A Level-Dependence Approach for Assessing De-Anchoring of Inflation Expectations: Evidence from Colombia

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Abstract

This study introduces a methodology for evaluating the de-anchoring of inflation expectations by proposing indicators to measure deviations in short- and long-term inflation expectations from the Central Bank's target, analyzing their dependency over time using traditional and hierarchical statistical copulas, the latter incorporating the effect of the monetary policy stance. Using data from the Colombian financial market, the findings reveal that during inflationary episodes (2008–2009, 2015–2016, and 2022–2023), the dependency between short- and long-term expectations increased, indicating de-anchoring. This pattern was also observed during periods when inflation was below the target (2013 and 2020). Conversely, in years such as 2006, 2010, 2014, 2017, and 2021, and towards the end of 2023, the decrease in this dependency suggests that expectations were anchoring. Additionally, when the monetary policy stance was considered, there was a strong negative dependency during contractionary episodes, while progressive interest rate reductions were associated with a positive dependency.

JEL Classification: E31, E58, E52.

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1 Introduction

Central banks closely monitor inflation expectations, paying special attention to de-anchoring episodes. These episodes arise when expectations persistently deviate from the target or when shocks simultaneously affect short- and long-term horizons, weakening the expectations channel of monetary policy and requiring stronger interest rate adjustments at a higher economic cost. This study proposes a methodological approach that combines two strategies: quantifying deviations of short- and long-term expectations from the target and estimating the dependence between them. Additionally, it examines how this dependence evolves under different monetary policy stances.

This analysis leverages traditional statistical copulas to capture potential non-linearities in the dependence structure and incorporates hierarchical copulas to explore the influence of monetary policy. Applying these methods to historical data, the study assesses the degree of inflation expectations' de-anchoring across multiple inflationary periods and examines how monetary policy stance shapes the dependence between short- and long-term expectations.

Since inflation expectations are not directly observable, they must be inferred from various sources, such as surveys (Candia *et al.*, 2024), financial market instruments (Joyce *et al.*, 2010), or social media data analyzed using Natural Language Processing techniques (Angelico *et al.*, 2022). These expectations reflect the perceptions, beliefs, and forecasts of households, businesses, and financial market agents regarding future price movements. In turn, they influence key economic decisions related to wages, investment, consumption, and pricing (Coibion *et al.*, 2018). As a result, expectations serve as a crucial signal of the Central Bank's credibility and play a fundamental role in shaping short-term inflation dynamics (Mester, 2022). This study uses Forward Breakeven Inflation (FBEIs), which takes advantage of their high-frequency and allow inference of expected inflation between two points in time.

The application of the proposed methodology to Colombian data reveals that in 2006, 2010, 2014, 2017, 2018, and 2021, expectations were anchored, as reflected by the decrease in dependency between short- and long-term expectations. In contrast, during episodes of high inflation, such as those observed in 2008–2009, 2011, 2015–2016, and 2022–2023, dependency increased notably, indicating de-anchoring. These findings complement those of Gamba-Santamaría *et al.* (2016) and Iregui-Bohórquez *et al.* (2021), who evaluate the anchoring of inflation expectations based on surveys, and the study by Escorcia-Arana (2017), which examines the impact of de-anchoring on monetary policy in Colombia. Additionally, when the monetary policy stance was considered, a strong negative dependence was observed during contractionary episodes, while progressive interest rate reductions correlated with positive dependency.

After this introduction, this paper is structured as follows: The second section explores the motivation for the study and addresses the relevant literature. The third section describes the methodology employed to estimate the degree of de-anchoring in inflation expectations. The fourth section presents the data to which the proposed methodology is applied. The fifth section presents and analyzes the results. Finally, the sixth section summarizes the key findings, discusses their implications, and offers the study's conclusions.

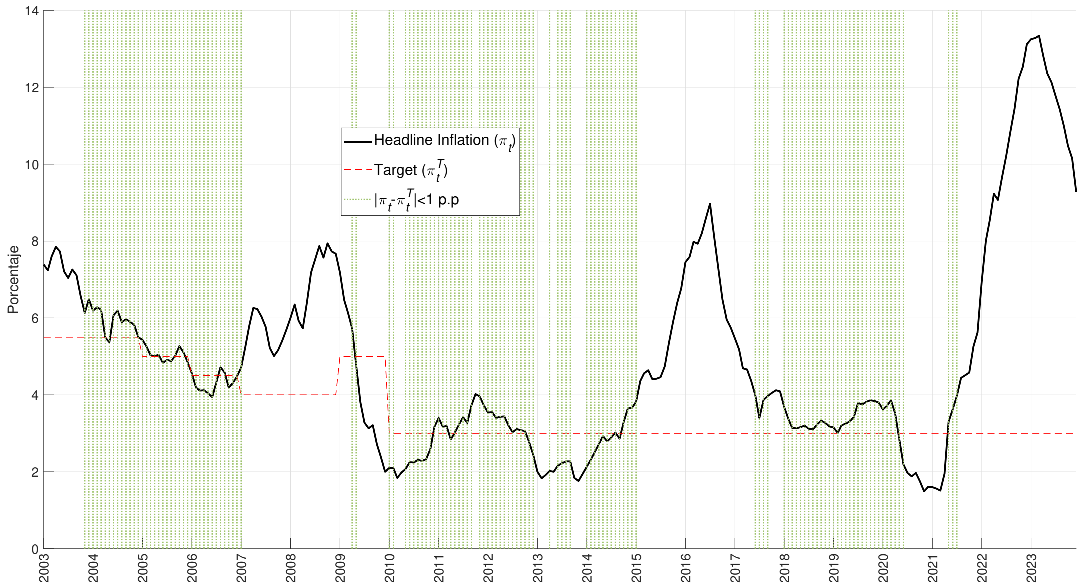
2 Motivation and Literature Review

In Colombia, the inflation targeting framework was implemented in 1999 as a means of stabilizing prices and promoting economic growth. Under this strategy, the inflation expectations of economic agents, along with the forecasts of the Central Bank, play a crucial role since they allow for the anticipation of inflationary pressures and the active adjustment of monetary policy. This approach aims to align actual inflation with a specific target, thereby strengthening the credibility of the Central Bank.

According to Dornbusch y Fischer (1993), Colombia's case is particularly relevant in the context of inflation. The country was considered an example of "moderate inflation par excellence" with inflation rates persistently around 20% between 1970 and the early 1990s. Following the Banco de la República's independence, established in the 1991 Constitution, and the adoption of the inflation targeting framework, inflation has fluctuated around ± 1 percentage point (p.p.) of the target and has remained in single digits for most of the 21st century (Figure 1).

To further investigate this topic, the study adopts the framework established by Goel y

Figure 1: Headline Inflation and Target Inflation



Source: Banco de la República

Tsatsaronis (2022), which divides the analysis of the degree of de-anchoring of inflation expectations into three research questions:

1. **What is the comovement between different horizons of expectations?** This approach is relevant because if the shocks faced by the economy similarly affect both short-term (ST) and long-term (LT) expectations, agents do not perceive that the shocks dissipate over time; rather, they consider them to be permanent in nature, indicating that expectations are de-anchored. On this issue, Antunes (2015) calculated the probability that LT expectations remain elevated, conditional on high values of ST expectations. Meanwhile, Gefang *et al.* (2008) estimated the degree of pass-through from ST to LT expectations using a mixture distribution model; Natoli y Sigalotti (2017a,b) proposed an indicator based on logistic regression that measures the probability that disinflationary shocks to ST expectations translate into significant declines in LT expectations.

2. **How far are inflation expectations from the long-term target?** This aspect analyzes the extent to which expectations deviate from the target inflation level. For example, Strohsal y Winkelmann (2015) estimated the anchor perceived by the market and its convergence strength relative to the inflation target. Strohsal *et al.* (2016)

explains inflation expectations based on past inflation, the inflation target, and previous expectations; Dash *et al.* (2020) evaluates how deviations in ST expectations from the target translate into deviations in LT expectations; Davis (2013) estimates a nested Phillips curve, considering the effect of past inflation on current inflation; and Apokoritis *et al.* (2019) calculates the impact of inflation surprises on household survey responses in the Netherlands.

3. **How much did expectations vary?** This line of research studies the probability distribution of the Dutch household survey according to socio-demographic characteristics, while Goel y Tsatsaronis (2022) measures the volatility of expectations as a fraction of the variability that their empirical model fails to explain.

This study aims to integrate the research approaches mentioned in questions (1) and (2), considering both deviations from the inflation target and the comovement between ST and LT horizons. In this sense, the work is similar to that of Dash *et al.* (2020), as it models deviations in expectations from the inflation target. However, unlike that study, instead of calculating the causal effect of ST expectations on LT expectations, it assesses the dependency between them using an approach similar to that proposed by Antunes (2015).[1]

3 Methodological Strategy

The methodology for estimating the degree of de-anchoring is described in detail below. First, inflation expectations are transformed into deviations from the inflation targets. Subsequently, their degree of dependency was evaluated by applying two statistical copulas structures.

In this context, Dash *et al.* (2020) analyze the pass-through of short-term (ST) to long-term (LT) inflation expectations of U.S. households using the following model:

$$(\pi^e_{t,LT} - \pi^T) = \alpha + \beta_t(\pi^e_{t,ST} - \pi^T) + \epsilon_t \tag{1}$$

where $\pi^e_{t,LT}$ represents LT inflation expectations in period t, $\pi^e_{t,ST}$ corresponds to ST inflation expectations formed in the same period, and π^T denotes the inflation target. The parameter

[1]Statistical dependence is defined as the probability that the occurrence of one event affects another. If two events A and B are independent, then $P(A|B) = P(A)$ and $P(B|A) = P(B)$.

β_t captures the degree of de-anchoring and varies over time.[2] However, this model has limitations in measuring the degree of de-anchoring when high-frequency and highly volatile expectation indicators are used. In the presence of transitory shocks, the estimator might suggest an increase in de-anchoring, potentially leading to overreaction by the Central Bank.

3.1 Short- and Long-Term Expectation Indicators

Considering the above, the following indicators are defined to calculate the weighted average of the deviations of expectations from the inflation target. These indicators assign a decreasing weight as the deviations move further back in time.[3] Their construction is detailed below:

$$\widehat{\mathsf{sti}}_t = \sum_{k=1}^{K} \frac{K-k+1}{K(K+1)/2} \left|\pi^e_{t-k+1,ST} - \bar{\pi}_{t-k+1}\right| \tag{2}$$

$$\widehat{\mathsf{lti}}_t = \sum_{k=1}^{K} \frac{K-k+1}{K(K+1)/2} \left|\pi^e_{t-k+1,LT} - \bar{\pi}_{t-k+1}\right| \tag{3}$$

Following the notation of Dash *et al.* (2020), $\pi^e_{t,ST}$ and $\pi^e_{t,LT}$ correspond to ST and LT inflation expectations, respectively, $\bar{\pi}_t$ represents the inflation target, and K denotes the number of periods considered in the calculation of the indicator. An increase in the values of the short-term ($\widehat{\mathsf{sti}}_t$) and long-term ($\widehat{\mathsf{lti}}_t$) indicators suggests that inflation expectations are persstiently further from the target.

In a scenario of complete credibility towards the Central Bank, where inflation expectations are perfectly anchored during the last K periods, the indicators $\widehat{\mathsf{sti}}t$ and $\widehat{\mathsf{lti}}t$ would take a value of 0. In the event of a transitory shock that deviates short- and long-term expectations from the target, the indicators would reflect these deviations in $\left|\pi^e_{t,st} - \bar{\pi}\right|$ and $\left|\pi^e_{t,lt} - \bar{\pi}\right|$ for period t. Since there are no deviations in the last K periods, the initial reaction of the indicators would be attenuated by past behavior. Additionally, as it is a transitory shock, its effect would disappear over time, which would be reflected in the convergence of the

[2]The value of β_t reflects the degree of de-anchoring of LT inflation expectations. A high β_t indicates greater de-anchoring and, consequently, a slower convergence of LT expectations towards the inflation target.

[3]Following the methodology of Martinez-Rivera y Hernandez-Bejarano (2012), the weighted sum of the last K expectations is calculated, i.e., the most recent data is assigned a weight of K, the previous one $K-1$, and so on until the first observation of the considered period, which is assigned a weight of 1. Each of these values is divided by $K(K+1)/2$, with K being the number of expectations considered for the calculation of the indicator

indicators towards 0, suggesting that expectations remain anchored. On the contrary, if the economy faces a permanent inflationary shock, the indicators would increase proportionally to the persstience of the shock, indicating a possible de-anchoring of expectations.

3.1.1 Degree of De-anchoring: Traditional Copulas

To estimate the degree of de-anchoring, Antunes (2015) analyzed the tail dependence of the distribution of $\pi^e_{t,ST}$ and $\pi^e_{t,LT}$.[4] By perform this calculation, it is essential to know the joint cumulative distribution functions $(G(X_1, X_2))$ corresponding to the marginal cumulative distribution functions $(F(X_1), F(X_2))$.[5] Formally:

$$G(X_1, X_2) = P(X_1 \leq x_1\ ,\ X_2 \leq x_2) \tag{4}$$

Intuitively, this function provides the probability that the realizations of both random variables are less than arguments x_1 and x_2. For example, $G(X_1 = 5, X_2 = 1)$ denote the probability of observing a value less than five in X_1 and one in X_2.

One way to estimate a joint cumulative function from two marginal cumulative distributions $(F(X_1), F(X_2))$ is to use Sklar's Theorem.[6]

$$G(X_1, X_2) = C_{X_1,X_2}(F(X_1), F(X_2)) \tag{5}$$

Where C_{X_1,X_2} corresponds to the copula function used to determine the joint cumulative distribution function of the two marginal cumulative distributions.[7] In other words, a copula is a function that connects two or more marginal cumulative distribution functions, allowing the construction of a joint cumulative distribution function. Function C_{X_1,X_2} can be expressed as

$$C_{X_1,X_2} = G(F^{-1}(X_1), F^{-1}(X_2)) \tag{6}$$

Where F^{-1} denotes the inverse of distribution functions $F(X_1)$ and $F(X_2)$. There are two requirements for using copulas: first, the variables modeled by the copula must be defined

[4] A more detailed explanation of conditional tail dependence is found in Appendix A.

[5] The marginal cumulative distribution of X_1 is defined as $F(X_1) = P(X_1 \leq x_1)$ and indicates the cumulative probability that the random variable X_1 is less than or equal to a certain value x_1 regardless of what happens in X_2. The same applies to $F(X_2)$.

[6] Antunes (2015) notes that a parametric function can be estimated for the distribution F, but highlights the issues of scale and domain in terms of the variables X_1 and X_2 in this procedure.

[7] It is possible to calculate a multivariate copula in cases of 3 or more variables.

in the interval $[0, 1]$; second, the marginal distributions of the variables must be uniform (Nelsen, 2006). When using a copula, the cumulative distribution functions of each variable are specified along with a function (copula) that links them. Thus, it is possible to separate the modeling of marginal distributions from the dependency structure between the two variables. See Appendix B for a detailed explanation of the copulas.

To measure the dependency between indicators $\widehat{\mathsf{sti}}_t$ and $\widehat{\mathsf{lti}}_t$, the copulas presented in Table 1 were used.[8] By performing this procedure, it is possible to estimate the degree of de-anchoring under different functional forms of dependency, emphasizing different quantiles of the distribution of the indicators, and allowing the capture of the positive or negative direction of the dependency.

Table 1: Range of Dependency Parameter and Direction for Different Copulas

Type of Copula	**Range of Dependency Parameter**	**Direction of Dependency**
Gaussian	$[-1, 1]$	Positive and negative
Clayton	$[0, \infty)$	Positive (in the left tail)
Frank	$(-\infty, \infty)$	Positive and negative
t-Student	$[-1, 1]$	Positive and negative
Gumbel	$[1, \infty)$	Positive (in the right tail)

3.1.2 Degree of De-anchoring: Hierarchical Copulas

An alternative strategy to traditional copulas is the use of hierarchical copulas or Vine Copulas.[9] This class of multivariate copulas allows for flexible modeling of the dependency between more than two random variables. Unlike traditional copulas, which simultaneously model the dependency between all variables, hierarchical copulas decompose the joint distribution into a series of bivariate copulas, facilitating their estimation and computational handling.

Vine Copulas are based on graphical representations known as vines that describe the relationships between variables (nodes). One category of these copulas is *C-Vine*, where a "central" node is selected that has a direct relationship with all other variables, and the rest

[8] For a more detailed explanation of types of copulas, see Patton (2009).
[9] The technical details can be found in Appendix B.

of the dependency relationships are constructed conditionally from that variable. The joint density function of d variables $X_1, X_2, \ldots, X_d$ using a *C-Vine* can be expressed as:

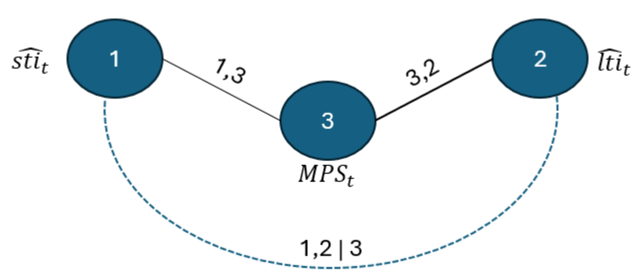
$$G(X_1, \ldots, X_d) = \prod_{i=1}^{d} F(X_i) \prod_{i=1}^{d-1} \prod_{j=i+1}^{d} C_{i,j|1,\ldots,i-1}(F(X_i|X_1, \ldots, X_{i-1}), F(X_j|X_1, \ldots, X_{i-1})) \quad (7)$$

where $c_{i,j|1,\ldots,i-1}$ are the conditional copulas that capture the dependency between variables X_i and X_j, given other variables.

This technique has been used to study the bubbles risk among 11 assets during COVID-19, identifying $S\&P$ 500 and gold as key nodes in the transmission of this risk (Yao *et al.*, 2023). On the other hand, Hamza *et al.* (2024) explore the strong dependency between 10-year US futures and other assets in the context of high inflation and the Russia-Ukraine war. This study takes advantage of the flexibility of the methodology to incorporate the effects of the contemporary monetary policy stance into the dependency analysis between $\widehat{\mathsf{sti}}_t$ and $\widehat{\mathsf{lti}}_t$. This framework offers valuable insights into the transmission mechanisms of monetary policy and its influence on expectations dynamics.

Figure 2 shows the proposed modeling structure using this type of copula, where node 1 corresponds to $\widehat{\mathsf{sti}}_t$, node 2 to $\widehat{\mathsf{lti}}_t$, and node 3 to the monetary policy stance at time t (MPS_t). The algorithm is developed as follows: i. the dependency between nodes 1 and 3 is calculated; ii. the dependency between nodes 3 and 2 is calculated; and iii. The dependency between nodes 1 and 2, conditioned on node 3, is calculated.

Figure 2: Hierarchical copula structure in a chain.



The copula's functional form is selected at each step using the Modified Bayesian Information Criterion for Vine Copulas (MBICv), following Nagler *et al.* (2019).

4 Inflation Expectations in Colombia

Inflation expectations in Colombia have been analyzed from various perspectives. Rincón-Torres *et al.* (2023) explore the rationality and degree of disagreement among inflation expectations. Iregui *et al.* (2021) study the efficiency of expectation revisions against observed inflation. Hernández-Montes *et al.* (2022) evaluate the forecasting ability of business expectations. Romero-Torres *et al.* (2023) examine whether the relationship between expectations and inflation in Colombia varies depending on how these expectations are measured, among other relevant studies.

These expectations can be measured through surveys, derived from financial market instruments, or from social media using Natural Language Processing techniques. Surveys inquire about ST forecast horizons and are usually low-frequency (monthly or quarterly).[10] On the other hand, expectations derived from the financial market, called break-even inflation (BEIs), are calculated from the nominal and real interest rates of sovereign debt securities, are high-frequency (daily), and have a broader forecast horizon corresponding to the term structure of the yield curve. Based on BEIs, it is possible to construct Forward Breakeven Inflation (FBEIs), which allow inference of expected inflation between two points in time and are detailed in Appendix C. Lastly, there is an alternative for measuring inflation expectations in real-time using social media. In Colombia, Muñoz-Martínez *et al.* (2025) adopted a similar approach.

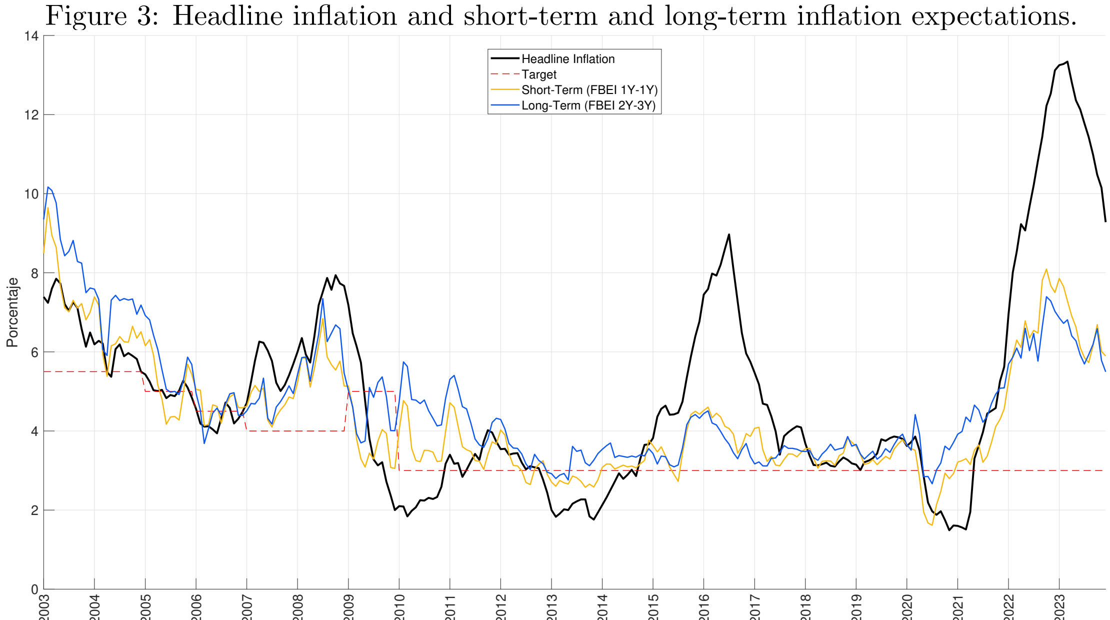
This study uses implicit inflation expectations from the financial market, because of their frequency and availability at different maturities. In particular, FBEI 1A-1A and FBEI 2A-3A are considered as short- and long-term expectations, respectively. However, the methodology is flexible for using other horizons. FBEIs reflect the aggregate market view of future inflation in specific range of time, as they incorporate the assessment of various economic agents, from financial institutions to individual investors. These are interpreted as the average expected inflation over a given period.[11]

[10] In Colombia, the Banco de la República conducts the Monthly Expectations Survey (EME), the Monthly Economic Business Survey (EMEE), and the Quarterly Expectations Survey (ETE). Additionally, there is the Financial Opinion Survey (EOF) conducted by Fedesarrollo and the Consensus Economics survey.

[11] For example, a FBEI 2A-3A rate of 5% would indicate that the inflation expectation for the next three years starting two year after the time when is created is, on average, 5%

Figure 3 shows the evolution of short- and long-term expectations. It also compares observed inflation with the target inflation ($\bar{\pi}_t$) set by the Banco de la República for the period from January 2003 to December 2023. Three inflationary episodes are observed: i. between 2007 and 2008 explained by oil price shocks; ii. the episode between 2015 and 2016 caused by the El Niño phenomenon, the truckers' strike, and decreases in oil prices (Bejarano-Salcedo *et al.*, 2020); and iii. the most recent inflationary period from 2022 to 2023 due to supply chain shocks, global uncertainty, and pent-up demand following COVID-19. Additionally, co-movement between inflation expectations and inflation dynamics is observed. The details of the data used are listed in Table 2.

Figure 3: Headline inflation and short-term and long-term inflation expectations.



Source: Banco de la República

Table 2: Data for the study period between 2003M1 - 2023M12

Variable	**Frequency**
Annual Total Inflation	Monthly
Annual LT Target	Monthly
FBEI 1Y-1Y (ST)	Daily
FBEI 2Y-3Y (LT)	Daily

Table 3 illustrates that ST expectations are generally higher during periods of high inflation (Panel A) than when inflation remains within $\pm 1\%$ of the target level (Panel B). In Panel

A, it is notable that during the 2022-2023 period, there is a greater difference between the average level of LT and ST expectations from the target, which could suggest a more persistent inflationary shock compared to the periods 2007-2008 and 2015-2016. Furthermore, ST expectations exhibit greater volatility than LT expectations in the three periods analyzed. In Panel B, ST expectations are, on average, close to the target; however, the distance from the target in LT expectations is slightly greater. Additionaly, the volatility of LT expectations is higher than ST in the episodes of 2003-2006 and 2010-2014.

Table 3: Descriptive statistics of inflation expectations during inflationary episodes (Panel A) and inflation within ±1 p.p of the target (Panel B).

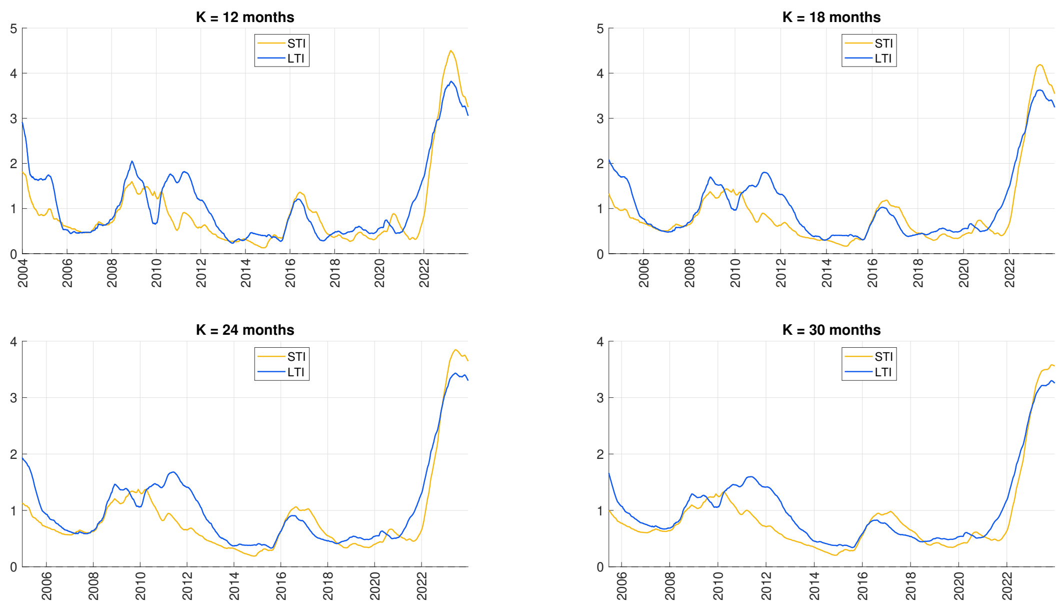
		FBEI 1Y - 1Y (ST)	**FBEI 2Y - 3Y (LT)**
		Panel A	
2007/01-2008/12	Mean	5.25%	5.49%
	Variance	0.73	0.67
2015/01-2016/07	Mean	3.89%	3.80%
	Variance	0.60	0.52
2022/01-2023/12	Mean	6.66%	6.31%
	Variance	0.88	0.59
		Panel B	
2003/11-2006/12	Mean	5.60%	6.10%
	Variance	1.00	1.20
2010/05-2014/12	Mean	3.25%	3.78%
	Variance	0.50	0.68
2017/01-2020/06	Mean	3.34%	3.47%
	Variance	0.43	0.31

Inflation expectations from the financial market and the proposed expectation indicators ($\widehat{\mathsf{sti}}_t$ and $\widehat{\mathsf{lti}}_t$) are characterized by high volatility. An alternative to eliminate spurious dependence caused by the persistence and heteroscedasticity of the series is to apply an AR(1) model to capture their conditional mean and a GARCH(1,1) model for the variance, following the proposal by (Cherubini *et al.*, 2016).

5 Degree of de-anchoring in Colombia

This section presents the main findings of applying the proposed methodology to inflation expectations derived from the Colombian financial market. Figure 4 illustrates the dynamics of $\widehat{\mathsf{sti}}_t$ and $\widehat{\mathsf{lti}}_t$ correspond to Equations 2 and 3, respectively. Indicator estimation is performed recursively depending on the number of periods considered (K = 12, 18, 24, and 30 months).[12]

Figure 4: Sensitivity of the indicator to different values of K



The smaller the value of K, the greater is the sensitivity of the indicators to recent inflationary shocks. For example, throughout 2023, the indicators reflect the peaks of inflation and their expectations between 2022 and 2023. In contrast, when K takes a value of 30 months, the magnitude of the indicator in 2023 is reduced because of the influence of historical values; in 2021, the expectations remained relatively controlled, which moderates the magnitude of the indicator.

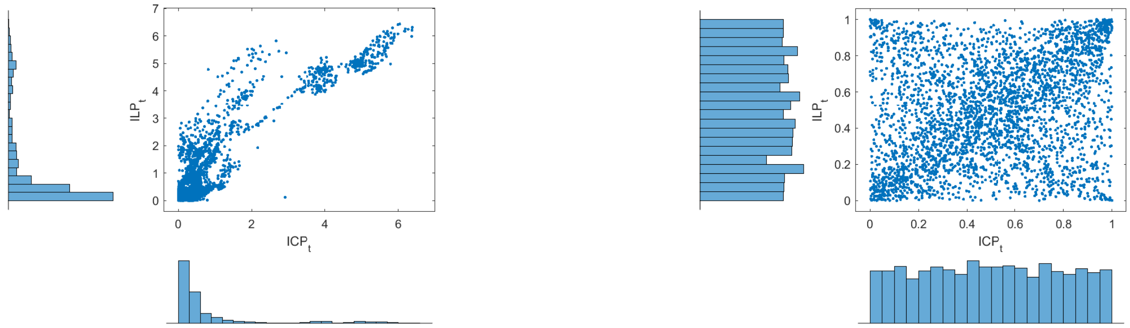
This section presents the indicator results for K = 12 months. The results for the other values of K are found in Appendix D. Figure 5 compares the empirical distributions of the observed indicators (a) and the pseudo-observations obtained by applying the Probability Integral Transformation to indicators (b) (Czado, 2019).[13] This ensures that the data are

[12]The value of K is chosen *ad-hoc*.

[13]This principle states that if we have a random variable (X) with a cumulative distribution function

bounded in the interval $[0, 1]$ and that their marginal distributions are uniform.

Figure 5: Comparison of raw vs transformed indicators



(a) Observed indicators. (b) Pseudo-observed indicators.

The estimates were made using rolling windows of 250 days (one year) and were divided into two parts. Panel (a) of Figure 6 presents the t-Student, Gaussian, and Frank copulas, which measure the positive and negative dependencies between variables. Panel (b) shows copulas that measure only positive dependency.[14] To facilitate the economic interpretation of the results, the following transformation is applied to the measures:

$$Estimation_i = \frac{Measure_i}{max(|Measure_i|)} \qquad where \;\; i = Frank, Gumbel, Clayton \tag{8}$$

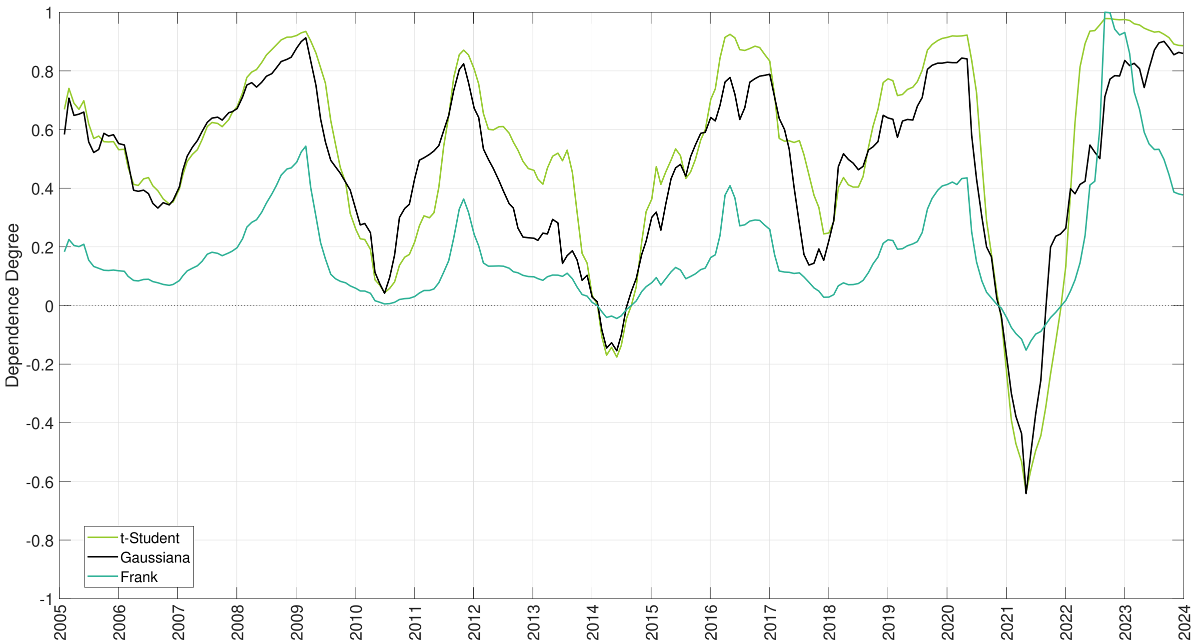
In the case of the t-Student and Gaussian copulas, no transformation is performed, as their results are bounded between -1 and 1. In episodes where the estimates are equal to 0, the indicators are independent; on the other hand, estimates of 1 and -1 suggest complete dependency, where the sign determines the direction.

In general, various measures show an increase in the degree of dependency during the inflationary episodes of 2008-2009, 2012, 2015-2016, and 2022-2023. Similarly expectations and inflation were lower than the target in 2013 and 2020. This indicates that both types of de-anchoring occur below and above the target. In contrast, a decrease in the degree of dependency was observed in 2007, 2014, 2017, and 2021, as well as a decreasing trend in 2023.

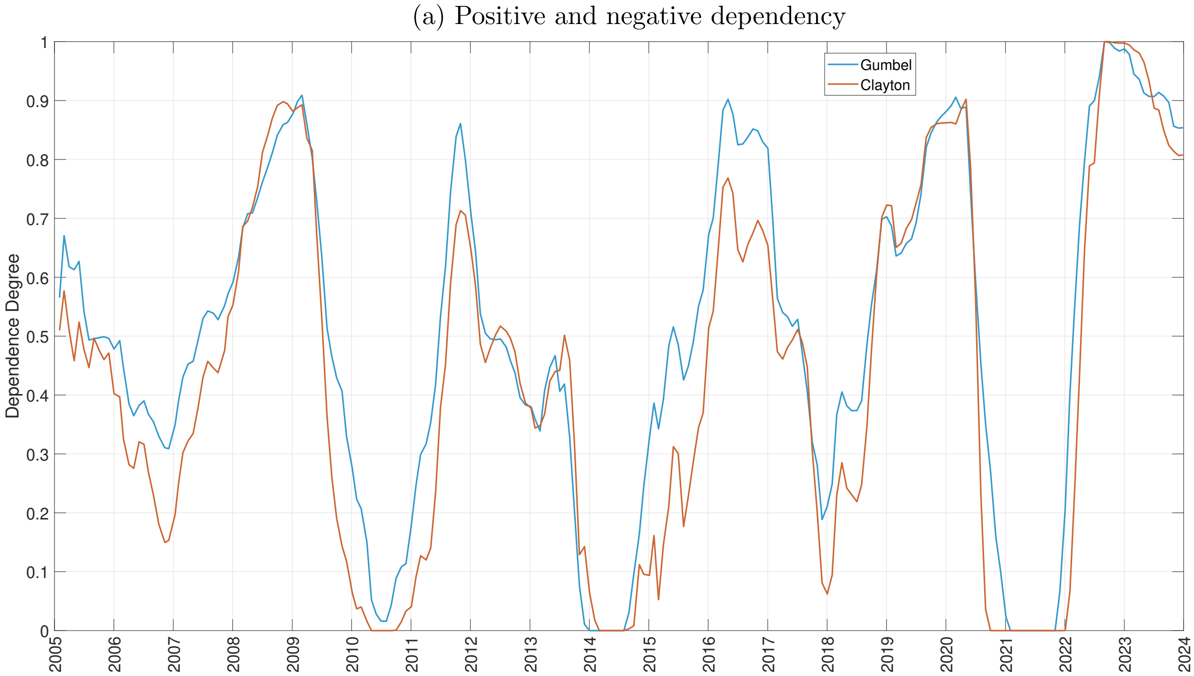
$(F(x))$, then the transformed variable (U = F(X)) follows a uniform distribution in the interval $[0, 1]$. In the R program, the function *pseudo_obs* facilitates this transformation by calculating empirical quantiles or relative positions of the data in the sample, assigning each observation a value between 0 and 1. Applying the cumulative distribution function of (X) to its values, we obtain values in the range of 0 to 1, thus achieving a uniform distribution

[14] See Table 1 for more details on the range of dependency parameters.

Figure 6: Estimation of dependency degree using traditional copulas



(a) Positive and negative dependency

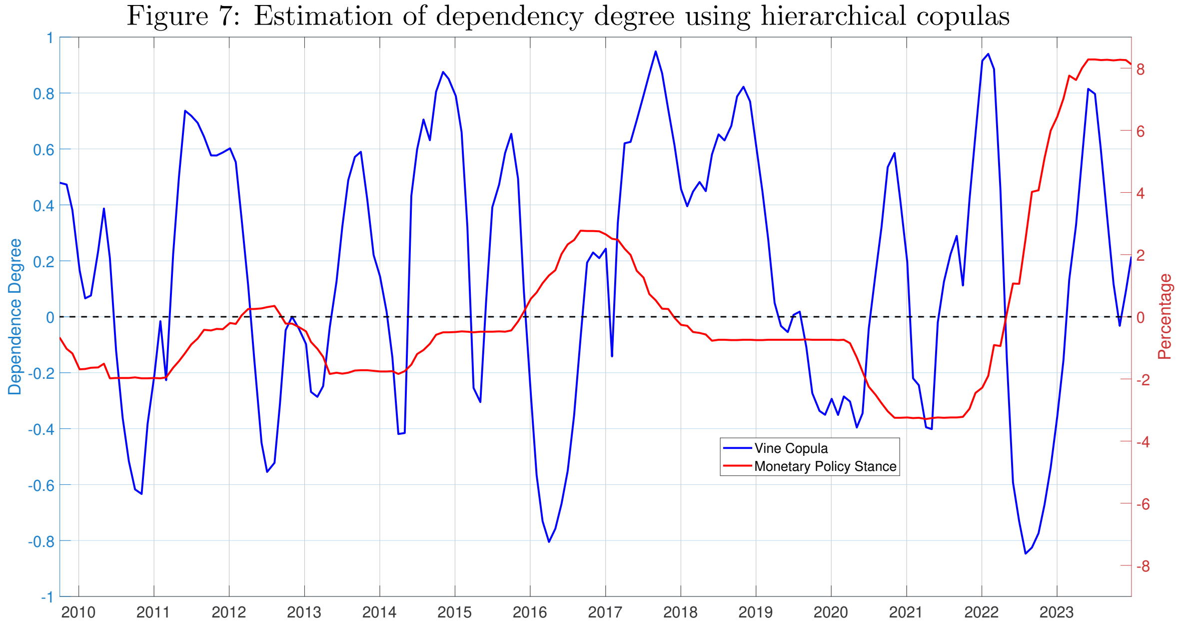


(b) Positive dependency

To complement this analysis, the interaction between expectations and the monetary policy stance is considered, defined as the difference between the Interbank Rate (IB) and the interest rate consistent with GDP at its equilibrium level and inflation at the 3% target. The latter, called the nominal neutral interest rate, is approximately 5% in a steady state, following González-Gómez *et al.* (2020).[15] As shown in Figure 2, the hierarchical copula models the dependence of each indicator on monetary policy stance. Subsequently, the dependency between the indicators conditioned on this stance is evaluated.

The intuition behind this exercise is that monetary policies should exhibit a significant degree of dependence on inflation expectations. When the economy is in a steady state with inflation aligned with the target, monetary policy should adopt a more contractionary (expansionary) stance in response to inflationary (disinflationary) pressures to counteract these shocks. Initially, the dependence should be positive. However, as monetary policy transmission channels begin to influence inflation expectations, the dependence should become negative, reflecting the process of expectation anchoring. Figure 7 presents the results of the hierarchical copula estimation, illustrating this dynamic:

Figure 7: Estimation of dependency degree using hierarchical copulas



The analysis reveals a high degree of dependency between the indicators, conditioned on the monetary policy stance, with a dependency pattern resembling a cyclical structure. Notably, when monetary policy began to adjust interest rates—either increasing or decreasing them—the dependency between the indicators was significantly affected. For instance, in 2016 and 2022, when monetary policy adopted a contractionary stance, a strong negative dependency

[15]Monetary policy is considered expansionary (contractionary) when the policy interest rate is below (above) the nominal neutral interest rate. Appendix E presents the stance of the monetary policy according to available data

between expectations was observed. This indicates that while short-term inflation expectations ($\widehat{\mathsf{sti}}_t$) continued to deviate from the target, long-term inflation expectations ($\widehat{\mathsf{lti}}_t$) remained closer to the target, signaling strong credibility in the monetary policy framework.

Conversely, in 2017 and 2020, characterized by a progressive reduction in interest rates, the estimation suggests a positive dependence between the indicators, reflecting a comovement in the same direction. During periods when interest rates remained stable, the dependency between indicators was driven solely by their intrinsic relationship.

Appendix D presents robustness checks using indicators for 18, 24, and 30 months. The results are broadly consistent with the main analysis, further supporting our findings.

6 Conclusions

The expectations channel is fundamental to the effectiveness of monetary policy as it influences the behavior of economic agents. Through this channel, monetary policy decisions, such as changes in the interest rate, affect future inflation expectations and shape decisions on consumption, investment, and the pricing of goods and services. When expectations are anchored around the inflation target set by the central bank, the credibility of the monetary authority is reinforced, allowing monetary policy decisions to be more effective and less costly in terms of economic activity. Conversely, if expectations become de-anchored, economic agents may expect inflationary pressures to persist over time, reducing the effectiveness of monetary policy measures and increasing the need for more aggressive adjustments, which, in turn, lead to higher costs for economic activity.

In this context, policymakers closely monitor the dynamics of expectations relative to the target and its co-movements to estimate the degree of de-anchoring. They assess whether inflationary shocks similarly affect short-term (ST) and long-term (LT) inflation expectations; if both horizons of expectations consistently deviate from the target, they are considered de-anchored.

This study proposes a methodology that integrates both approaches, analyzing the deviations of short-term (ST) and long-term (LT) expectations from the target and their dependence over time. To this end, we develop ST and LT indicators that consider both contemporary and historical deviations and recognize the relevance of the past. Subsequently, we estimate the dependency between both indicators in rolling windows of one year, using two statistical copula structures: traditional copulas, which allow the evaluation of tail and average dependency of the distribution, and a hierarchical structure that measures the dependency between ST and LT expectations, conditioned on the monetary policy stance.[16]

The empirical application to Colombia's FBEIs reveals a significant increase in the dependency between ST and LT expectations during the inflationary episodes of 2008–2009, 2015–2016, and 2022–2023, suggesting an increase in the degree of de-anchoring. A similar pattern is observed in 2013 and 2020, when inflation and expectations were below the target. On the other hand, a decrease in dependency is detected in the years 2006, 2010, 2014, 2017, and 2021, as well as a declining trend toward the end of 2023.

When the dependency between expectations is conditioned on the monetary policy stance, the results confirm a strong dependency—both positive and negative—throughout the analysis period. This suggests that the monetary policy stance is closely synchronized with the evolution of expectations, reflecting consistent interaction in both high- and low-inflation contexts. Notably, when monetary policy began to adjust interest rates—either increasing or decreasing them—the dependency between the indicators was significantly affected. For instance, in 2016 and 2022, during contractionary monetary policy periods, a strong negative dependency between expectations was observed. This indicates that while short-term inflation expectations ($\widehat{\mathsf{sti}}_t$) continued to deviate from the target, long-term inflation expectations ($\widehat{\mathsf{lti}}_t$) remained closer to the target, signaling strong credibility in the monetary policy framework. Conversely, in 2017 and 2020, which were characterized by progressive interest rate reductions, the estimation suggests a positive dependency, indicating a co-movement in the same direction. During periods when interest rates remained stable, the dependency between indicators was driven solely by their intrinsic relationship.

In conclusion, this study provides a methodological tool that expands the range of key elements for analyzing inflation expectations derived from the financial market. The findings

[16] It is important to note that both exercises are not intended to establish causality. The copula framework was used to characterize the dependence structure, allowing for non-linearities.

highlight the importance of examining both deviations from the inflation target and the interdependence between short- and long-term expectations. The results underscore periods of increased de-anchoring, suggesting that greater efforts to communicate the Central Bank's commitment to meeting the inflation target may be warranted. Additionally, the estimates enable the evaluation of monetary policy effectiveness and facilitate real-time monitoring, leveraging the high frequency of data.

For future work, it would be useful to leverage vine copulas to incorporate variables such as liquidity measures and inflation risk premia, among others, to complement dependency analysis. Based on this new set of variables, alternative dependency structures can be explored using D-Vine copulas.

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A Conditional Tail Dependence

Conditional tail dependence is defined as:

$$\lambda_U = \lim_{k \to 1} Pr(Y > y_k \mid X > x_k) \tag{A1}$$

$$\lambda_L = \lim_{k \to 0} Pr(Y \leq y_k \mid X \leq x_k) \tag{A2}$$

where λ_U and λ_L represent the upper and lower tails of the joint cumulative distribution, X and Y represent two random variables, and y_k and x_k represent the values of the distribution associated with quantile k. In the case of the upper tail, the relevant quantile k tends to the maximum. Intuitively, λ_U measures the asymptotic probability of having high realizations in variable Y, conditioned on high observed realizations for variable X. Similarly for λ_L.

Antunes (2015) studies the revisions of ST and LT expectations by performing the following calculation:

$$\Delta X = X_t - X_{t-1} \quad \text{(A3)} \qquad\qquad \Delta Y = Y_t - Y_{t-1} \quad \text{(A4)}$$

where ΔY and ΔX represent the changes in ST and LT expectations, respectively. After performing this calculation, the heteroscedasticity issues of the series are removed using a GARCH(1,1) model, and the tail dependence of the distributions is calculated following equations (A1) and (A2) to focus the analysis on episodes where the most relevant shocks in the economy materialize.

B Statistical Copulas

Copulas allow representing a joint distribution of correlated random variables from their individual marginal distributions. Formally, a copula is a function that links univariate marginal distributions to construct a multivariate distribution, thus capturing the dependency structure between variables.

Given a pair of random variables X and Y, with marginal distribution functions $F_X(x)$ and $F_Y(y)$, the joint distribution function $F_{X,Y}(x,y)$ can be expressed in terms of a copula $C(u,v)$, where $u = F_X(x)$ and $v = F_Y(y)$. The relationship between the joint distribution and the copula is:

$$F_{X,Y}(x,y) = C_{x,y}(F_X(x), F_Y(y)), \tag{B1}$$

where the copula C specifically describes the dependency structure between the two variables. If $C(x,y) = x \cdot y$, then the variables X and Y are independent; in other cases, the copula captures different dependency structures. Below are some common copulas. For a more exhaustive discussion, see Nelsen (2006) and Patton (2009).

Gumbel Copula

The Gumbel copula is used to model dependencies in the upper tails of the distribution. It is an asymmetric copula, allowing it to capture situations where one variable exhibits greater dependency than the other. Its formula is:

$$C(x,y) = \exp\left(-\left[(-\ln x)^\theta + (-\ln y)^\theta\right]^{1/\theta}\right) \tag{B2}$$

where $\theta \geq 1$ is the dependency parameter. When $\theta = 1$, the variables are independent, and as $\theta \to \infty$, the dependency approaches being perfect.

Clayton Copula

The Clayton copula is particularly useful for modeling dependencies in the lower tail, being suitable in contexts where both variables tend to exhibit low values simultaneously. The functional form of the Clayton copula is:

$$C(x,y) = \left[\max\left(x^{-\theta} + y^{-\theta} - 1, 0\right)\right]^{-1/\theta} \tag{B3}$$

where the dependency parameter $\theta > 0$ measures the intensity of the dependency in the lower tail. Large values of θ indicate stronger dependency in that tail.

Gaussian Copula

The Gaussian copula is derived from the multivariate normal distribution. If the random variables X and Y follow a standard normal distribution, their copula is given by:

$$C(x,y) = \Phi_\rho(\Phi^{-1}(x), \Phi^{-1}(y)) \tag{B4}$$

where Φ^{-1} is the inverse function of the standard normal distribution and Φ_ρ represents the bivariate joint distribution function with a correlation coefficient ρ. This copula is symmetric and does not exhibit tail dependence, making it suitable for modeling moderate linear dependencies.

t-Student Copula

The t-Student copula is an extension of the Gaussian copula, but with the advantage of capturing dependencies in both tails of the distribution, which is relevant in contexts where the variables can exhibit extreme values jointly. The function of this copula is:

$$C(x,y) = t_{\nu,\rho}(t_\nu^{-1}(x), t_\nu^{-1}(y)) \tag{B5}$$

where t_ν^{-1} is the inverse function of the univariate t-Student distribution with ν degrees of freedom, and $t_{\nu,\rho}$ is the bivariate joint distribution function of the t-Student with correlation coefficient ρ and ν degrees of freedom.

The main advantage of the t-Student copula lies in its ability to model tail dependence, both in the upper and lower tails. The smaller the value of ν, the greater the tail dependence. This makes the t-Student copula especially useful in financial risk applications, where joint extreme events are important to model.

Vine Copulas

Vine Copulas are multivariate copulas that model the dependency between multiple random variables flexibly, decomposing the joint distribution into bivariate copulas to facilitate their estimation and computational handling. They use graphical representations called *vines*, which show the hierarchical relationships between the variables. They can be divided into two types:

C-Vine

In a *C-Vine*, a 'central' variable is selected that has a direct relationship with all other variables, and the rest of the dependency relationships are constructed conditionally from that variable. The joint density function of d variables $X_1, X_2, \ldots, X_d$ using a *C-Vine* can be expressed as:

$$f(x_1, \ldots, x_d) = \prod_{i=1}^{d} f(x_i) \prod_{i=1}^{d-1} \prod_{j=i+1}^{d} c_{i,j|1,\ldots,i-1}(F(x_i|x_1, \ldots, x_{i-1}), F(x_j|x_1, \ldots, x_{i-1})) \quad \text{(B6)}$$

where $c_{i,j|1,\ldots,i-1}$ are the conditional copulas that capture the dependency between the variables X_i and X_j, given other variables.

D-Vine

In a *D-Vine*, the dependency relationships between the variables are modeled as a sequence of bivariate copulas. In this case, the joint density function is decomposed in terms of conditional bivariate copulas that link each pair of variables. The joint density of d variables using a *D-Vine* is expressed as:

$$f(x_1, \ldots, x_d) = \prod_{i=1}^{d} f(x_i) \prod_{k=1}^{d-1} \prod_{i=1}^{d-k} c_{i,i+k|1,\ldots,i-1}(F(x_i|x_1, \ldots, x_{i-1}), F(x_{i+k}|x_1, \ldots, x_{i-1})) \quad \text{(B7)}$$

This approach allows modeling complex conditional dependencies between pairs of variables.

C Construction of Inflation Expectations Derived from the Financial Market

Breakeven inflations (BEIs), or implied inflation rate, refer to the difference between the yield of a nominal bond and the yield of an inflation-indexed bond with the same maturity. This rate represents the inflation that investors expect over the bond's horizon, such that if actual inflation equals the BEI rate, investors will receive comparable returns on both nominal and real (or indexed) bonds.

They are constructed as follows:

$$\pi_t^{e,m} = \frac{1 + i_t^m}{1 + r_t^m} - 1 \tag{C1}$$

where i_t^m and r_t^m correspond to the yields of a nominal bond and a real bond, respectively. These bonds must have the same credit quality and common maturity.

Forward Break-even Inflations (FBEIs) represent the expected inflation rate for a specific future period, calculated from the implicit expectations in the BEI rates of bonds with different maturities. These rates allow inferring the expected inflation between two points in time, using bonds with different maturities.

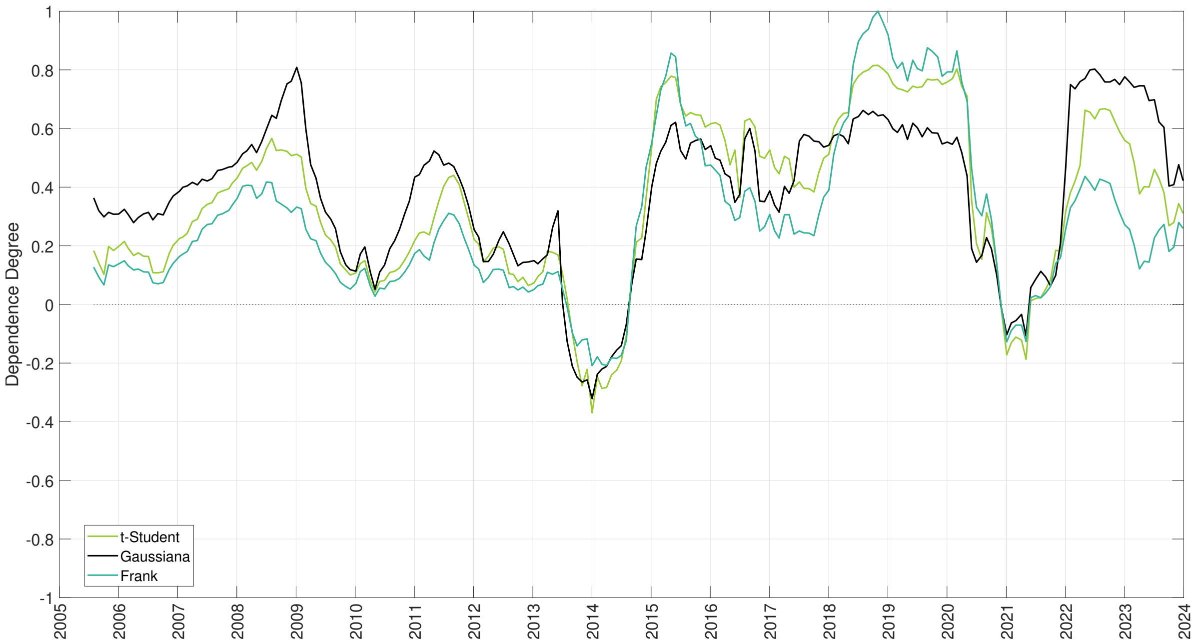
The FBEI is constructed as follows:

$$\pi_t^{e,(m,n)} = \frac{(1 + i_t^n)}{(1 + i_t^m)} \cdot \frac{(1 + r_t^m)}{(1 + r_t^n)} - 1 \tag{C2}$$

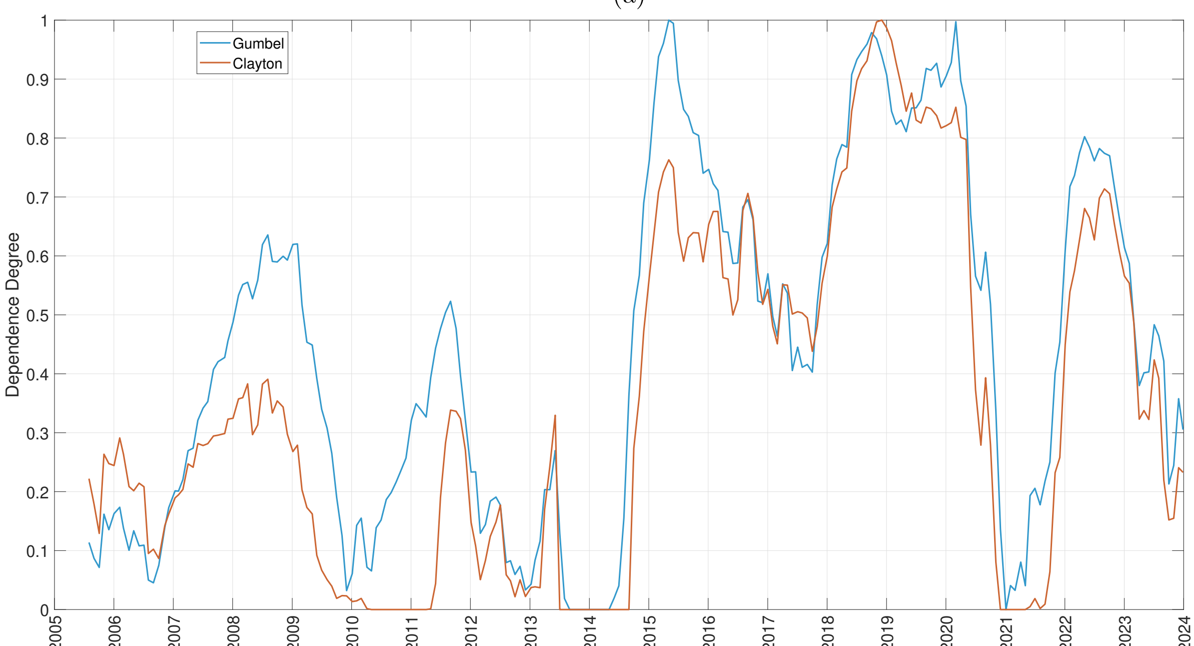
where i_t^m and i_t^n represent the nominal yields of bonds with maturities m and n, respectively, and r_t^m and r_t^n the real yields of bonds with the same maturities. This formula provides the expected inflation between periods m and n, allowing investors to evaluate their inflation expectations for specific future periods, conditioned on the common maturity and comparable credit quality of both bonds.

D Estimation Results for Different Horizons of $\widehat{sti}$ and $\widehat{lti}$

Figure section.1: Estimation of Dependency Degree Using Traditional Copulas. **K=18 months**



(a)



(b)

Figure section.2: Estimation of Dependency Degree Using Hierarchical Copulas. **K=18 months**

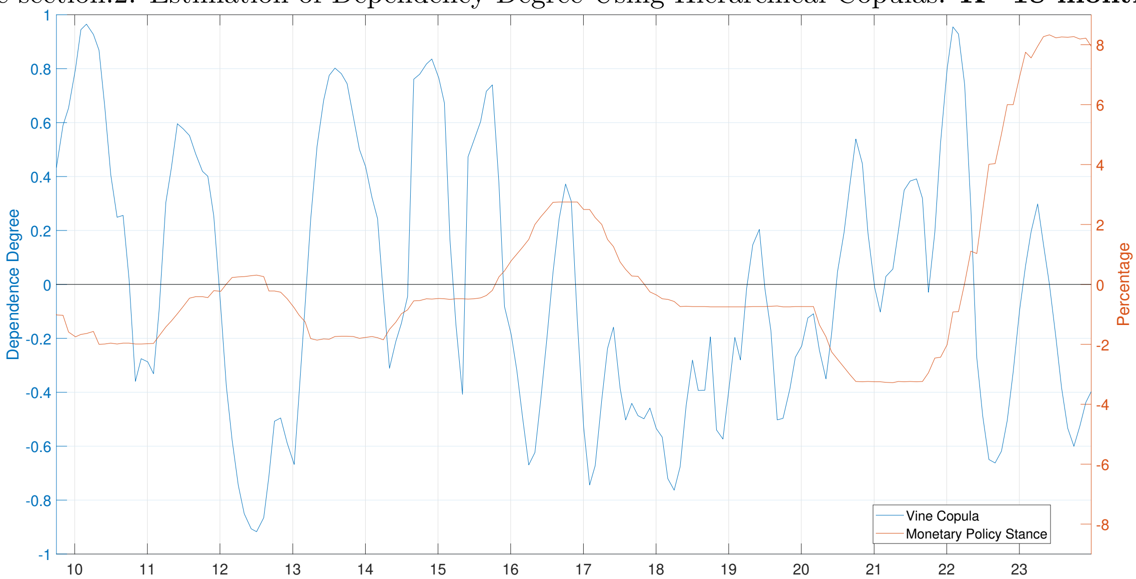
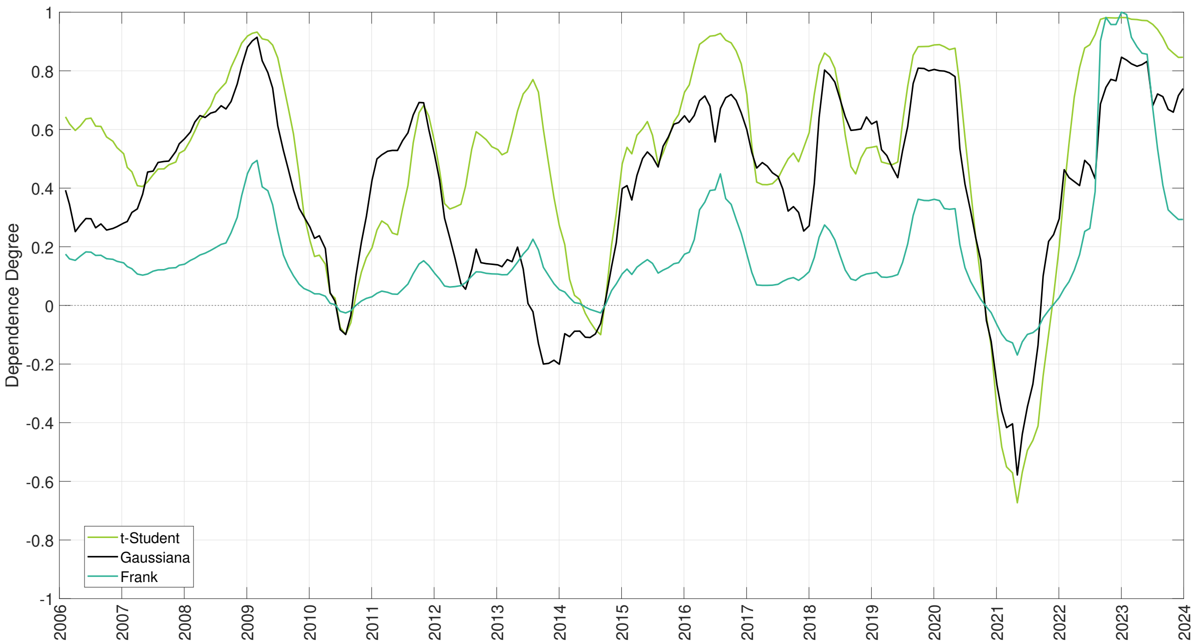
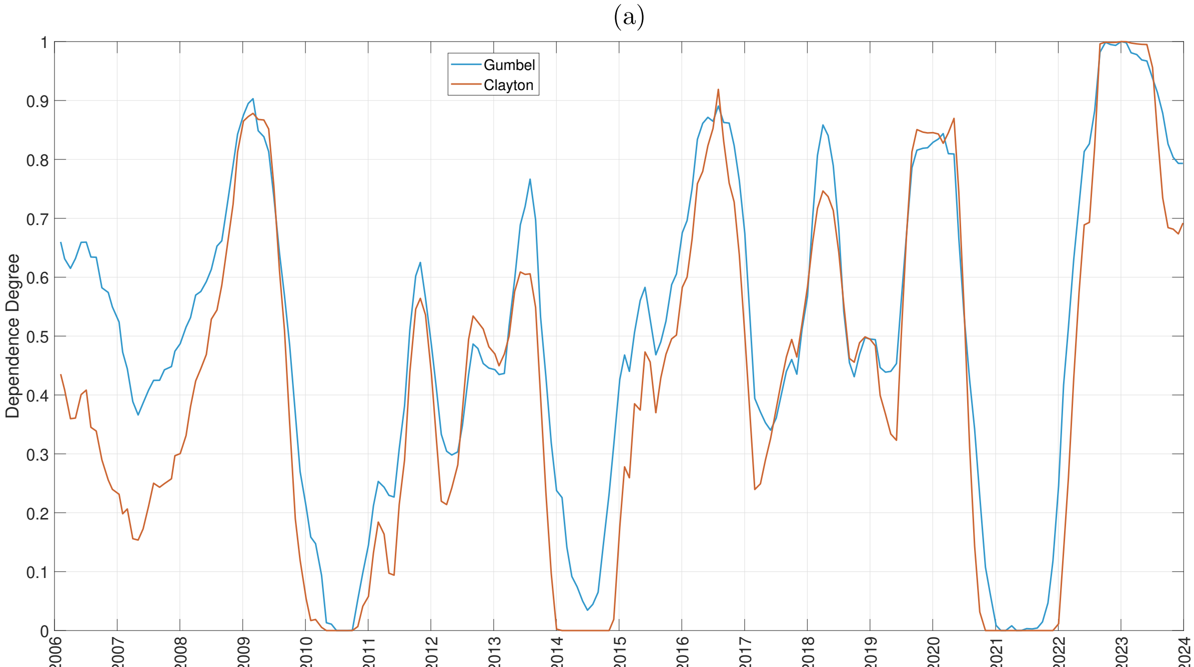


Figure section.3: Estimation of Dependency Degree Using Traditional Copulas. **K=24 months**



(a)



(b)

Figure section.4: Estimation of Dependency Degree Using Hierarchical Copulas. **K=24 months**

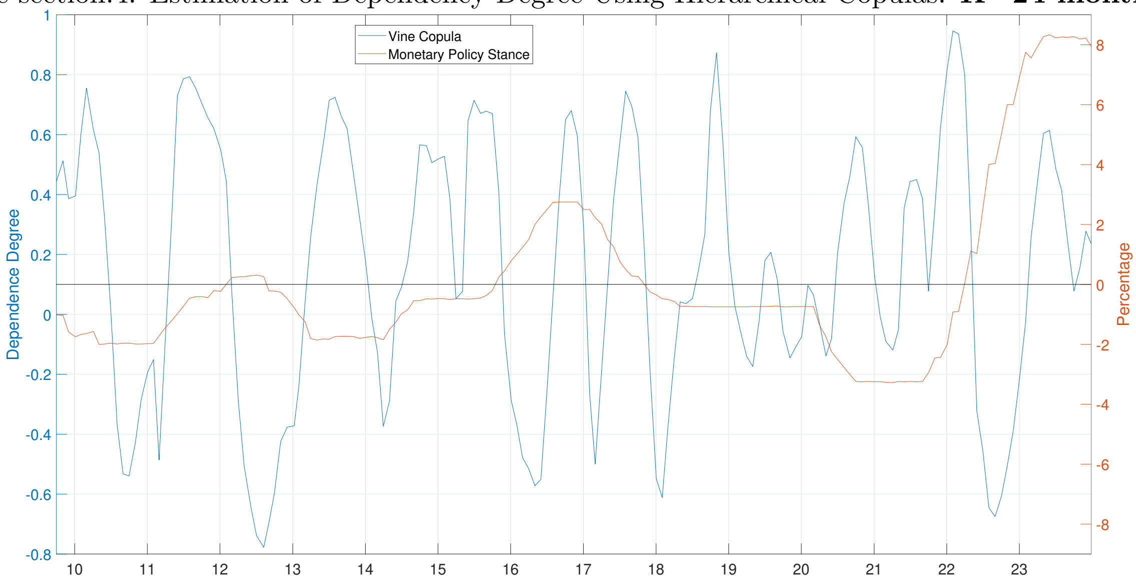
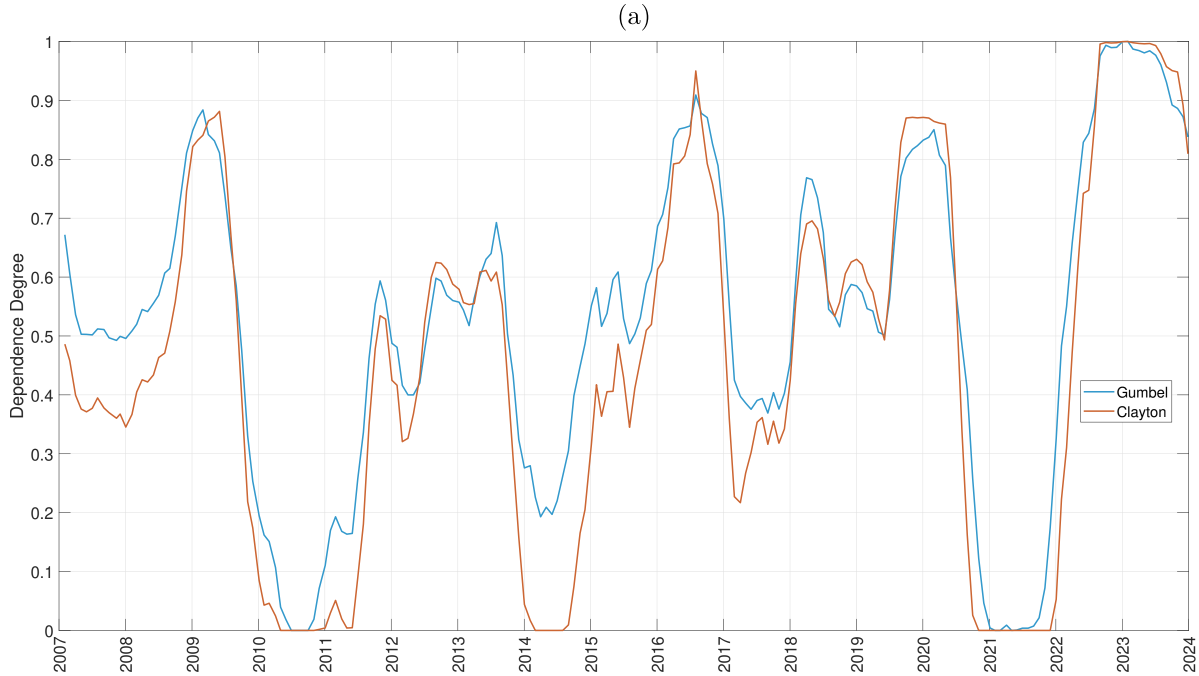


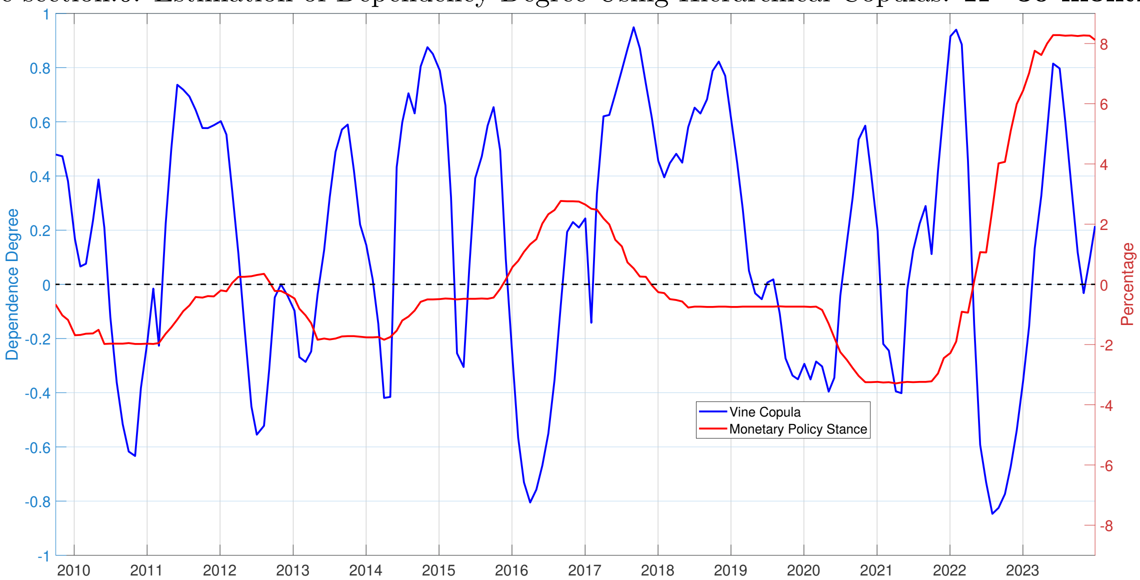
Figure section.5: Estimation of Dependency Degree Using Traditional Copulas. **K=30 months**

(a)



(b)

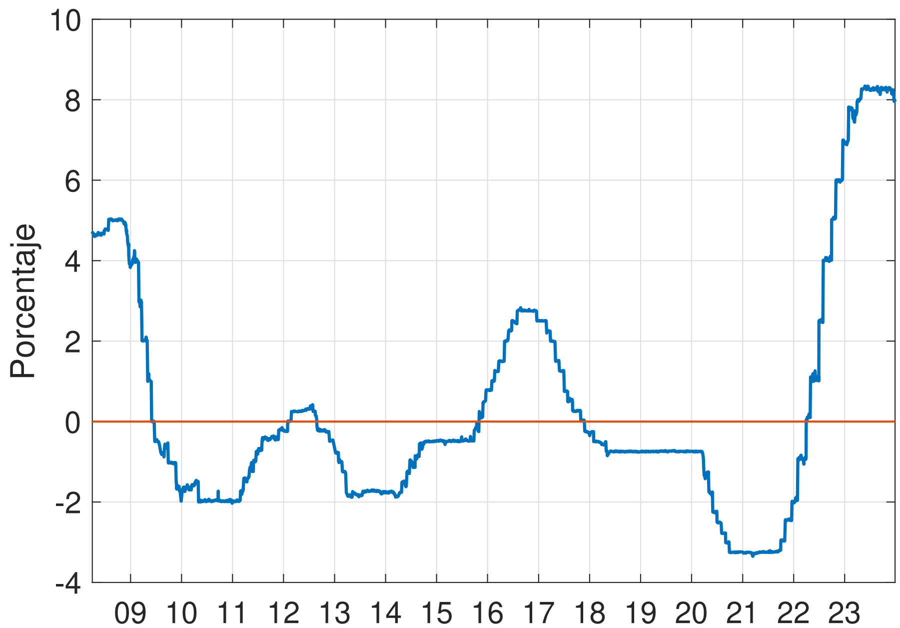
Figure section.6: Estimation of Dependency Degree Using Hierarchical Copulas. **K=30 months**



E Monetary Policy Stance

The data for the Interbank Interest Rate is available from April 2, 2008. The following figure shows its dynamics:

Figure section.1: Monetary Policy Stance.



Negative (positive) values indicate an expansionary (contractionary) stance.