit, CBDC), depending on the value of η and Δ.

Such a loan contract can be interpreted as bundled services. When the bank offers loans, it may require the entrepreneur to open and use a bank account for business. Thus, the bank forces the entrepreneur to use Debit by bundling loans and payment instruments. However, forcing the entrepreneur to actually use a particular payment method in business transactions may not be easy. Even if the entrepreneur opens a bank account, it can still accept cash or CBDC. Then, it will be ex-post inefficient for the bank not to continue the investment. We thus restrict the bank's ability to commit so that the bank cannot make such a time-inconsistent action, which in turn, implies that the bank cannot effectively bundle the services.

4.3.3 Bank competition

Our model also provides a perspective on how bank competition affects the equilibrium result. For example, the diversion fraction, λ, can alternatively be interpreted as the bargaining power of the entrepreneur, while $1 - \lambda$ represents the bargaining power or profitability of banks. One consequence of intensified bank competition is the decrease in bank profitability, or conversely, an increase in λ. As explored above, this scenario is likely to cause the (CBDC, Cash) equilibrium to disappear, rendering CBDC cash-like. While this discussion considers the macro level, at a micro level, in Section 5, we explore an individual bank's adjustment of λ, such as modifications to monitoring intensity.

4.3.4 Social cost of diversion

Diverting funds is often suggested as a costly action, e.g., an efficiency loss, significantly reducing the value of a unit of funds diverted to the entrepreneur.[19] Integrating this aspect into our framework is straightforward. Earlier, we identified a range of optimal anonymity levels. When factoring in the social cost of diversion, the optimal level of anonymity shifts to the lower end of this range. For example, Proposition 7 becomes

Proposition 8. ***(Optimal design of anonymity with social diversion cost).*** *When $\lambda \leq {\eta^I}^{-1}(\underline{\eta})$, the optimal level of CBDC anonymity $\eta^* = \eta^I(\lambda)$; when $\lambda \geq {\eta^I}^{-1}(\underline{\eta})$, the optimal level of CBDC anonymity $\eta^* = \widehat{\eta}$.*

While the main conclusions from our analyses remain consistent, considering the social cost of diversion provides a more precise determination of the optimal level of anonymity.

5 Strategic response of the bank

We finally examine strategic interactions between the central bank and banks. In the previous sections, we assumed that λ, the degree of anonymity in Debit, was a parameter, which

[19] See, for example, Holmstrom and Tirole (2011).

rendered the bank passive regarding the equilibrium outcomes. However, an equilibrium that the central bank desires to implement by changing η may not be in the best interest of the bank. For example, a (CBDC, Cash) equilibrium may be less desirable than a (Debit, Cash) equilibrium because the H-type may divert more. In this section, we explore the possibility of banks strategically responding to the introduction of CBDCs, thereby influencing the resulting equilibrium outcomes. Specifically, we now allow the bank to choose the value of λ. In fact, it is plausible that the bank can influence the extent to which entrepreneurs divert funds when they use Debit. For example, banks might intensify their loan scrutiny, tighten credit standards, or even re-evaluate their risk assessment models. Such proactive measures enable the bank to counteract the introduction of CBDC, potentially preventing outcomes that are not in their favor.

To examine such a strategic response, we consider two possible cases: First, banks choosing λ before η is chosen; second, banks choosing λ after η is chosen. The first case aligns with situations where commercial banks foresee the introduction of a CBDC and proactively adjust their monitoring strategies. This might occur in contexts where central banks engage in extensive communication with commercial banks regarding the introduction of a CBDC, perhaps via pilot projects and developing public-private partnerships. Such communication could help the commercial banks to more accurately predict the introduction of a CBDC.[20] The second case reflects situations where the launch of a CBDC is a surprise to commercial banks. If its central bank were to suddenly issue a CBDC without extensive prior indication, commercial banks in the country would find themselves in a reactive position. They might

[20] Sweden, known for its high digital payment adoption rate with its Swish system, likely exemplifies this case. Considering its advanced financial infrastructure and the close relationship between the Riksbank, Sweden's central bank, and major commercial banks, it is plausible that commercial banks like Handelsbanken or SEB would probably be well-informed and prepared. If the Riksbank were to announce an upcoming CBDC issuance, these commercial banks might preemptively adjust their loan monitoring strategies.

be compelled to recalibrate their loan monitoring processes in response to unforeseen shifts in borrower behavior brought on by the CBDC.

Formally, we now assume that the bank can choose a value for λ before making the loan. The bank chooses λ to maximize the (expected) return that it can receive, while the central bank chooses η to maximize welfare (expected total net output). The pair (λ, η) is determined first, and each combination creates a subsequent signaling game between the entrepreneur and the bank. We will study two separate game structures as illustrated above: the bank determines λ before the central bank chooses η, and the central bank decides on η before the bank chooses λ.[21] Each of these sequential-move games precedes the signaling game discussed earlier. In either case, the central bank can choose any $\eta \in [0, 1]$ and the bank can choose any $\lambda \in [\frac{r+\Delta}{r+R_h}, \frac{r}{r+R_l}]$. The restriction on the strategy space for the bank comes from Assumption 3.

5.1 Bank as the first mover

Our first focus is the case where the bank first chooses λ and then the central bank chooses η. In such a case, if the bank anticipates that the central bank will choose η to implement a less favorable equilibrium for the bank, it may be able to strategically set λ to preclude such an outcome. More pointedly, in situations where the bank and the central bank favor different equilibrium outcomes, the bank has the capability to steer the game towards the equilibrium it deems preferable.

To establish a conflict of interest between the central bank and the bank, we introduce an additional constraint on Δ.

[21] While we are not presenting the simultaneous-move game where the bank and the central bank make decisions at the same time, the equilibrium outcome of it is the same as the second scenario where the central bank moves first.

Assumption 5. $\Delta < \frac{(1-\theta)r}{3\theta-1}$,

where we also assume that $\theta > \frac{1}{3}$ so that the denominator of the bound is positive. Note that this is a sufficient condition. Under this assumption, the bank prefers (Debit, CBDC) to (CBDC, Cash) because

$$\begin{aligned}\Pi^B(\text{Debit, CBDC}) &= \theta(1-\lambda)(r+R_h) + (1-\theta)(1-\eta)(r+\Delta) \\ &> \theta(1-\eta)(r+R_h) + \theta(1-\eta)2\Delta \\ &= \Pi^B(\text{CBDC, Cash}).\end{aligned}$$

On the other hand, the central bank prefers (CBDC, Cash) to (Debit, CBDC) to (Debit, Cash).

We now show that by moving first, the bank can implement the equilibrium outcome (Debit, CBDC). The mechanism here is that the bank can choose λ such that $\underline{\eta} \leq \eta^I(\lambda)$ so that the central bank's best response will be to choose $\eta \in [\widehat{\eta}, 1]$. In particular, because its payoff is decreasing in λ, the bank chooses the lowest possible level of λ that achieves $\underline{\eta} \leq \eta^I(\lambda)$.[22]

Proposition 9. ***(Bank as first mover)***. *When the bank moves first, the equilibrium is the bank choosing* $\lambda^* = \min\{{\eta^I}^{-1}(\underline{\eta}), \frac{r+\Delta}{r+R_h}\}$ *and the central bank choosing* $\eta \in [\widehat{\eta}, 1]$. *The equilibrium of the subsequent signaling game consists of (Debit, CBDC).*

This proposition shows that the bank's strategic response leads to the (Debit, CBDC) equilibrium although the (CBDC, Cash) equilibrium yields higher welfare. Thus, when the bank has freedom in its choices and acts very fast, the central bank might not get the best result.

[22]Specifically, if $\eta^I(\frac{r+\Delta}{r+R_h}) > \underline{\eta}$, the bank chooses chooses $\frac{r+\Delta}{r+R_h}$ because λ is assumed to be greater than $\frac{r+\Delta}{r+R_h}$ b Assumption 3. Otherwise, the bank chooses λ such that $\eta^I(\lambda) = \underline{\eta}$.

We call this result the *from Debit to Cash* shift of CBDC: banks see a CBDC that might replace debit, so they adjust debit to be more like CBDC (by increasing λ) to keep it relevant, which in turn, makes CBDC act more like cash (by increasing η).

5.2 Central Bank as the First Mover

We lastly explore the case where the sequence of decisions is reversed: the central bank determines η before the bank chooses λ. We retain Assumption 5 to ensure the conflict of interest between the bank and the central bank. Additionally, we assume that Δ is substantially small so that (CBDC, Cash) can constitute an equilibrium for some combination of (λ, η). Remember that a (CBDC, Cash) equilibrium exists if $\eta^I(\lambda) \leq \underline{\eta}$. Because we have assumed that the bank's choice of λ falls within the range $[\frac{r+\Delta}{r+R_h}, \frac{r}{r+R_l}]$, we have to make sure that $\eta^I(\frac{r+\Delta}{r+R_h}) < \underline{\eta}$. Since this lower bound is increasing over Δ and $\underline{\eta}$ is decreasing over Δ, the sufficiently small Δ can guarantee the existence of (CBDC, Cash) equilibria. Specifically, we let $\bar{\Delta}$ solve $\eta^I(\frac{r+\bar{\Delta}}{r+R_h}) = \underline{\eta}(\bar{\Delta})$, or

$$\frac{r+\Delta}{r+R_h+2\Delta} = \frac{r}{r+R_l+2\Delta},$$

so that, when $\Delta \leq \bar{\Delta}$, there exists (CBDC, Cash) equilibrium, guaranteeing the conflict of interest.

When there is the conflict of interest, the central bank chooses a lower η to limit the potential equilibrium outcomes to (CBDC, Cash) and (Debit, Cash); determining which of them constitutes an equilibrium depends on the bank's response. Given the low η and positive Δ, which render (CBDC, Cash) more preferable than (Debit, Cash), the bank chooses λ to implement the (CBDC, Cash) equilibrium. However, if Δ increases to a level that eliminates the conflict of interest, the central bank chooses a higher η, aiming to implement the (Debit, CBDC) equilibrium. The bank's incentive is aligned with the central bank. Notice that, in either case, the central bank can achieve its most desirable outcome for given Δ.

Proposition 10. ***(Central bank as first mover)*** *If $\Delta \leq \bar{\Delta}$, the equilibria are characterized by $\eta \in [\eta^I(\frac{r+\Delta}{r+R_h}), \underline{\eta}]$ and $\lambda(\eta) = [\frac{r+\Delta}{r+R_h}, \frac{r+R_h+2\Delta}{r+R_h}\eta]$, and the equilibrium outcome of the subsequent signaling game is (CBDC, Cash); if $\Delta > \bar{\Delta}$, the equilibria are characterized by $\eta \in [\widehat{\eta}, 1]$, $\lambda(\eta) = \frac{r+\Delta}{r+R_h}$, and the equilibrium outcome of the subsequent signaling game is (Debit,CBDC).*

We conclude this section with two observations. First, the two sets of analyses suggest the first-mover advantage. In each case, the first mover can influence which equilibrium outcomes will be played in the subsequent signaling game, achieving their favorable outcome. Second, the strategic interaction between the central bank and the bank can be interpreted as competition between deposit and a CBDC. The most desirable outcome features (CBDC, Cash) where debit is not used in transactions. The bank's strategic response in Section 5.1 is to sustain the market share of Debit as (Debit, CBDC) is the preferred outcome for the bank. Thus, the optimal design of a CBDC is influenced by such competition, and if the bank can move first, the competition may hinder the most socially desirable outcome.

6 Concluding remarks

We have explored how the user anonymity of CBDC affects bank lending to characterize its optimal design of CBDC. Specifically, by modeling payment methods as signaling tools in this context, we have shown how choices of payment instrument determine the bank's lending decision, and furthermore, how adding a CBDC changes the equilibrium outcomes.

Our findings suggest that a modest level of anonymity can cause credit misallocation, as it might lead both high- and low-productive entrepreneurs to adopt a CBDC, leaving the bank with no option but to finance both. Thus, maximum adoption does not necessarily imply efficiency. The optimal level of anonymity hinges on what the CBDC, compared to the other payment methods, can offer to business users. If it has a large benefit for sales, CBDC should be cash-like so that low-productive entrepreneurs adopt CBDC. If, instead, it has a small benefit for sales, CBDC should be deposit-like so that high-productive entrepreneurs adopt CBDC. However, we have also shown that this deposit-like CBDC may not be straightforward to implement if the bank preemptively reacts to the introduction of CBDC, which could make the most desirable outcome unattainable. Our result highlights that (i) any advantageous features of a CBDC should be carefully examined to optimally design user anonymity of CBDC and (ii) the competition between deposits and CBDC may undermine the implementation of the most desirable outcome.

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A Proofs for selected results

Proof of Proposition 2. We show this result by examining equilibrium outcomes over $\eta \in [0,1]$. Recall that Lemma 1 implies that potential pure-strategy separating equilibrium outcomes are (Debit, Cash), (CBDC, Cash), and (Debit, CBDC). Examining these equilibrium outcomes over η is thus sufficient to identify all equilibrium outcomes.

1. Let $\underline{\eta}$ be such that L-type is indifferent between cash and CBDC (if choosing CBDC allows the L-type to be continued):

$$\underline{\eta}(r + R_l) = r.$$

2. When $\eta \in [\underline{\eta}, 1]$, (Debit, Cash) is supported by the belief that, if the bank observes the CBDC, it believes that it is chosen by a L-type. Meanwhile, (CBDC, Cash) cannot be an equilibrium on this region, because L-type will mimic H-type:

$$\eta(r + R_l) > r.$$

   Similarly, (Debit, CBDC) cannot be equilibrium on this region, because L-type will deviate to Cash.

3. When $\eta \in (\lambda, \underline{\eta}]$, the CBDC is only profitable for H-type. The bank's belief when observing the CBDC should be

$$\mu(\text{type} = H|\text{CBDC}) = 1.$$

   Given this belief, (Debit, Cash) fails the Cho-Kreps Intuitive Criterion. (Debit, CBDC) cannot be an equilibrium because L-type will deviate to Cash. Only (CBDC, Cash) can be an equilibrium.

   - Note that if $\lambda > \underline{\eta}$, such equilibrium does not exist.

4. At $\eta = \lambda$, both (Debit, Cash) and (CBDC, Cash) survive

5. When $\eta \in [0, \lambda)$, (CBDC, Cash) fails the Cho-Kreps intuitive criterion because H-type can be strictly better off by deviating to Debit if the bank believes it is a H-type, but

this is not true for L-type:

$$\lambda(r + R_h) > \eta(r + R_h);$$

$$r > \lambda(r + R_l).$$

(Debit, CBDC) cannot be an equilibrium on this region again because L-type will deviate to Cash. (Debit, Cash) is an equilibrium.

This completes the proof. □

Proof of Proposition 3. Pooling equilibrium exists for $\eta \in [\underline{\eta}, \bar{\eta}]$. We let $\bar{\eta}$ denote the threshold where the bank is willing to lend to both types at time 1 such that

$$(1 - \bar{\eta})(\theta R_h + (1 - \theta)R_l) = k.$$

Each equilibrium survives the intuitive criteria because no type has profitable deviation. □

Proof of Proposition 5. We first show the case where $\lambda \leq \eta$. We prove the result in several steps

1. Let $\widehat{\eta}$ be such that L-type is indifferent between Cash and CBDC (if choosing CBDC deters the L-type from continuation):

$$\widehat{\eta}(r + \Delta) = r.$$

2. When $\eta \in [\widehat{\eta}, 1]$,(Debit, CBDC) is an equilibrium. H-type's incentive constraint is satisfied thanks to the assumption:

$$\eta(r + \Delta) \leq r + \Delta \leq \lambda(r + R_h)$$

   and (Debit, Cash) is not. (CBDC, Cash) is not equilibrium because L-type will mimic the H-type.

3. Let $\underline{\eta}$ be such that L-type is indifferent between Cash and CBDC (if continued)

$$\underline{\eta}(r + R_l + 2\Delta) = r.$$

4. When $\eta \in [\underline{\eta}, \widehat{\eta}]$, (Debit, Cash) is an equilibrium with belief that CBDC is chosen by L-type. (CBDC, Cash) is not an equilibrium because the L-type will mimic the H-type because $\eta > \underline{\eta}$.

5. When $\eta \in (\lambda, \underline{\eta}]$, CBDC is only profitable for H-type. The bank's belief when observing CBDC should be
$$\mu(\text{type} = G|\text{CBDC}) = 1.$$
Given this belief, (Debit, Cash) fails the intuitive Criterion. (Debit, CBDC) cannot be an equilibrium again because L-type will deviate to Cash. Only (CBDC, Cash) can be an equilibrium.

   - Note that if $\lambda > \underline{\eta}$, such an equilibrium does not exist.

6. At $\eta = \lambda$, both (Debit, Cash) and (CBDC, Cash) survive the intuitive criterion.

7. When $\eta \in [0, \lambda)$, let $\tilde{\eta}$ be such that
$$\tilde{\eta}(r + R_h + 2\Delta) = r,$$
then (CBDC, Cash) exists for $\eta \in [\tilde{\eta}, \lambda]$ supported by the belief that Debit is from L-type. H-type will not mimic L-type because $\eta \geq \tilde{\eta}$ and L-type will not mimic H-type because $\eta \leq \underline{\eta}$. In the meantime, (Debit, Cash) exists on $[0, \lambda)$ supported by the belief that the CBDC is from L-type. The CBDC is dominated by Cash for both types. However, let $\eta^I(\lambda)$ be such that
$$\eta^I(r + R_h + 2\Delta) = \lambda(r + R_h),$$
we argue that when $\eta < \eta^I(\eta)$, (CBDC, Cash) fails the Cho-Kreps, because the H-type wants to deviate to Debit if the bank believes it is from Debit, but this is not true for L-type:
$$\eta(r + R_h + 2\Delta) < \lambda(r + R_h).$$
We also argue that (Debit, Cash) exists and only exists for $\eta \in [0, \eta^I(\lambda)]$: when $\eta > \eta^I(\lambda)$, it fails the Cho-Kreps, because H-type can strictly profit by deviating to

the CBDC but L-type cannot ($\eta < \underline{\eta}$); when $\eta \le \eta^I(\lambda)$, it is supported by the belief that the CBDC is from L-type.

We then study when $\eta \ge \underline{\eta}$. When $\lambda < {\eta^I}^{-1}(\underline{\eta})$, (CBDC, Cash) exists on $[\eta^I(\lambda), \underline{\eta}]$. The analysis is almost idential to the above case, the only difference being that when $\eta \in [\underline{\eta}, \lambda]$, (Debit, cash) survives Cho-Kreps because no type can strictly profit by deviating to the CBDC. When $\lambda \ge {\eta^I}^{-1}(\underline{\eta})$, (CBDC, Cash) no longer exists because when $\eta > \underline{\eta}$, L-type will mimic the H-type. □

Proof of Proposition 6. When $\lambda \le \underline{\eta}$, the argument is the same as in $\Delta = 0$. When $\underline{\eta} \le \lambda \le {\eta^I}^{-1}(\underline{\eta})$ (CBDC, CBDC) also survives Cho-Kreps because H-type cannot profit deviating to Debit. When $\lambda \ge {\eta^I}^{-1}(\underline{\eta})$, if $\eta \in [\eta^I(\lambda), \bar{\eta}]$, the argument is the same; if $\eta \in [\underline{\eta}, \eta^I(\lambda)]$, (CBDC, CBDC) fails Cho-Kreps, because H-type wants to deviate to Debit but not L-type. □

Proof of Proposition 9. We discuss the following cases.

1. $\frac{r+\Delta}{r+R_h} \le \underline{\eta}$. If this is true,
   (a) If the bank chooses $\lambda \le \underline{\eta}$, then $\eta^I(\lambda) \le \underline{\eta}$, the central bank can then choose $\eta \in [\eta^I(\lambda), \underline{\eta}]$ and the equilibrium outcome is (CBDC, Cash).
   (b) If the bank chooses λ such that $\eta^I(\lambda) \le \underline{\eta} < \lambda$, the central bank can again choose $\eta \in [\eta^I(\lambda), \underline{\eta}]$ and the equilibrium outcome is (CBDC, Cash).
   (c) If the bank chooses λ such that $\underline{\eta} \le \eta^I(\lambda)$, the central bank's best response is to choose $\eta \in [\widehat{\eta}, 1]$ and the equilibrium outcome is (Debit, CBDC). And this is feasible if

$$\underline{\eta} \le \eta^I\left(\frac{r}{r+R_l}\right),$$

   that is, η^I evaluated at the highest value of λ must be greater than or equal to $\underline{\eta}$. This condition simplies as

$$\frac{r+R_h}{r+R_l} > \frac{r+R_h+2\Delta}{r+R_l+2\Delta},$$

   and is always true.

2. $\frac{r+\Delta}{r+R_h} \geq \underline{\eta}$. If this is true,

   (a) If the bank chooses λ such that $\eta^I(\lambda) \leq \lambda < \underline{\eta}$, the central bank can again choose $\eta \in [\eta^I(\lambda), \underline{\eta}]$ and the equilibrium outcome is (CBDC, Cash).

   (b) If the bank chooses λ such that $\underline{\eta} \leq \eta^I(\lambda)$, the central bank's best response is to choose $\eta \in [\widehat{\eta}, 1]$ and the equilibrium outcome is (Debit, CBDC).

The bank is hence always able to implement (Debit, CBDC) and will choose the lowest level of λ possible, which solves

$$\eta^I(\lambda) = \underline{\eta}$$

if possible, otherwise the bank chooses $\lambda = \frac{r+\Delta}{r+R_h}$. □

Proof of Proposition 10. The central bank chooses $\eta \in [0, 1]$ first to maximize total net output, and then the bank responds by choosing λ to maximize its payoff in period 2 subject to $\frac{r+\Delta}{r+R_h} \leq \lambda \leq \frac{r}{r+R_l}$, further followed by the signaling game we have established. The bank's payoff is

$$\theta(1-\eta)(r + R_h + 2\Delta)$$

in (CBDC, Cash), and

$$\theta(1-\lambda)(r + R_h)$$

in (Debit, Cash) and

$$\theta(1-\lambda)(r + R_h) + (1-\theta)(1-\eta)(r + \Delta)$$

in (Debit, CBDC).

1. If the central bank chooses $\eta \in [0, \underline{\eta}]$, we don't know which one is larger, $\frac{r+\Delta}{r+R_h}$ or $\underline{\eta}$.

   (a) $\frac{r+\Delta}{r+R_h} \leq \underline{\eta}$.

      i. $\eta \leq \frac{r+\Delta}{r+R_h} \leq \underline{\eta}$, in this case any λ the bank chooses will be greater than or equal to η. The bank chooses $\lambda \in [\frac{r+\Delta}{r+R_h}, \frac{r}{r+R_l}]$. Can the bank choose λ such that $\eta^I(\lambda) \leq \eta$? Since $\frac{r+\Delta}{r+R_h} \leq \underline{\eta}$, we have $\eta^I(\frac{r+\Delta}{r+R_h}) < \frac{r+\Delta}{r+R_h} \leq \underline{\eta}$, hence the choice set is non-empty. The bank can choose any $\frac{r+\Delta}{r+R_h} \leq \lambda \leq \frac{r+R_h+2\Delta}{r+R_h}\eta$, the central bank chooses $\eta \in [\eta^I(\frac{r+\Delta}{r+R_h}), \underline{\eta}]$, and the equilibrium outcome is (CBDC, Cash). The bank has a higher payoff in this case.

ii. $\frac{r+\Delta}{r+R_h} \leq \eta \leq \underline{\eta} = \frac{r}{r+R_l+2\Delta}$. It is possible that the bank can choose λ such that $\eta^I(\lambda) \leq \eta$ because

$$\eta^I(\frac{r+\Delta}{r+R_h}) < \frac{r+\Delta}{r+R_h} \leq \eta \leq \underline{\eta}.$$

The bank can choose any $\frac{r+\Delta}{r+R_h} \leq \lambda \leq \frac{r+R_h+2\Delta}{r+R_h}\eta$, and the equilibrium outcome is (CBDC, Cash). The bank has higher payoff in this case.

(b) $\frac{r+\Delta}{r+R_h} \geq \underline{\eta}$.

- If $\eta^I(\frac{r+\Delta}{r+R_h}) > \underline{\eta}$, or

$$\frac{r+\Delta}{r+R_h+2\Delta} > \frac{r}{r+R_l+2\Delta},$$

it is not possible for the bank to choose λ such that

$$\eta^I(\lambda) \leq \eta \leq \underline{\eta},$$

and the equilibrium is (Debit, Cash), and $\lambda = \frac{r+\Delta}{r+R_h}$ will generate the highest payoff. The bank receives $\theta(1-\frac{r+\Delta}{r+R_h})(r+R_h)$.

- If $\eta^I(\frac{r+\Delta}{r+R_h}) \leq \underline{\eta}$, the bank can choose any $\frac{r+\Delta}{r+R_h} \leq \lambda \leq \frac{r+R_h+2\Delta}{r+R_h}\eta$, the central bank chooses $\eta \in [\eta^I(\frac{r+\Delta}{r+R_h}), \underline{\eta}]$, and the equilibrium outcome is (CBDC, Cash). The bank has a higher payoff in this case.

Note that $\frac{r+\Delta}{r+R_h+2\Delta} > \frac{r}{r+R_l+2\Delta}$ is stricter than $\frac{r+\Delta}{r+R_h} \geq \underline{\eta}$, and can be written as

$$\Delta \geq \bar{\Delta}$$

where $\bar{\Delta}$ solves

$$\frac{r+\Delta}{r+R_h+2\Delta} = \frac{r}{r+R_l+2\Delta}.$$

2. If the central bank chooses $\eta \in [\underline{\eta}, \bar{\eta}]$, it will generate (Debit, Cash) as the separating equilibrium. The bank will choose the lowest level of $\lambda = \frac{r+\Delta}{r+R_h}$. At this level,

$$\eta^I(\lambda) = \frac{r+R_h}{r+R_h+2\Delta} * \frac{r+\Delta}{r+R_h+2\Delta}.$$

If this is greater than $\underline{\eta}$, we have pooling equilibria from $[\eta^I(\lambda), \bar{\eta}]$; if it is smaller, we have $[\underline{\eta}, \bar{\eta}]$ as pooling equilibria.

3. If the central bank chooses $\eta \in [\bar{\eta}, \widehat{\eta}]$, the choice of λ cannot affect equilibrium because $\lambda \leq \frac{r}{r+R_l} < \bar{\eta} \leq \eta \leq \widehat{\eta}$. The equilibrium is (Debit, Cash). The bank chooses the lowest level of λ which is $\frac{r+\Delta}{r+R_h}$.

4. If the central bank chooses $\eta \in [\widehat{\eta}, 1]$, similarly, the equilibrium is (Debit, CBDC), and the bank chooses $\lambda = \frac{r+\Delta}{r+R_h}$.

In sum, the output is maximized when (CBDC, Cash) is the equilibrium and the bank prefers (CBDC, Cash) to (Debit, Cash). If (CBDC, Cash) is not implementable, the central bank will choose $\eta \in [\widehat{\eta}, 1]$ to generate the second-highest output. But in this game, the bank cannot control whether (CBDC, Cash) is implementable; this question depends only on Δ and other project return parameters. □