A New Keynesian DSGE Model for Monetary Policy and Forecasting in Colombia

Óscar I. Ávila-Montealegre* Fredy A. Castañeda-Valdés† José D. Pulido-Pescador‡

Juan A. Rincón-Galvis§

Macroeconomic Modeling Department¶
Banco de la República

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Abstract

We develop a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model for monetary policy analysis and forecasting in Colombia. The model is calibrated with long-run steady-state ratios, and structural parameters and shocks are estimated using Bayesian techniques. Key features include Rotemberg pricing, wage rigidities, investment adjustment costs, and the use of both domestic and imported inputs in consumption and investment. In addition, trade adjustment costs are incorporated to reduce short-run pass-through effects and capture the gradual adjustment of trade flows. To account for the unprecedented deviations caused by the Covid-19 pandemic, we introduce special *Covid shocks* that absorb disruptions in key exogenous variables, ensuring the model appropriately filters data beyond its estimation period (2000Q1–2019Q4) up to 2024Q3. The shock decomposition reveals that before 2008, Colombia's economy grew above its potential level, driven by trend shocks and strong foreign demand. The 2008 crisis led to a slowdown due to declining foreign demand and prices, though U.S. financial conditions provided some support. A subsequent boom in 2011–2012 was fueled by rising oil prices. During the Covid-19 pandemic, sharp contractions in demand and supply led to heightened macroeconomic volatility, with pandemic shocks explaining most of the output collapse in 2020Q2 and contributing to the observed declines in inflation and the policy rate. Impulse response analysis confirms standard economic channels: productivity shocks boost output and lower inflation, while demand shocks raise both. Foreign shocks play a dominant role in economic fluctuations, highlighting the economy's sensitivity to external conditions.

Keywords: DSGE; New Keynesian Model; Small Open Economy; Monetary Policy; Business Cycles

JEL Classification: E32; E37; E47; E52

*Researcher, Macroeconomic Modeling Department, Banco de la República, e-mail: oavilamo@banrep.gov.co

†Specialized Economist, Macroeconomic Modeling Department, Banco de la República, e-mail: fcastava@banrep.gov.co

‡Head of Model Development Section, Macroeconomic Modeling Department, Banco de la República, e-mail: jpulidpe@banrep.gov.co

§Economist, Macroeconomic Modeling Department, Banco de la República, e-mail: jrincoga@banrep.gov.co

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1 Introduction

This document describes the new dynamic stochastic general equilibrium (DSGE) model developed by the Department of Macroeconomic Modeling at Banco de la República, the Colombian Central Bank. The model is primarily used for monetary policy analysis and forecasting. It builds on the benchmark business cycle literature for emerging economies, such as Gali and Monacelli (2005); Garcia-Cicco et al. (2010), and incorporates additional mechanisms from the trade literature, including a dynamic trade elasticity from Drozd et al. (2021), which reduces pass-through in the short run. The model is calibrated and estimated using Colombian data for the period 2000-2019, and aims to update the main DSGE model currently used by the technical staff, based on González et al. (2011). Additionally, to account for the unprecedented disruptions caused by the Covid-19 pandemic, the model includes a filtering approach that extends the observed data up to 2024Q3. By introducing *Covid shocks* as in Faria-e-Castro (2024) and Ferroni et al. (2022), which absorb deviations in key exogenous variables, the model allows for a clearer separation between pandemic-related fluctuations and structural economic dynamics.

Colombia is a small open economy (SOE) with a GDP representing an average of 0.38% of global output in the last ten years (World Bank, 2024), making it exposed to external shocks. Trade openness, measured by the ratio of trade balance to GDP, stands at a deficit of −4.4% (DANE, 2025b), reflecting the deep integration of the country into global markets. A key feature of Colombia's external sector is its reliance on commodity exports, with oil accounting for an average of 32% of total exports in the last 30 years (DANE, 2025a), making the economy particularly vulnerable to fluctuations in global commodity prices. Since the 1991 constitutional reform, Colombia's Central Bank —Banco de la República— has operated independently of the government, implementing an inflation-targeting regime of an annual inflation of 3% in the long run, with a policy interest rate that responds to macroeconomic conditions.

Table 1.1 presents a snapshot of key business cycle moments for various macroeconomic variables, including their volatilities, correlations, and first-order auto-correlations. As shown in the table, consumption exhibits a volatility similar to that of GDP, while investment, exports, and imports show more than twice its volatility, while net exports show nearly half. As can be observed, consumption, investment, imports, and exports are procyclical, whereas net exports are countercyclical. These stylized facts about the Colombian business cycle exhibit a behavior similar to that of emerging economies described by Uribe and Schmitt-Grohé (2017).[1]

Table 1.1: Business Cycle Moments: Data

	Moments of data				
Variables (QoQ)	$\overline{x}$	σ_x	σ_x/σ_y	$Corr(x,y)$	$Corr(x_t,x_{t-1})$
GDP	0.009	0.009	1	1.000	-0.02
Consumption	0.010	0.008	0.86	0.530 ***	-0.09
Investment	0.016	0.042	4.84	0.490 ***	-0.06
Imports	0.017	0.038	4.38	0.410 ***	-0.03
Exports	0.008	0.037	4.21	0.340 ***	-0.23

Note: For columns $Corr(x,y)$ and $Corr(x_t,x_{t-1})$, * p<0.1, ** p<0.05, *** p<0.01

Given these structural characteristics, our model is designed to replicate the key macroeconomic stylized facts of the Colombian economy and provide reliable short-term forecasts over a monetary policy horizon of eight quarters. By incorporating a Taylor Rule to describe Banco de la República's monetary policy response, as well as essential open-economy features, the model not only aims to enhance the understanding of GDP fluctuations, its

[1] Uribe and Schmitt-Grohé (2017) show that, on average across countries, consumption is equally volatile than output; but that its relative volatility is lower in high-income countries, suggesting more consumption smoothing compared to poor countries. In this regard, Colombia appears comparable to a typical middle-income country. On the other hand, Colombia exhibits the same countercyclicality of the trade balance as the average country.

components, and the dynamics of inflation, but also supports monetary policy analysis by offering insights into the effects of different shocks and monetary policy adjustments on the Colombian economy.

The main components of the model can be summarized as follows (see Figure 1.1 for a graphical representation of the model structure). A representative household makes decisions regarding consumption, labor supply, investment in capital, and asset allocation (both foreign and domestic). Following Garcia-Cicco et al. (2010); Schmitt-Grohé and Uribe (2003), we close the model with a debt-elastic interest rate.

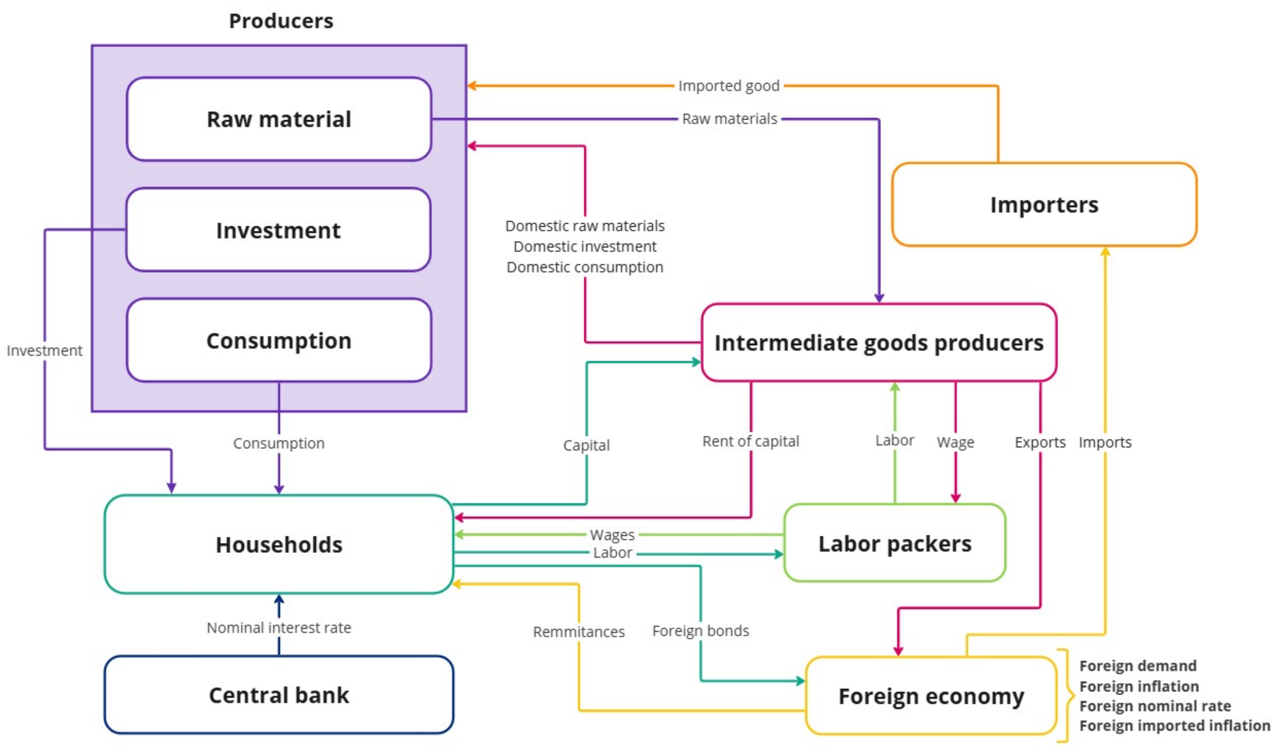


Figure 1.1: Model structure

Labor is heterogeneous and managed by a labor union that operates under monopolistic competition and faces wage rigidities (see Christiano et al. (2005)). Domestic production occurs in multiple stages. First, a continuum of heterogeneous firms, operating under monopolistic competition and subject to price rigidities, employs capital, labor, and materials to produce intermediate goods. In the next stage, a representative competitive firm aggregates these domestic intermediates into a homogeneous good, which is then used for consumption, investment, materials, and exports.

Consumption, investment, and materials are produced using specific technologies that combine domestic and imported goods. These firms operate in competitive markets and incur input adjustment costs, generating a dynamic trade elasticity, as in Avila-Montealegre and Mix (2024); Drozd et al. (2021). Imported goods are also produced in two stages. First, a continuum of firms, operating under monopolistic competition and subject to price rigidities, produces heterogeneous import varieties. Then, a perfectly competitive firm aggregates these varieties into a homogeneous good, which is subsequently used for consumption, investment, and materials.

Finally, exports depend on the relative price of exports and foreign demand. The central bank sets the nominal interest rate on domestic bonds following a Taylor rule that responds to deviations in output and inflation. For tractability, we model price rigidities using Rotemberg adjustment costs. As is well known, the Calvo and Rotemberg models of price and wage setting are isomorphic in a first-order approximation (e.g., Born and Pfeifer, 2020; Keen and Wang, 2007; Lombardo and Vestin, 2008). This equivalence, along with our use of first-order approximations, makes Rotemberg pricing more convenient for our purposes.

The model is solved, simulated, and estimated using Dynare (Adjemian et al., 2024). We calibrate some parameters of the model to replicate the average long-term relations of Colombia for the pre-COVID period, including the shares of consumption, investment, exports and the trade balance over GDP, as well as inflation, real interest rates, real GDP growth, debt over GDP, and the current account balance. We also estimate shock processes and other parameters using Bayesian techniques. Finally, the shock processes of foreign variables are estimated outside the model.

Using the estimated parameters, we filter the data and analyze the behavior of the Colombian economy through the lens of the model. The results suggest that prior to the global financial crisis of 2008, the Colombian economy grew above its potential level, mainly driven by trend shocks. During these years, we also observe a positive contribution from foreign demand and export prices, especially between 2003 and 2006. A counteracting force during this period was the foreign interest rate. Around the time of the global financial crisis, the drop in output growth was mainly explained by the decline in foreign demand, foreign prices, and transitory TFP shocks. During these quarters, relaxed financial conditions in the U.S. acted as a positive force on output growth. Around 2011–2012, Colombia experienced another economic boom, this time driven by a positive shock in export prices, especially oil.

In addition to these forces, consumption and investment dynamics are influenced by global financial conditions and import price fluctuations. During the first part of the sample period (2003–2006), declining import prices also supported growth in these components. However, starting in 2014, the rapid rise in import prices became a drag on consumption and investment.

The Covid-19 pandemic introduced an unprecedented shock to both demand and supply, leading to sharp contractions in economic activity. To capture these disruptions, we introduce *Covid shocks* that allow the model to filter the observed data beyond its estimation period (2000Q1–2019Q4) up to 2024Q3. The shock decomposition reveals that during the 2020Q2 contraction, pandemic-related shocks accounted for most of the decline in output growth, contributing to the heightened volatility observed during the Covid quarters. These shocks also help explain the sharp drop in inflation and the policy rate as the central bank responded to collapsing demand and economic uncertainty. By incorporating this approach, the model provides a clearer distinction between pandemic-induced fluctuations and structural economic dynamics, ensuring a more accurate assessment of Colombia's post-pandemic recovery.

The model's impulse response functions also deliver the standard mechanisms in which the different domestic and external shocks are transmitted to the whole economy. We find that a positive foreign demand shock boosts economic activity, primarily through exports. The positive income effect boosts consumption and investment. In contrast, higher import prices and foreign interest rates cause output to fall. Regarding domestic shocks, higher productivity reduces marginal costs, boosting economic activity while lowering inflation and the policy interest rate. Similarly, a positive demand shock increases consumption, investment, and inflation, prompting the Central Bank to raise the interest rate.

The document is organized into five sections, including this introduction. Section Two presents the theoretical model. Then, in Section Three, we describe the dataset, explain the steady-state calibration, and report estimation results. In Section Four, we present the impulse response functions from the model and the shock decomposition of Colombia's main macroeconomic variables. We analyze key episodes through the lens of the model and assess its forecasting power. Finally, in Section Five, we conclude.

2 Model

In this section, we present the main mathematical derivations of the DSGE model. We start by describing the sources of exogenous growth in the model, such as population and labor-augmenting technological progress. Next, we present the optimization problem of the representative household, which includes the decisions of the labor union (packer). We then analyze the optimization problems of final goods producers, including consumption and investment, and raw material producers. After that, we describe the aggregation problem of the domestic homogeneous good producer, followed by the decision problem of heterogeneous variety producers, who operate in monopolistic competition and face price rigidities. Similar problems are then described for importers. The last part of the model includes exports, monetary policy, and macroeconomic aggregates.

2.1 Sources of growth

As observed in the data section, Colombian time series for GDP and its components are non-stationary, since they present a positive trend. To account for this, in the model we consider two exogenous sources of growth. On the one hand, we assume a technological process that evolves according to equations (2.1) and (2.2). These equations are consistent with Garcia-Cicco et al. (2010), who argue that trend shocks are a significant source of business cycle fluctuations in emerging economies. On the other hand, we consider that the population grows at a constant rate, equation (2.3).

$$\mathcal{Z}_t = (1 + g_t^z)\mathcal{Z}_{t-1} \tag{2.1}$$

$$g_t^z = \rho_z g_{t-1}^z + (1 - \rho_z)g^z + \epsilon_t^z \tag{2.2}$$

$$\mathcal{N}_t = (1 + \bar{n})\mathcal{N}_{t-1} \tag{2.3}$$

In order to convey a balanced growth path in a stationary model, all real variables are de-trended using the sources of growth, $\mathcal{Z}_t$ and $\mathcal{N}_t$. Hence, by definition, variables in per-capita terms are written as:

$$\widehat{X}_t \equiv \frac{\mathcal{X}_t}{\mathcal{N}_t}\,. \tag{2.4}$$

where $\mathcal{X}_t$ is the variable in levels. Similarly, the variable in efficient units of labor is given by:

$$x_t \equiv \frac{\mathcal{X}_t}{\mathcal{Z}_t\mathcal{N}_t} = \frac{\widehat{X}_t}{\mathcal{Z}_t}\,. \tag{2.5}$$

2.2 Households

The representative household maximizes the present value of utility, which depends on consumption per member, $\widehat{C}$, and hours worked, h (adjusted by technological progress, $\mathcal{Z}_t^{1-\sigma}$). The instantaneous utility function per household member is given by:

$$u(\widehat{C}_t, \widehat{C}_{t-1}, h_t) = \frac{1}{1-\sigma}\left(\widehat{C}_t - \hbar\widehat{C}_{t-1}\right)^{1-\sigma} - \mathcal{Z}_t^{1-\sigma}\frac{\psi_h}{1+\eta}h_t^{1+\eta} \tag{2.6}$$

where σ is the (inverse of the) intertemporal elasticity of substitution (IES), $\hbar$ is the habit formation parameter, and ψ_h is the labor disutility parameter. Given this utility function, the welfare of the representative household is equal to the present discounted value of utility per member multiplied by the size of the household, $\mathcal{N}_t$:

$$U(\mathcal{N}_t, \widehat{C}_t, \widehat{C}_{t-1}, h_t) = \sum_{t=0}^{\infty} \beta^t \mathcal{N}_t \zeta_t^u \left[\frac{1}{1-\sigma} \left(\widehat{C}_t - \hbar \widehat{C}_{t-1} \right)^{1-\sigma} - \mathcal{Z}_t^{1-\sigma} \frac{\psi_h}{1+\eta} h_t^{1+\eta} \right] \tag{2.7}$$

where ζ_t^u is a shock to the discount factor and follows an AR(1) process:

$$\zeta_t^u = \rho_u \zeta_{t-1}^u + (1-\rho_u)\zeta^u + \epsilon_t^u \tag{2.8}$$

Using equation (2.3), normalizing the initial level of population to 1, ($\mathcal{N}_0 = 1$), and expressing the variables in efficient units of labor, $c_t \equiv \frac{\widehat{C}_t}{\mathcal{Z}_t}$, household's welfare can be rewritten as:

$$U(c_t, c_{t-1}, h_t) = \sum_{t=0}^{\infty} \beta^t (1+\bar{n})^t \zeta_t^u \mathcal{Z}_t^{1-\sigma} \left[\frac{1}{1-\sigma} \left(c_t - \hbar \frac{c_{t-1}}{1+g_t^z} \right)^{1-\sigma} - \frac{\psi_h}{1+\eta} h_t^{1+\eta} \right] \tag{2.9}$$

The next step in the household's problem is to define the budget constraint. Households use their income to consume $\mathcal{C} = \mathcal{N}\widehat{C}$, invest on physical capital $\mathcal{I} = \mathcal{N}\widehat{I}$, and buy (nominal) domestic and foreign bonds $\mathcal{B} = \mathcal{N}\widehat{B}, \mathcal{B}^\star = \widehat{B}^\star$. On the income side, households are paid for their labor h, they get rents on capital and bonds $k, b, b^\star$, profits from firms' ownership Π, and transfers (remittances). More specifically, the aggregate budget constraint in efficient units of labor is given by:

$$\zeta_t^D P_t c_t + \zeta_t^D \zeta_t^i P_t^I i_t + P_t b_t + s_t P_t^\star b_t^\star =$$
$$w_t(l) h_t(l) + \frac{R_t^K (u_t^k k_{t-1})}{(1+g_t^{nz})} + \frac{(1+i_{t-1}^{nom})}{(1+g_t^{nz})} P_{t-1} b_{t-1} + \frac{(1+i_{t-1}^{nom,\star})}{(1+g_t^{nz})} s_t P_{t-1}^\star b_{t-1}^\star + \Pi_t^{nom} + \tau_t^{nom}$$

where P_t, P_t^I, and $P_t^\star$ are the prices of consumption, investment, and foreign bonds, s_t is the nominal exchange rate, w_t is the real wage, R_t^K is the gross return on capital, $i^{nom}, i^{*,nom}$ are the nominal returns on domestic and foreign bonds, u^k is capital utilization and Π, T are profits and transfers. Notice that wages and labor in the budget constraint are indexed by l. This indexation captures labor heterogeneity that will be more explicit in the labor union problem, which allows for wage rigidities in the short run. The price of investment includes the term ζ_t^i that captures shocks to the investment demand. Similarly, ζ_t^D captures a demand shock that affects both consumption and investment. These shocks follow an AR(1) processes:

$$\zeta_t^i = \rho_i \zeta_{t-1}^i + (1-\rho_i)\zeta^i + \epsilon_t^i \tag{2.10}$$
$$\zeta_t^D = \rho_D \zeta_{t-1}^D + (1-\rho_D)\zeta^D + \epsilon_t^D \tag{2.11}$$

The capital law of motion includes endogenous depreciation, as in Schmitt-Grohé and Uribe (2012), and adjustment costs as in Chari et al. (2000). A higher utilization of capital depreciates capital faster, while capital adjustment costs make investment less volatile in the short run. To facilitate mathematical notation we will define $\delta_t \equiv \delta(u_t^k)$.

$$\delta(u_t^k) = \bar{\delta}_0 + \delta_1(u_t^k - 1) + \frac{\delta_2}{2}(u_t^k - 1)^2 \tag{2.12}$$
$$k_t = \left[1 - \delta_t - \frac{\kappa}{2} \left(\frac{i_t(1+g_t^{nz})}{k_{t-1}} - (\delta + g^{nz}) \right)^2 \right] \frac{k_{t-1}}{1+g_t^{nz}} + i_t \tag{2.13}$$

To simplify the notation, let's define the aggregate growth factor as $1 + g_t^{nz} = (1 + \bar{n})(1 + g_t^z)$, and the variation of the real exchange rate Δz_t as:

$$\Delta z_t = \Delta s_t + \pi_t^\star - \pi_t \tag{2.14}$$

Where foreign inflation $\pi_t^\star$ follows and AR(1) process:

$$\pi_t^\star = (\pi_{t-1}^\star)^{\rho_{\pi^\star}} (\pi^\star)^{1-\rho_{\pi^\star}} (1 + \epsilon_t^{\pi^\star}) \tag{2.15}$$

From the household's optimization problem, we find the first-order conditions (FOCs) with respect to consumption, capital, and bonds. The labor problem is later introduced due to the presence of a labor packer and wage rigidities. Given this, the Lagrange multiplier (marginal utility of consumption) is:

$$\zeta_t^D \lambda_t = \left(c_t - \frac{\hbar c_{t-1}}{1 + g_t^z} \right)^{-\sigma} - \mathbb{E}_t \frac{\hbar \tilde{\beta}_{t+1}}{1 + g_{t+1}^z} \left(\frac{\zeta_{t+1}^u}{\zeta_t^u} \right) \left(c_{t+1} - \frac{\hbar c_t}{1 + g_{t+1}^z} \right)^{-\sigma} \tag{2.16}$$

Define $\tilde{\beta}_t$ as the discount factor adjusted for population and technology growth, similarly the stochastic discount factor (SDF) can be expressed as:

$$\tilde{\beta}_t \equiv \beta(1 + \overline{n})(1 + g_t^z)^{1-\sigma} \tag{2.17}$$

$$SDF_{t,t+1} \equiv \tilde{\beta}_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \tag{2.18}$$

Given these definitions, the FOCs with respect to domestic and foreign bonds are:

$$1 = \mathbb{E}_t \frac{SDF_{t,t+1}}{(1 + g_{t+1}^{nz})} \left(\frac{\zeta_{t+1}^u}{\zeta_t^u} \right) \left(\frac{1 + i_t^{nom}}{1 + \pi_{t+1}} \right) \tag{2.19}$$

$$1 = \mathbb{E}_t \frac{SDF_{t,t+1}}{(1 + g_{t+1}^{nz})} \left(\frac{\zeta_{t+1}^u}{\zeta_t^u} \right) \left(\frac{1 + i_t^{\star,nom}}{1 + \pi_{t+1}^\star} \right) \frac{z_{t+1}}{z_t} \tag{2.20}$$

Equations (2.19) and (2.20) represent the arbitrage condition between domestic and foreign bonds, usually known as the uncovered interest parity condition. Finally, the FOCs with respect to capital utilization, capital and investment are:

$$r_t^k = \frac{\lambda_t^i}{\lambda_t} (\delta_1 + \delta_2 (u_t^k - 1)) \tag{2.21}$$

$$\lambda_t^i = \mathbb{E}_t (\beta(1 + \bar{n})) \frac{\zeta_{t+1}^u}{\zeta_t^u} (1 + g_{t+1}^z)^{1-\sigma} \Bigg\{ \lambda_{t+1} \left(\frac{r_{t+1}^k u_{t+1}^k}{1 + g_{t+1}^{nz}} \right)$$
$$+ \lambda_{t+1}^i \left[\kappa \left(\frac{i_{t+1}(1 + g_{t+1}^{nz})}{k_t} - (\delta + g^{nz}) \right) \frac{i_{t+1}}{k_t} + \left(1 - \delta_{t+1} - \frac{\kappa}{2} \left(\frac{i_{t+1}(1 + g_{t+1}^{nz})}{k_t} - (\delta + g^{nz}) \right)^2 \right) \frac{1}{1 + g_{t+1}^{nz}} \right] \Bigg\} \tag{2.22}$$

$$\lambda_t \zeta_t^i \zeta_t^D p_t^i = \lambda_t^i \left[1 - \kappa \left(\frac{i_t (1 + g_t^{nz})}{k_{t-1}} - (\delta + g^{nz}) \right) \right] \tag{2.23}$$

These set of conditions represent the Euler Equations for capital and investment, and show the trade off that consumers face at the moment of sacrificing current consumption and increasing the level of capital.

2.2.1 Labor packer

We introduce a labor packer or union that combines different types of labor into a composite labor good that is then supplied to firms at a nominal wage rate W_t. We first consider the problem of the competitive labor packing firm, and then we go back to the household's optimization problem. In particular, the representative labor packer maximizes profits choosing across labor varieties according to:

$$\max_{h_t(l)} W_t Z_t^w \left(\int_0^1 h_t(l)^{\frac{\varepsilon_{w,t}-1}{\varepsilon_{w,t}}} dl \right)^{\frac{\varepsilon_{w,t}}{\varepsilon_{w,t}-1}} - \zeta_t^w \int_0^1 W_t(l) h_t(l) dl$$

where $\varepsilon_{w,t}, Z_t^w, \zeta_t^w$, are shocks to the markup, the labor-packer technology, and the labor cost, and follow AR(1) processes:

$$\varepsilon_{w,t} = (\varepsilon_{w,t-1})^{\rho_{\varepsilon w}} (\varepsilon_w)^{1-\rho_{\varepsilon w}} (1 + \epsilon_t^{\varepsilon_w}) \tag{2.24}$$

$$Z_t^w = (Z_{t-1}^w)^{\rho_{Z^w}} (1 + \epsilon_t^{Z^w}) \tag{2.25}$$

$$\zeta_t^w = (\zeta_{t-1}^w)^{\rho_{\zeta^w}} (1 + \epsilon_t^{\zeta^w}) \tag{2.26}$$

From the F.O.C with respect to $h_t(l)$ we get the optimal demand for labor varieties, and the aggregate wage in real and efficient units of labor:

$$h_t(l) = \left(\frac{Z_t^w W_t}{\zeta_t^w W_t(l)} \right)^{\varepsilon_{w,t}} \frac{H_t}{Z_t^w} \tag{2.27}$$

$$\varpi_t^{1-\varepsilon_{w,t}} = \left(\frac{Z_t^w}{\zeta_t^w} \right)^{\varepsilon_{w,t}} \frac{1}{Z_t^w} \int_0^1 \varpi_t(l)^{1-\varepsilon_{w,t}} dl \tag{2.28}$$

Notice that there is an equivalence between the productivity and the labor cost shocks.

2.2.2 Rotemberg wage adjustment

Now, consider that households face quadratic adjustment costs to change wages, and nominal wages are indexed to past (total) inflation and past technological progress. This cost is paid in units of the final good. Expressing it in efficient units of labor we have:

$$\Upsilon_t^w = \frac{\phi_w}{2} \left(\frac{\varpi_t(l)(1+\pi_t)(1+g_t^z)}{\varpi_{t-1}(l)(1+\pi_{t-1})(1+g_{t-1}^z)} - 1 \right)^2 \tag{2.29}$$

Let us define the nominal adjustment factor as:

$$(1+\pi_t)(1+g_t^z) = 1 + \pi_t^z \tag{2.30}$$

Where π_t is total inflation defined as the weighted average between food, regulated, and non-food-and-regulated (NFR) inflation:

$$\pi_t^{foo} = \rho_{\pi^{foo}}\pi_{t-1}^{foo} + (1-\rho_{\pi^{foo}})\pi^{foo} + \epsilon_t^{\pi^{foo}} \tag{2.31}$$
$$\pi_t^{reg} = \rho_{\pi^{reg}}\pi_{t-1}^{reg} + (1-\rho_{\pi^{reg}})\pi^{reg} + \epsilon_t^{\pi^{reg}} \tag{2.32}$$
$$\pi_t = \phi^{\pi^{NFR}}\pi_t^{NFR} + \phi^{\pi^{foo}}\pi_t^{foo} + (1-\phi^{\pi^{NFR}}-\phi^{\pi^{foo}})\pi_t^{reg} + \epsilon_t^{\pi} \tag{2.33}$$

Now, we can re-write Household's optimization problem, after replacing the optimal labor demand from the labor packer, equation (2.27), and focus only on the wage decision. From the FOC, we find the *New Keynesian wage Phillips Curve*:

$$\begin{aligned}\left(\frac{H_t}{Z_t^w}(\zeta_t^w)^{\frac{\varepsilon_{w,t}}{\varepsilon_{w,t-1})}}\right)^{1+\eta}\frac{\psi_h\varepsilon_{w,t}}{\varpi_t\lambda_t} &= (\varepsilon_{w,t}-1)\frac{H_t}{Z_t^w}(\zeta_t^w)^{\frac{\varepsilon_{w,t}}{\varepsilon_{w,t}-1}} \\ &+ \phi_w\left(\frac{\varpi_t}{\varpi_{t-1}}\frac{(1+\pi_t^z)}{(1+\pi_{t-1}^z)}-1\right)\frac{(1+\pi_t^z)}{\varpi_{t-1}(1+\pi_{t-1}^z)}p_t^q q_t \\ &- \mathbb{E}_t SDF_{t,t+1}\frac{\zeta_{t+1}^u}{\zeta_t^u}\left[\phi_w\left(\frac{\varpi_{t+1}(1+\pi_{t+1}^z)}{\varpi_t(1+\pi_t^z)}-1\right)\frac{\varpi_{t+1}(1+\pi_{t+1}^z)}{\varpi_t^2(1+\pi_t^z)}p_{t+1}^q q_{t+1}\right]\end{aligned} \tag{2.34}$$

2.2.3 Foreign interest rate

Following Schmitt-Grohé and Uribe (2003), we consider that the foreign interest rate responds to deviations of the foreign debt-to-GDP ratio from its steady-state value. Additionally, we assume that the relevant interest rate for Colombia can be decomposed into two components: the risk-free foreign interest rate, $i_t^{nom,FED}$, and a risk premium, i_t^{prem}. Specifically, we model this relationship as follows:

$$(1+i_t^{nom,\star}) = (1+i_t^{nom,FED})(1+i_t^{prem}) \tag{2.35}$$

where the debt-elastic risk premium increases exponentially with the deviation of debt to annual output as

$$i_t^{prem} = \zeta_t^{i^{prem}} + \psi^{i^{prem}}\left[\exp\left(z_t\frac{b_t^\star}{y_t} - z\frac{b^\star}{y}\right)-1\right]. \tag{2.36}$$

We assume that both the risk-free foreign rate and the observable risk premium follow AR(1) processes of the form:

$$i_t^{nom,FED} = \left(i_{t-1}^{nom,FED}\right)^{\rho_{i^\star,FED}}\left(i^{nom,FED}\right)^{1-\rho_{i^\star,FED}}\left(1+\epsilon_t^{i^{nom,FED}}\right) \tag{2.37}$$
$$\zeta_t^{i^{prem}} = \left(\zeta_{t-1}^{i^{prem}}\right)^{\rho^{i^{prem}}}\left(\zeta^{i^{prem}}\right)^{1-\rho^{i^{prem}}}(1+\epsilon_t^{i^{prem}}) \tag{2.38}$$

2.3 Producers of consumption (final) goods

Consumption goods are produced by a firm that operates in perfect competition and combines domestic (c_t^D) and foreign intermediates (c_t^{IM}). Following the trade literature, Drozd et al. (2021) and Avila-Montealegre and Mix (2024), we introduce input adjustment costs that make it difficult for the firm to change the input composition in the short run. As a result, the Armington trade elasticity is smaller in the short run. In particular, we consider the following structure for the production of consumption goods:

$$c_t + \frac{\phi_d^c}{2}\left(\frac{c_t^D}{c_{t-1}^D} - 1\right)^2 + \frac{\phi_{im}^c}{2}\left(\frac{c_t^{IM}}{c_{t-1}^{IM}} - 1\right)^2 = Z_t^c\left[(\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^D\right)^{\frac{\omega_c-1}{\omega_c}} + (1-\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^{IM}\right)^{\frac{\omega_c-1}{\omega_c}}\right]^{\frac{\omega_c}{\omega_c-1}} \tag{2.39}$$

where γ^C is the home bias parameter, ω_c is the elasticity of substitution between home and foreign varieties and Z_t^c is an efficiency consumption shock.

$$Z_t^c = (Z_{t-1}^c)^{\rho_{Z^c}}(1 + \epsilon_t^{Z^c}) \tag{2.40}$$

The optimization problem is given by:

$$\max_{c_t^D, c_t^{IM}} \mathbb{E}_0 \sum_{t=0}^{\infty} SDF_{0,t}\zeta_t^u\left\{P_t\left[Z_t^c\left((\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^D\right)^{\frac{\omega_c-1}{\omega_c}} + (1-\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^{IM}\right)^{\frac{\omega_c-1}{\omega_c}}\right)^{\frac{\omega_c}{\omega_c-1}}\right.\right.$$
$$\left.\left. - \frac{\phi_D^c}{2}\left(\frac{c_t^D}{c_{t-1}^D} - 1\right)^2 - \frac{\phi_{IM}^c}{2}\left(\frac{c_t^{IM}}{c_{t-1}^{IM}} - 1\right)^2\right] - P_t^Q c_t^D - P_t^{IM} c_t^{IM}\right\}$$

where P_t^Q and P_t^{IM} are the prices of domestic and foreign intermediates (in domestic currency). From the F.O.Cs we find the relative demands for domestic and imported inputs:

$$Z_t^c\left((\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^D\right)^{\frac{\omega_c-1}{\omega_c}} + (1-\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^{IM}\right)^{\frac{\omega_c-1}{\omega_c}}\right)^{\frac{1}{\omega_c-1}}(\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^D\right)^{-\frac{1}{\omega_c}} = p_t^q + \phi_D^c\left(\frac{c_t^D}{c_{t-1}^D} - 1\right)\frac{1}{c_{t-1}^D}$$
$$-\mathbb{E}_t SDF_{t,t+1}\frac{\zeta_{t+1}^u}{\zeta_t^u}(1+\pi_{t+1})\left(\phi_D^c\left(\frac{c_{t+1}^D}{c_t^D} - 1\right)\frac{c_{t+1}^D}{(c_t^D)^2}\right) \tag{2.41}$$

$$Z_t^c\left((\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^D\right)^{\frac{\omega_c-1}{\omega_c}} + (1-\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^{IM}\right)^{\frac{\omega_c-1}{\omega_c}}\right)^{\frac{1}{\omega_c-1}}(1-\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^{IM}\right)^{-\frac{1}{\omega_c}} = p_t^{IM} + \phi_{IM}^c\left(\frac{c_t^{IM}}{c_{t-1}^{IM}} - 1\right)\frac{1}{c_{t-1}^{IM}}$$
$$-\mathbb{E}_t SDF_{t,t+1}\frac{\zeta_{t+1}^u}{\zeta_t^u}(1+\pi_{t+1})\left(\phi_{IM}^c\left(\frac{c_{t+1}^{IM}}{c_t^{IM}} - 1\right)\frac{c_{t+1}^{IM}}{(c_t^{IM})^2}\right) \tag{2.42}$$

Notice that without input adjustment costs $\phi_D^c = \phi_{IM}^c = 0$ these demand functions are standard, and they only depend on static relative prices and aggregate demand. Once we introduce adjustment costs, these demands also depend on dynamic factors.

2.4 Producers of investment (final) goods

Investment goods follow an aggregation similar to that of consumption goods. A firm that acts in perfect competition uses domestic and foreign intermediates to produce investment goods according to:

$$i_t + \frac{\phi_d^i}{2}\left(\frac{i_t^D}{i_{t-1}^D} - 1\right)^2 + \frac{\phi_{im}^i}{2}\left(\frac{i_t^{IM}}{i_{t-1}^{IM}} - 1\right)^2 = Z_t^i\left[\left(\gamma^i\right)^{\frac{1}{\omega_i}}\left(i_t^D\right)^{\frac{\omega_i-1}{\omega_i}} + \left(1-\gamma^i\right)^{\frac{1}{\omega_i}}\left(i_t^{IM}\right)^{\frac{\omega_i-1}{\omega_i}}\right]^{\frac{\omega_i}{\omega_i-1}} \tag{2.43}$$

where γ^I is the home bias parameter, ω_I is the elasticity of substitution between home and foreign varieties and Z_t^i represents an efficiency investment shock as in Fisher (2006); Mandelman et al. (2011).

$$Z_t^i = (Z_{t-1}^i)^{\rho_{Z^i}}(1+\epsilon_t^{Z^i}) \tag{2.44}$$

Following a similar procedure, as in consumption, we find the relative demands for investment inputs:

$$\begin{aligned} Z_t^i \left(\left(\gamma^i\right)^{\frac{1}{\omega_i}} \left(i_t^D\right)^{\frac{\omega_i-1}{\omega_i}} + \left(1-\gamma^i\right)^{\frac{1}{\omega_i}} \left(i_t^{IM}\right)^{\frac{\omega_i-1}{\omega_i}} \right)^{\frac{1}{\omega_i-1}} \left(\gamma^i\right)^{\frac{1}{\omega_i}} \left(i_t^D\right)^{-\frac{1}{\omega_i}} &= \frac{p_t^q}{p_t^i} + \phi_D^i \left(\frac{i_t^D}{i_{t-1}^D}-1\right)\frac{1}{i_{t-1}^D} \\ &- \mathbb{E}_t SDF_{t,t+1}\frac{\zeta_{t+1}^u}{\zeta_t^u}\frac{p_{t+1}^i}{p_t^i}(1+\pi_{t+1})\left(\phi_D^i\left(\frac{i_{t+1}^D}{i_t^D}-1\right)\frac{i_{t+1}^D}{(i_t^D)^2}\right) \end{aligned} \tag{2.45}$$

$$\begin{aligned} Z_t^i \left(\left(\gamma^i\right)^{\frac{1}{\omega_i}} \left(i_t^D\right)^{\frac{\omega_i-1}{\omega_i}} + \left(1-\gamma^i\right)^{\frac{1}{\omega_i}} \left(i_t^{IM}\right)^{\frac{\omega_i-1}{\omega_i}} \right)^{\frac{1}{\omega_i-1}} \left(1-\gamma^i\right)^{\frac{1}{\omega_i}} \left(i_t^{IM}\right)^{-\frac{1}{\omega_i}} &= \frac{p_t^{IM}}{p_t^i} + \phi_{IM}^i \left(\frac{i_t^{IM}}{i_{t-1}^{IM}}-1\right)\frac{1}{i_{t-1}^{IM}} \\ &- \mathbb{E}_t SDF_{t,t+1}\frac{\zeta_{t+1}^u}{\zeta_t^u}\frac{p_{t+1}^i}{p_t^i}(1+\pi_{t+1})\left(\phi_{IM}^i\left(\frac{i_{t+1}^{IM}}{i_t^{IM}}-1\right)\frac{i_{t+1}^{IM}}{(i_t^{IM})^2}\right) \end{aligned} \tag{2.46}$$

where p_t^{IM} is the relative price of foreign intermediates in domestic currency.

2.5 Producers of raw materials

Raw materials are similarly produced by aggregating domestic and foreign intermediates, by a firm that acts in perfect competition according to the following production function:

$$m_t + \frac{\phi_d^m}{2}\left(\frac{m_t^D}{m_{t-1}^D}-1\right)^2 + \frac{\phi_{im}^m}{2}\left(\frac{m_t^{IM}}{m_{t-1}^{IM}}-1\right)^2 = Z_t^m \left[(\gamma^m)^{\frac{1}{\omega_m}} \left(m_t^D\right)^{\frac{\omega_m-1}{\omega_m}} + (1-\gamma^m)^{\frac{1}{\omega_m}} \left(m_t^{IM}\right)^{\frac{\omega_m-1}{\omega_m}} \right]^{\frac{\omega_m}{\omega_m-1}} \tag{2.47}$$

where γ^m is the home bias parameter and ω_m is the elasticity of substitution between home and foreign varieties and Z_t^m represents an efficiency raw materials shock.

$$Z_t^m = (Z_{t-1}^m)^{\rho_{Z^m}}(1+\epsilon_t^{Z^m}) \tag{2.48}$$

Following a similar procedure, as in consumption and investment, we find the relative demands for raw materials inputs:

$$\begin{aligned} Z_t^m \left((\gamma^m)^{\frac{1}{\omega_m}} \left(m_t^D\right)^{\frac{\omega_m-1}{\omega_m}} + (1-\gamma^m)^{\frac{1}{\omega_m}} \left(m_t^{IM}\right)^{\frac{\omega_m-1}{\omega_m}} \right)^{\frac{1}{\omega_m-1}} (\gamma^m)^{\frac{1}{\omega_m}} \left(m_t^D\right)^{-\frac{1}{\omega_m}} &= \frac{p_t^q}{p_t^m} + \phi_D^m \left(\frac{m_t^D}{m_{t-1}^D}-1\right)\frac{1}{m_{t-1}^D} \\ &- \mathbb{E}_t SDF_{t,t+1}\frac{\zeta_{t+1}^u}{\zeta_t^u}\frac{p_{t+1}^m}{p_t^m}(1+\pi_{t+1})\left(\phi_D^m\left(\frac{m_{t+1}^D}{m_t^D}-1\right)\frac{m_{t+1}^D}{(m_t^D)^2}\right) \end{aligned} \tag{2.49}$$

$$Z_t^m \left((\gamma^m)^{\frac{1}{\omega_m}} \left(m_t^D\right)^{\frac{\omega_m-1}{\omega_m}} + (1-\gamma^m)^{\frac{1}{\omega_m}} \left(m_t^{IM}\right)^{\frac{\omega_m-1}{\omega_m}} \right)^{\frac{1}{\omega_m-1}} (1-\gamma^m)^{\frac{1}{\omega_m}} \left(m_t^{IM}\right)^{-\frac{1}{\omega_m}} = \frac{p_t^{IM}}{p_t^m} + \phi_{IM}^m \left(\frac{m_t^{IM}}{m_{t-1}^{IM}} - 1 \right) \frac{1}{m_{t-1}^{IM}} \quad (2.50)$$

$$-\mathbb{E}_t SDF_{t,t+1} \frac{\zeta_{t+1}^u}{\zeta_t^u} \frac{p_{t+1}^m}{p_t^m} (1+\pi_{t+1}) \left(\phi_{IM}^m \left(\frac{m_{t+1}^{IM}}{m_t^{IM}} - 1 \right) \frac{m_{t+1}^{IM}}{(m_t^{IM})^2} \right)$$

2.6 Production of domestic intermediate homogeneous goods

Domestic intermediates goods are produced by a perfect competitive firm that aggregates across different varieties of domestic inputs $q(j)$ according to the following production function. For simplicity, quantities are already expressed in efficient units of labor (dividing by $\mathcal{N}_t \mathcal{Z}_t$). The optimization problem is given by:

$$\max_{q_t(j)} p_t^q \left[\int_0^1 q_t(j)^{\frac{\varepsilon_{q,t}-1}{\varepsilon_{q,t}}} dj \right]^{\frac{\varepsilon_{q,t}}{\varepsilon_{q,t}-1}} - \int_0^1 p_t^q(j) q_t(j) dj \quad (2.51)$$

where $\varepsilon_{q,t}$ is the elasticity of substitution across varieties and follows an AR process.

$$\varepsilon_{q,t} = (\varepsilon_{q,t-1})^{\rho_q} (\varepsilon_q)^{1-\rho_q} (1+\epsilon_t^q) \quad (2.52)$$

p_t^q is the relative price of domestic intermediates in terms of consumption goods, P_t^q/P_t. From the F.O.C we find the demand for a variety j in function of the relative price:

$$q_t(j) = \left(\frac{p_t^q(j)}{p_t^q} \right)^{-\varepsilon_{q,t}} q_t \quad (2.53)$$

and the aggregate price index:

$$p_t^q = \left(\int_0^1 (p_t^q(j))^{1-\varepsilon_{q,t}} dj \right)^{\frac{1}{1-\varepsilon_{q,t}}} \quad (2.54)$$

In equilibrium, total production is equal to the input demand of domestic intermediates for consumption, investment, raw materials, exports, and adjustment costs (Υ_t):

$$q_t = c_t^D + i_t^D + m_t^D + ex_t + \Upsilon_t \quad (2.55)$$

where Υ_t is defined as the sum of the individual price adjustment costs to wages, domestic production and imported goods:

$$\Upsilon_t \equiv \underbrace{\frac{\phi_w}{2} \left(\frac{\varpi_t(l)(1+\pi_t^z)}{\varpi_{t-1}(l)(1+\pi_{t-1}^z)} - 1 \right)^2}_{\Upsilon_t^{\varpi}} + \underbrace{\frac{\phi_q}{2} \left(\frac{p_t^q(j)(1+\pi_t)}{p_{t-1}^q(j)(1+\pi_{t-1})} - 1 \right)^2}_{\Upsilon_t^q} + \underbrace{\frac{\phi_{IM}}{2} \left(\frac{p_t^{IM}(j)(1+\pi_t)}{p_{t-1}^{IM}(j)(1+\pi_{t-1})} - 1 \right)^2}_{\Upsilon_t^{IM}}$$

$$\Upsilon_t \equiv \Upsilon_t^{\varpi} + \Upsilon_t^q + \Upsilon_t^{IM} \quad (2.56)$$

2.7 Producers of domestic heterogeneous inputs

Producers of heterogeneous inputs operate under monopolistic competition. These firms are a continuum of size one. The representative firm chooses capital, labor, raw materials, and prices to maximize its profits. We divide the optimization problem in two stages. First, we consider the static problem of the firm, which chooses production factors. Then, we analyze the dynamic pricing decision assuming a Rotemberg set up. The production function of the representative firm is given by:

$$Q_t(j) = \frac{Z_t^q \psi_q}{Z^{\pi^{nfr}}} (\mathcal{K}_{t-1}^s(j))^\alpha (\mathcal{M}_t(j))^\mu (\mathcal{Z}_t \mathcal{N}_t h_t(j))^{1-\alpha-\mu} \tag{2.57}$$

where Z_t^q is an aggregate productivity shock that follows an AR(1) process, and ψ_q is a scale parameter that guarantees that GDP equals to 1 in steady state:

$$Z_t^q = (Z_{t-1}^q)^{\rho_{z,q}} (Z^q)^{1-\rho_{z,q}} (1 + \epsilon_t^{z,q}) \tag{2.58}$$

and $Z^{\pi^{nfr}}$ serves as both, an inflationary shock, and an exogenous process to explain Non food and regulated (NFR) inflation as a residual from headline, food and regulated inflation rates:

$$Z_t^{\pi^{nfr}} = \left(Z_t^{\pi^{foo}}\right)^{\phi^{\pi^{foo}}} \left(Z_t^{\pi^{reg}}\right)^{1-\phi^{\pi^{foo}}} . \tag{2.59}$$

Z^{π^j} with $j \in \{foo, reg\}$ follow a multiplicative AR(1) process. The aggregate equilibrium implies that total factor demand equals total factor demand $u_t^k \mathcal{K}_{t-1} = \int_0^1 \mathcal{K}_{t-1}^s(j)dj$, $\mathcal{M}_t = \int_0^1 \mathcal{M}_t(j)dj$ and $H_t = \int_0^1 h_t(j)dj$. Now, we express equation (2.57) in efficient units of labor:

$$q_t(j) = \frac{Z_t^q \psi_q}{Z^{\pi^{nfr}}} \left(\frac{k_{t-1}^s(j)}{1 + g_t^{nz}}\right)^\alpha (m_t(j))^\mu (h_t(j))^{1-\alpha-\mu} \tag{2.60}$$

For the static optimization problem we minimize total costs subject to the technological constraint (all in efficient units of labor). From the FOCs we find the demands for factors:

$$r_t^k = \alpha(1 + g_t^{nz}) \left(mc_t^q \frac{q_t}{k_{t-1}^s}\right) \tag{2.61}$$

$$k_t^s = u_{t+1}^k k_t \tag{2.62}$$

$$p_t^m = \mu \left(mc_t^q \frac{q_t}{m_t}\right) \tag{2.63}$$

$$\varpi_t = (1 - \alpha - \mu) \left(mc_t^q \frac{q_t}{h_t}\right) \tag{2.64}$$

and the real marginal cost, in units of consumption:

$$q_t(j) = \frac{Z_t^q \psi_q}{Z^{\pi^{nfr}}} mc_t^q q_t(j) \left\{ \left(\frac{\alpha}{r_t^k}\right)^\alpha \left(\frac{\mu}{p_t^m}\right)^\mu \left(\frac{(1 - \alpha - \mu)}{\varpi_t}\right)^{1-\alpha-\mu} \right\}$$

Notice that the marginal cost is the same for all firms j, $mc_t^q(j) = mc_t^q$. This implies that the demand for factors of production is the same for all firms.

$$mc_t^q = \frac{Z^{\pi^{nfr}}}{Z_t^q \psi_q} \left(\frac{r_t^k}{\alpha}\right)^\alpha \left(\frac{p_t^m}{\mu}\right)^\mu \left(\frac{\varpi_t}{1 - \mu - \alpha}\right)^{1-\mu-\alpha} \tag{2.65}$$

Before formulating the dynamic optimization problem, we introduce a simplified indexation rule. This rule assumes that if firms do not optimally adjust their prices, those prices are instead updated based on past total inflation. Under this assumption, price adjustment costs arise due to deviations from optimal pricing behavior. These costs, expressed in units in real terms (units of consumption goods), can be formulated as follows:

$$\Upsilon_t^q = \frac{\phi_q}{2}\left(\frac{p_t^q(j)(1+\pi_t)}{p_{t-1}^q(j)(1+\pi_{t-1})} - 1\right)^2 \tag{2.66}$$

Now, we can write the dynamic optimization problem:

$$\max_{p_t^q(j)} \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta(1+\overline{n}))^t \left(\frac{\mathcal{Z}_t}{\mathcal{Z}_0}\right)^{1-\sigma} \frac{\lambda_t}{\lambda_0} \zeta_t^u \left[p_t^q(j) q_t(j) - mc_t^q(j) q_t(j) - \frac{\phi_q}{2}\left(\frac{p_t^q(j)(1+\pi_t)}{p_{t-1}^q(j)(1+\pi_{t-1})} - 1\right)^2 p_t^q q_t\right]$$

subject to equation (2.53):

$$q_t(j) = \left(\frac{p_t^q}{p_t^q(j)}\right)^{\varepsilon_{q,t}} q_t \tag{2.53}$$

From the FOC of this problem we find the *New Keynesian domestic intermediate goods Phillips Curve*:

$$\begin{aligned} \frac{\varepsilon_{q,t} mc_t}{p_t^q} = (\varepsilon_{q,t} - 1) + \left[\phi_q \left(\frac{p_t^q (1+\pi_t)}{p_{t-1}^q (1+\pi_{t-1})} - 1\right)\left(\frac{p_t^q}{p_{t-1}^q}\frac{(1+\pi_t)}{(1+\pi_{t-1})}\right)\right] \\ - \mathbb{E}_t SDF_{t,t+1} \frac{\zeta_{t+1}^u}{\zeta_t^u}\left[\phi_q \left(\frac{p_{t+1}^q(1+\pi_{t+1})}{p_t^q(1+\pi_t)} - 1\right)\left(\left(\frac{p_{t+1}^q}{p_t^q}\right)^2 \frac{(1+\pi_{t+1})}{(1+\pi_t)}\right)\frac{q_{t+1}}{q_t}\right] \end{aligned} \tag{2.67}$$

Given that p^q is a relative price, we can define the inflation of domestic varieties as:

$$\frac{p_t^q}{p_{t-1}^q} = \frac{1+\pi_t^q}{1+\pi_t} \tag{2.68}$$

Finally, we can find the aggregate profits function, in real and efficient units of labor as:

$$\Pi_t^q = \left(p_t^q - mc_t^q\right) q_t - \Upsilon_t^q p_t^q q_t \tag{2.69}$$

2.8 Producers of imported goods

Imported goods are produced in two stages. In the first stage, a continuum of firms that operate under monopolistic competition produce an import variety (j). These firms import the foreign good from the rest of the world and price it in the domestic market. In a second stage, a perfect competitive firm aggregates the different varieties and produces an homogeneous good that is assigned to consumption, investment, and raw materials. These two problems are described in the following sections.

2.8.1 Import aggregation

Total imports are produced by a representative firm that operates under perfect competition. This firm aggregates heterogeneous varieties according to the following production function, already expressed in efficiency units of labor:

$$im_t = \left(\int_0^1 (im_t(j))^{\frac{\varepsilon_{im,t}-1}{\varepsilon_{im,t}}} dj\right)^{\frac{\varepsilon_{im,t}}{\varepsilon_{im,t}-1}} \tag{2.70}$$

The optimization problem is given by:

$$\max_{im_t(j)} P_t^{IM} \left(\int_0^1 (im_t(j))^{\frac{\varepsilon_{im,t}-1}{\varepsilon_{im,t}}} dj\right)^{\frac{\varepsilon_{im,t}}{\varepsilon_{im,t}-1}} - \int_0^1 P_t^{IM}(j) im_t(j) dj$$

where P^{IM} is the price of imports in local currency and $\varepsilon_{im,t}$ is the elasticity of substitution across variates and follows an AR(1) process:

$$\varepsilon_{im,t} = (\varepsilon_{im,t-1})^{\rho_{im}} (\varepsilon_{im})^{1-\rho_{im}} (1 + \epsilon_t^{im}) \tag{2.71}$$

From the F.O.C. we find the demand for input $im(j)$ depends on the relative price and total imports:

$$im_t(j) = \left(\frac{P_t^{IM}}{P_t^{IM}(j)}\right)^{\varepsilon_{im,t}} im_t \tag{2.72}$$

and the aggregate price:

$$P_t^{IM} = \left(\int_0^1 (P_t^{IM}(j))^{1-\varepsilon_{im,t}} dj\right)^{\frac{1}{1-\varepsilon_{im,t}}} \tag{2.73}$$

In equilibrium, total imports are equal to import demand for consumption, investment, and raw materials:

$$im_t = c_t^{IM} + i_t^{IM} + m_t^{IM} \tag{2.74}$$

2.8.2 Import varieties

To produce one unit of import variety (j) the representative firm pays $P_t^{IM,\star}$ in foreign currency, due to the law of one price the cost in domestic currency is $s_t P_t^{IM,\star}$. This firm operates in monopolistic competition and chooses the domestic price $P_t^{IM}(j)$ to maximize the present value of profits, subject to the relative demand from equation (2.72) and considering Rotemberg adjustment costs for prices. Static profits expressed in units of consumption goods is given by:

$$\frac{\Pi_t^{IM}(j)}{P_t} = p_t^{IM}(j) im_t(j) - z_t \frac{P_t^{IM,\star}}{P_t^{\star}} im_t(j) - \frac{\phi_{IM}}{2}\left(\frac{p_t^{IM}(j)(1+\pi_t)}{p_{t-1}^{IM}(j)(1+\pi_{t-1})} - 1\right)^2 p_t^q q_t \tag{2.75}$$

and the adjustment cost:

$$\Upsilon_t^{IM} = \frac{\phi_{IM}}{2}\left(\frac{p_t^{IM}(j)(1+\pi_t)}{p_{t-1}^{IM}(j)(1+\pi_{t-1})} - 1\right)^2 \tag{2.76}$$

where $p_t^{IM,*}$ is the relative price of imports in foreign currency and behaves as:

$$p_t^{IM,\star} = (p_{t-1}^{IM,\star})^{\rho^{p^{IM,\star}}}(p^{IM,\star})^{1-\rho^{p^{IM,\star}}}(1+\epsilon_t)^{\rho^{p^{IM,\star}}} \tag{2.77}$$

From the FOC we find the *New Keynesian Phillips Curve* for imported goods:

$$\begin{aligned}\left[(1-\varepsilon_{im,t})im_t + \varepsilon_{im,t}z_t\frac{p_t^{IM,\star}}{p_t^{IM}}im_t\right] &= \phi_{IM}\left(\frac{p_t^{IM}(1+\pi_t)}{p_{t-1}^{IM}(1+\pi_{t-1})}-1\right)p_t^q q_t\frac{(1+\pi_t)}{p_{t-1}^{IM}(1+\pi_{t-1})}\\ &- \mathbb{E}_t SDF_{t,t+1}\frac{\zeta_{t+1}^u}{\zeta_t^u}\phi_{IM}\left(\frac{p_{t+1}^{IM}(1+\pi_{t+1})}{p_t^{IM}(1+\pi_t)}-1\right)p_{t+1}^q q_{t+1}\frac{p_{t+1}^{IM}(1+\pi_{t+1})}{(p_t^{IM})^2(1+\pi_t)}\end{aligned} \tag{2.78}$$

Finally, importer's profits are given by:

$$\Pi_t^{im} = (p_t^{im} - z_t p_t^{im,\star})im_t - \Upsilon_t^{im}p_t^q q_t \tag{2.79}$$

2.9 Export demand

Consider that total exports depend on export prices and foreign demand. These prices come from the terms of trade data base built by the Central Bank of Colombia and capture any changes in the export bundle. In particular, real exports behave as follows:

$$ex_t^{real} = Z_t^{ex}\left(\frac{z_t p_t^{ex,\star}}{p_t^q}\right)^{\mu_x}(y_t^\star)^{\mu_y} \tag{2.80}$$

where $y_t^\star$ is the foreign demand, Z_t^{ex} is an export demand shock, and $p_t^{ex,\star} = \frac{P_t^{ex,\star}}{P_t^\star}$ denotes the price of exports relative to the foreign price index. All three variables follow AR(1) processes:

$$y_t^\star = (y_{t-1}^\star)^{\rho_{y^\star}}(y^\star)^{1-\rho_{y^\star}}(1+\epsilon_t^{y^\star}) \tag{2.81}$$

$$Z_t^{ex} = (Z_{t-1}^{ex})^{\rho_{Z^{ex}}}(1+\epsilon_t^{Z^{ex}}) \tag{2.82}$$

$$p_t^{ex,\star} = (p_{t-1}^{ex,\star})^{\rho_{p^{ex,\star}}}(p^{ex,\star})^{1-\rho_{p^{ex,\star}}}(1+\epsilon_t^{p^{ex,\star}}) \tag{2.83}$$

Finally, the value of exports is given by:

$$ex_t = p_t^q ex_t^{real} \tag{2.84}$$

2.10 Monetary policy

Regarding monetary policy we consider a standard Taylor-type Rule where the nominal domestic interest rate, i_t^{nom}, responds to deviations of the inflation and output gaps:

$$i_t^{nom} = \phi_i i_{t-1}^{nom} + (1-\phi_i)\left[i_t^{nom,n} + \phi_\pi\widetilde{\pi}_t + \phi_y\widetilde{y}_t\right] + \epsilon_t^{i^{nom}} \tag{2.85}$$

where ϕ_i is the own process' persistence to the previous period's value, and determines a convex combination with respect to the value of the neutral nominal interest rate $i_t^{nom,n}$, the inflation gap $\widetilde{\pi}_t$, with its marginal response ϕ_π, and the output gap $\widetilde{y}_t$, with its marginal response ϕ_y. $\epsilon_t^{i^{nom}}$ corresponds to a monetary policy shock.

The neutral nominal interest rate is defined as

$$i_t^{nom,n} = \frac{(1+\bar{\pi})(1+g_t^z)^{\sigma}}{\beta} - 1 \tag{2.86}$$

responding to variations of the domestic trend growth g_t^z. The long-run inflation-target is defined as $\bar{\pi}$, set to an annual value of 3%.

On the other hand, the inflation gap responds to the annual expected inflation four quarters ahead respect to the time-varying annualized inflation target $\bar{\pi}_t^A$, while the output gap responds to the deviations of the output level from its steady-state value:

$$\widetilde{\pi}_t = \mathbb{E}_t \pi_{t+4}^{NFR,A} - \overline{\pi}_t^A\,, \tag{2.87}$$

$$\widetilde{y}_t = y_t - y\,. \tag{2.88}$$

As part of the historical adjustments with the implementation of the inflation target regime, the time-varying inflation target follows a multiplicative AR-1 process of the form

$$\bar{\pi}_t = \bar{\pi}_{t-1}^{\rho_{\bar{\pi}}} \bar{\pi}^{(1-\rho_{\bar{\pi}})}(1+\epsilon_t^{\bar{\pi}}) \tag{2.89}$$

Where $\epsilon_t^{\bar{\pi}}$ is the inflation target shock.

2.11 Aggregates

We finally define some aggregate variables and equilibrium conditions. The trade balance in units of consumption goods is the difference between exports and imports in domestic currency:

$$tb_t = ex_t - z_t p_t^{IM,*} im_t \tag{2.90}$$

The real exchange rate variation is:

$$\frac{z_t}{z_{t-1}} = 1 + \Delta z_t \tag{2.91}$$

Remittances follow and exogenous AR(1) process:

$$rem_t = rem_t^{\rho_{rem}} rem_{ss}^{(1-\rho_{rem})}(1+\epsilon_t^{rem}) \tag{2.92}$$

The current account is defined as the change in foreign assets position:

$$ca_t = -z_t b_t^{\star} + z_t \frac{b_{t-1}^{\star}}{(1+\pi_t^{\star})(1+g_t^{nz})} \tag{2.93}$$

Given these definitions the trade balance equilibrium is given by:

$$-tb_t = z_t rem_t - ca_t - z_t \frac{i_{t-1}^{nom,\star} b_{t-1}^{\star}}{(1+\pi_t^{\star})(1+g_t^{nz})} \tag{2.94}$$

GDP on the production side is equal to the value added in the production of domestic intermediate goods in domestic currency:

$$GDP_t = p_t^q q_t - p_t^m m_t + \Pi_t^{im} \tag{2.95}$$

Similarly, GDP on the demand side is equal to the sum of consumption, investment, and the trade balance:

$$y_t = c_t + p_t^i i_t + tb_t \tag{2.96}$$

Transfers are defined in efficient units of labor and in terms of consumption goods measured in domestic currency, as follows:

$$\tau_t = z_t rem_t \tag{2.97}$$

Finally, the definition of total profits is:

$$\Pi_t = \Pi_t^q + \Pi_t^{im} \tag{2.98}$$

The full set of equations is reported in Appendix A.1.

3 Data to the model

We use quarterly data for Colombia and the U.S. from 2000Q1 to 2019Q4 to set the calibration targets and full-information methods estimation. We extend the database up to 2024Q3 to filter the model and produce unconditional forecasts. We parametrize the model using a combination of literature, steady-state calibration, and normalization. Parameter values are reported in Table B.3.

The model is calibrated to replicate the moments reported in Table 3.1. These targets correspond to the mean of the series for the capital-output ratio, national account ratios such as nominal imports and nominal GDP, nominal investment and nominal GDP, current account/GDP, and nominal trade balance and nominal GDP, the imported shares of consumption, investment, and raw materials, and the average share of working hours. The data series comes from the National Statistics Department (DANE), the Central Bank, and the Penn World Tables (PWT).

In particular, we build the long-term relations according to:

1. The **capital-to-GDP ratio** is computed as the mean of the ratio between net capital stock by economic activity and nominal GDP.
2. The **nominal investment-to-GDP ratio** is calculated as the mean of the ratio between gross fixed capital formation and nominal GDP. Data are taken in chained 2015 prices and are seasonally adjusted.
3. The **nominal imports-to-GDP ratio** is computed as the mean of the ratio between imports and GDP. Data are taken in current prices and are seasonally adjusted.
4. The **trade balance-to-GDP ratio** is computed as the average ratio of the trade balance (exports minus imports) to GDP. Data are taken in current prices and are seasonally adjusted.
5. The **current-account-to-GDP data** are taken from the statistics of the Banco de la República. The calculation is computed as the mean of this series.

6. The **imported consumption-to-total consumption ratio** is computed as the ratio of imported consumer goods (according to the Economic Use and Destination Classification, CUODE, in Spanish) to total consumption.
7. The **imported investment-to-total investment ratio** is computed as the ratio of capital goods and construction materials (according to the CUODE classification) to total investment.
8. The **imported raw materials-to-total raw materials ratio** is computed using the Input-Output Matrix. Imported raw materials come from the uses component of the imports matrix for total imports CIF across all sectors, while total raw materials are taken from total purchases at purchaser prices across all sectors.
9. The **average labor time** is calculated as the mean of the average annual hours worked by persons engaged from the PWT.
10. The **value-added-to-gross output ratio** is computed using the Input-Output Matrix. Value added is computed as the sum of value added for product groupings' national accounts according to CPC. Gross output is computed as the sum of total production for product groupings' national accounts according to CPC.

Name	Target	Value	Sample	Source
Capital /Annual GDP	K/Y^{Ann}	2.47	2000-2019	DANE - Annual
Investment / GDP	I/Y	0.21	2000Q1-2019Q4	DANE - Quarterly
Imports/GDP	IM/Y	0.18	2000Q1-2019Q4	DANE - Quarterly
Trade balance/GDP	TB/Y	-0.03	2000Q1-2019Q4	DANE - Quarterly
Current account/GDP	CA/Y	-0.03	2000-2019	BanRep - Annual
Imported Consumption / Consumption	C^{IM}/C	0.04	2000-2019	DANE - Annual
Imported Investment / Investment	I^{IM}/I	0.25	2000-2019	DANE - Annual
Imported raw materials / Raw materials	RM^{IM}/RM	0.14	2005, 2009 , 2015, 2017, 2019	DANE - Annual
Average labor time	H	0.23	2000-2019	PWT - Annual
Value Added / Gross Output	M/Y	1.95	2015, 2017, 2019	DANE - Annual

Table 3.1: Targeted ratios in the steady state

These moments are used to pin down some of the parameters of the model, such as the home-bias for consumption, investment, and raw materials ($\gamma_c, \gamma_i, \gamma_m$), the depreciation rate of capital (δ, δ_1), the shares of capital and materials in the production function (α, μ), the foreign debt in the long run ($b^\star$), the level of technology in the domestic production function (ψ^q), the dis-utility of labor (ψ^h), and the level of remittances (rem_{ss}). We describe the calibration of these parameters in Appendix B. In this appendix, we also report the analytical steady state.

Other parameters of the model are calibrated or fixed according to empirical projections, observed data, and monetary policy objectives. These include, real growth rate of GDP($g^{n,z}$), population growth rate ($\bar{n}$), technological progress (g^z), domestic inflation (π consistent with the inflation target from Banco de la República), foreign inflation ($\pi^\star$), domestic and foreign real interest rates ($i^{real}, i^{real,\star}$), the discount factor (β), and the Premium risk (i^{prem}).

3.1 Model observables and measurement equations

In this section, we describe the observed variables used for the model estimation and forecast. In particular, we report the data series, sources and transformations consistent with the model definitions.

The model considers the following variables as exogenous:

1. **Foreign GDP ($y^\star$)**: observed as the growth rate of the Real Gross Domestic Product (*GDPC1*) seasonally adjusted from the Federal Reserve Bank of St. Louis (FRED).

2. **Foreign inflation ($\pi^\star$)**: observed as the growth rate of the Consumer Price Index for All Urban Consumers (*CPIAUCSL*) seasonally adjusted from the FRED.

3. **Nominal foreign interest rate ($i^{nom,FED}$)**: observed as the Shadow rate from Wu and Xia (2016). The foreign policy rate for the quarter is equivalent to the mean of three months of the quarter.

4. **Risk premium (i^{prem})**: observed as the credit default swaps (CDS) at five years.

5. **Imported inflation in foreign currency ($\pi^{IM,\star}$)**: We compute this variable as the growth rate of the Imported Index of the Terms of Trade published by the Banco de la República divided by the Consumer Price Index for All Urban Consumers seasonally adjusted. Relative prices of imports in foreign currency $p^{ex\star}$.

6. **Exported inflation in foreign currency ($\pi^{EX,\star}$)**: We compute this variable as the growth rate of the Exported Index of the Terms of Trade published by the Banco de la República divided by the Consumer Price Index for All Urban Consumers seasonally adjusted. Relative prices of exports in foreign currency $p^{ex,\star}$.

7. We estimate the persistence of food and regulated inflation using the quarterly growth rate of the CPI from Banco de la República, the data were seasonally adjusted.

3.1.1 Measurement equations

The domestic observable variables include national accounts data, such as, GDP, Consumption, Investment, Exports and Imports, inflation data, including total CPI, NFR CPI, Food inflation, and Regulated inflation. Also, the time-varying inflation target is included. Finally, rates include the policy interest rate from the central bank and the risk premium.

National Accounts: We observe the percentage change of the GDP components in constant prices as

$$\Delta\%GDP_t^{COL,real} = \frac{GDP_t^{COL,real}}{GDP_{t-1}^{COL,real}} - 1\,.$$

In terms of model notation, we define the observable percentage change as

$$\Delta y_t^{obs} \equiv \Delta\%GDP_t^{COL,real}\,.$$

Hence, the measurement equation for GDP is

$$\Delta y_t^{obs} = \frac{y_t}{y_{t-1}}(1+g_t^{nz}) - 1 \qquad \text{(GDP growth)}$$

where the right-hand-side consists of the variables in efficiency units. Similarly, for the output components, the measurement equations take the form:

$$\Delta c_t^{obs} = \frac{c_t}{c_{t-1}}(1+g_t^{nz}) - 1 + B\Delta c_t^{y,obs} \quad \text{(Cons. growth)}$$

$$\Delta i_t^{obs} = \frac{i_t}{i_{t-1}}(1+g_t^{nz}) - 1 + B\Delta i_t^{y,obs} \quad \text{(Inv. growth)}$$

$$\Delta im_t^{obs} = \frac{im_t}{im_{t-1}}(1+g_t^{nz}) - 1 + B\Delta im_t^{y,obs} \quad \text{(Imports growth)}$$

$$\Delta ex_t^{r,obs} = \frac{ex_t^r}{ex_{t-1}^r}(1+g_t^{nz}) - 1 + B\Delta ex_t^{y,obs} \quad \text{(Exports growth)}$$

Here, $B\Delta x_t^{y,obs}$ are discrepancies that account for historical differences in growth rates between variable x and output (Faria-e-Castro, 2024). They take the form

$$B\Delta x_t^{y,obs} = B\Delta\bar{x}^{y,obs} + \epsilon_t^{B\Delta x} \tag{3.1}$$

where $B\Delta\bar{x}^{y,obs}$ is a calibrated constant defined as the historical difference between the growth rate of x and output (g^{nz}). Table 3.2 shows the calibrated values for the discrepancies.

Variable	Description	Discrepancy Value (Quarterly)
$B\Delta\bar{c}^{y,obs}$	Consumption growth	0.00008
$B\Delta\bar{i}^{y,obs}$	Investment growth	0.00795
$B\Delta\overline{ex^r}^{y,obs}$	Exports growth	−0.00078
$B\Delta\overline{im}^{y,obs}$	Imports growth	0.00820

Table 3.2: Long-run differences between GDP components and output growth rates

Trend Growth: To guide the balanced growth path of the economy, we observe a measure for the growth rate of the output trend $\Delta\bar{y}_t^{obs}$. It is constructed as the difference of the trend component from a one-sided Hodrick-Prescott filter to the logarithm of real GDP, effectively making it a growth rate. Its measurement equation is

$$\Delta\bar{y}_t^{obs} = g_t^{nz}\,. \tag{3.2}$$

Foreign Variables: We take foreign variables as fully exogenous, then they map their respective AR-1 processes.

$$\Delta y_t^{\star,obs} = \frac{y_t^\star}{y_{t-1}^\star}(1+g^\star) - 1 \quad \text{(Foreign demand growth)}$$

$$i_t^{Fed,obs} = (1+i_t^{Fed})^4 - 1 \quad \text{(Federal funds rate)}$$

$$\pi_t^{\star,obs} = \pi_t^\star \quad \text{(Foreign inflation)}$$

$$p_t^{im,\star,obs} = p_t^{im,\star} \quad \text{(Relative price of imports in USD)}$$

$$p_t^{ex,\star,obs} = p_t^{ex,\star} \quad \text{(Relative price of imports in USD)}$$

Inflation Rates: Regarding price variations, we map the model's produced inflation to the observed headline inflation (CPI variation), and observe its component baskets as exogenous. So, in model notation we define

$$\pi_t \equiv \frac{CPI_t^{COL}}{CPI_{t-1}^{COL}} - 1\,.$$

Hence, the measurement equations are

$$\pi_t^{obs} = \pi_t \qquad \text{(Headline inflation)}$$
$$\pi_t^{NFR,obs} = \pi_t^{NFR} \qquad \text{(Non food and regulated inflation)}$$
$$\pi_t^{foo,obs} = \pi_t^{foo} \qquad \text{(Food inflation)}$$
$$\pi_t^{reg,obs} = \pi_t^{reg} \qquad \text{(Regulated inflation)}$$

we also observe the time-varying inflation target to account for periods where the observed inflation was far from the current target of 3% annual.

$$\bar{\pi}_t^{obs} = \bar{\pi}_t \qquad \text{(Time-varying inflation target)}$$

Rates We observe the annual nominal policy rate, and the risk premium as the quarterly average of the 5 year maturity CDS.

$$i_t^{nom,obs} = (1 + i_t^{nom})^4 - 1 \qquad \text{(Nominal policy rate)}$$
$$i_t^{prem,obs} = (1 + i_t^{prem})^4 - 1 \qquad \text{(Risk premium)}$$

3.2 Estimation

We estimate the model parameters in two stages. First, we perform an in-model estimation of the persistence and standard deviations of exogenous variables, including those from the foreign block and food and regulated inflation rates. Second, we conduct a full-information estimation, beginning with maximum likelihood estimation and then refining the parameter estimates using Bayesian methods, which incorporate prior information and the likelihood function to obtain a posterior distribution of the parameters. For the prior distributions and initial values for parameters we follow Smets and Wouters (2007) and Faria-e-Castro (2024).

3.2.1 In-Model estimation for exogenous processes

In this case, we use the time series of foreign variables, as well as food and regulated inflation rates observed by the model (see Section 3.1). Since the exogenous variables are modeled as AR(1) processes, we estimate each one individually using its corresponding shock. For each variable, we perform a maximum likelihood estimation of its persistence parameter and standard deviation. The resulting estimates are then used as calibrated values in the model.

The computation of each exogenous variable is as follows:

1. We compute the quarterly real growth rate from the U.S. Real Gross Domestic Product (*GDPC1*) from FRED to compute the persistence of foreign demand.

2. We compute the persistence of the FED funds rate using the Shadow rate from Wu and Xia (2016). The quarterly rate is equivalent to the three-month average.
3. We compute the persistence of foreign inflation using the quarterly growth rate of the Consumer Price Index for All Urban Consumers (*CPIAUCSL*) seasonally adjusted and the index value from the end of period from FRED. The AR(1) model has a dummy between 2008Q4 and 2009Q1.
4. We compute the persistence of food and regulated inflation using the quarterly growth rate of the CPI from Banco de la República, the data were seasonally adjusted.

The results of this exercise are contained in Table 3.3:

Parameter	Description	Value
Persistence		
$\rho_{y^\star}$	Persistence of foreign demand	0.983
$\rho_{i^{nom,FED}}$	Persistence of FED funds rate	0.982
$\rho_{\pi^\star}$	Persistence of foreign inflation	0.100
$\rho_{p^{im,\star}}$	Persistence of foreign import prices	0.800
$\rho_{p^{ex,\star}}$	Persistence of foreign export prices	0.953
$\rho_{\pi^{foo}}$	Persistence of food inflation	0.310
$\rho_{\pi^{reg}}$	Persistence of regulated inflation	0.630
Standard Deviations		
$\epsilon_{y^\star}$	Std. dev. of foreign demand	0.0057
$\epsilon_{i^{nom,FED}}$	Std. dev. of FED funds rate	0.1645
$\epsilon_{\pi^\star}$	Std. dev. of foreign inflation	0.0070
$\epsilon_{p^{im,\star}}$	Std. dev. of foreign import prices	0.0138
$\epsilon_{p^{ex,\star}}$	Std. dev. of foreign export prices	0.0557
$\epsilon_{\pi^{foo}}$	Std. dev. of food inflation	0.0170
$\epsilon_{\pi^{reg}}$	Std. dev. of regulated inflation	0.0097

Table 3.3: In-model ML estimation of exogenous variables persistence and standard deviations

3.2.2 Bayesian Estimation

We use Bayesian techniques to estimate the rest of the parameters. These include the processes persistence and standard deviations for structural shocks such as the risk premium, technological growth, output efficiency, aggregate demand, investment demand, investment combination efficiency, imports price efficiency and exports demand. Also, we estimate parameters related to adjustment costs for capital, Rotemberg price adjustments in wage, domestic price, and imported goods prices, risk premium, and elasticities from the exported goods demand. Table 3.4 summarizes the prior distributions and the posterior results for parameters, and Table 3.5 the results for the standard deviations for structural shocks.

We follow Smets and Wouters (2007) and Faria-e-Castro (2024) for the prior distributions and their initial values. Bayesian estimation runs a Random Walk Metropolis Hastings algorithm to generate 40,000 draws from 2 parallel Markov-Chains in which the first 20,000 are discarded and use the remainder for the posterior distribution approximation. The algorithm is tuned for an acceptance ration of around 30%.

Table 3.4: Results from Metropolis-Hastings (parameters)

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
Persistence							
$\rho_{\zeta^{i,prem}}$	β	0.500	0.1000	0.702	0.0358	0.6420	0.7576
ρ_{g^z}	β	0.500	0.1000	0.865	0.0169	0.8325	0.8901
ρ_{Z^q}	β	0.500	0.1000	0.312	0.0317	0.2609	0.3692
ρ_{ζ^d}	β	0.500	0.1000	0.685	0.0202	0.6511	0.7190
ρ_{ζ^i}	β	0.500	0.1000	0.356	0.0229	0.3217	0.3967
$\rho_{Z_{p}^{im}}$	β	0.500	0.1000	0.492	0.0261	0.4521	0.5336
ρ_{Z^i}	β	0.500	0.1000	0.694	0.0382	0.6361	0.7617
$\rho_{Z^{ex}}$	β	0.500	0.1000	0.577	0.0205	0.5406	0.6109
Adjustment parameters							
κ	Γ	30.000	50.0000	0.410	0.9578	0.0000	0.9480
ϕ^w	Γ	2.000	2.0000	0.372	0.2122	0.0001	0.6591
ϕ_q	Γ	2.000	2.0000	14.069	0.6648	12.8978	15.1266
ϕ_{im}	Γ	2.000	2.0000	0.088	0.0937	0.0000	0.2060
μ^x	Γ	0.500	0.7500	0.232	0.0537	0.1453	0.3164
μ^y	Γ	1.000	2.0000	1.925	0.4056	1.3286	2.5940
$\psi^{i^{prem}}$	Γ	0.100	0.0100	0.050	0.0020	0.0463	0.0527
μ^{nfr}	Γ	0.500	0.7500	0.067	0.0506	0.0000	0.1459

Table 3.5: Results from Metropolis-Hastings (standard deviation of structural shocks)

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
$\epsilon^{\zeta^{i,prem}}$	Γ^{-1}	0.010	0.0500	0.184	0.0084	0.1695	0.1965
$\epsilon^{\overline{\pi}^{Tar}}$	Γ^{-1}	0.010	0.0500	0.355	0.0197	0.3231	0.3841
ϵ^{g^z}	Γ^{-1}	0.010	0.0500	0.002	0.0002	0.0017	0.0023
ϵ^{Z_q}	Γ^{-1}	0.010	0.0500	0.007	0.0004	0.0065	0.0080
$\epsilon^{z,d}$	Γ^{-1}	0.010	0.0500	0.020	0.0017	0.0171	0.0227
ϵ^{ζ^i}	Γ^{-1}	0.010	0.0500	0.006	0.0023	0.0026	0.0092
ϵ^{Z_i}	Γ^{-1}	0.010	0.0500	0.021	0.0023	0.0179	0.0245
$\epsilon^{Z^{ex}}$	Γ^{-1}	0.010	0.0500	0.034	0.0024	0.0299	0.0379
$\epsilon^{Z_{p}^{im}}$	Γ^{-1}	0.010	0.0500	0.075	0.0056	0.0668	0.0850
$\epsilon^{i^{nom}}$	Γ^{-1}	0.010	0.0500	0.014	0.0009	0.0119	0.0150

Figures 3.1 and 3.2 show the time series for smoothed observable variables in estimation and estimated shocks, respectively.

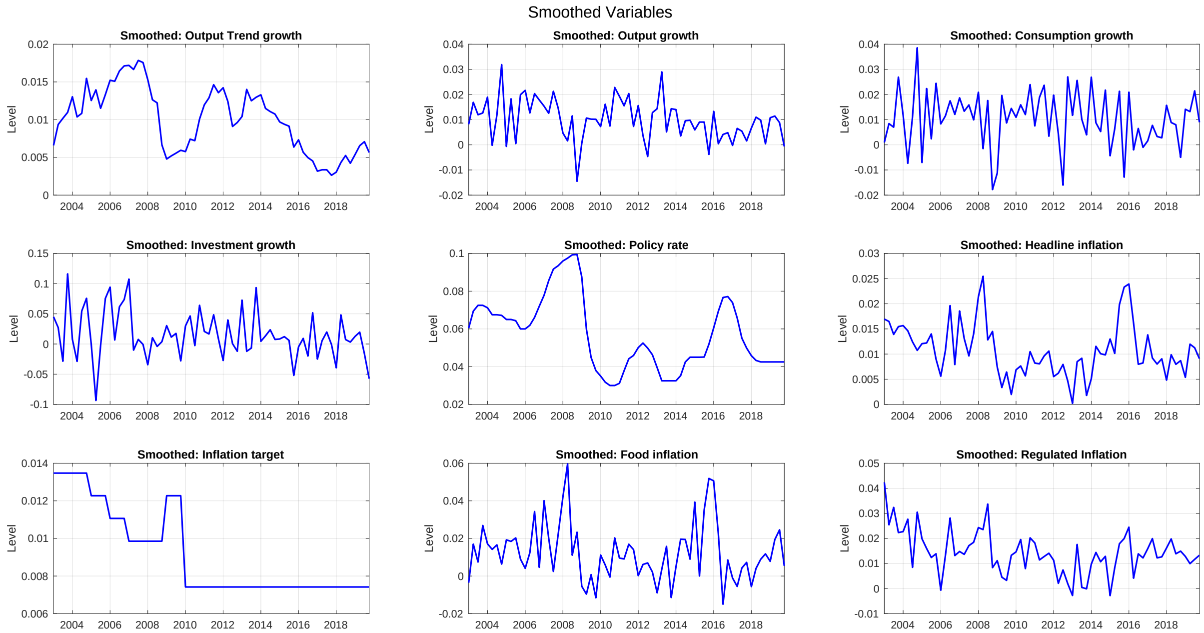


Figure 3.1: Time series for smoothed observed variables in estimation.

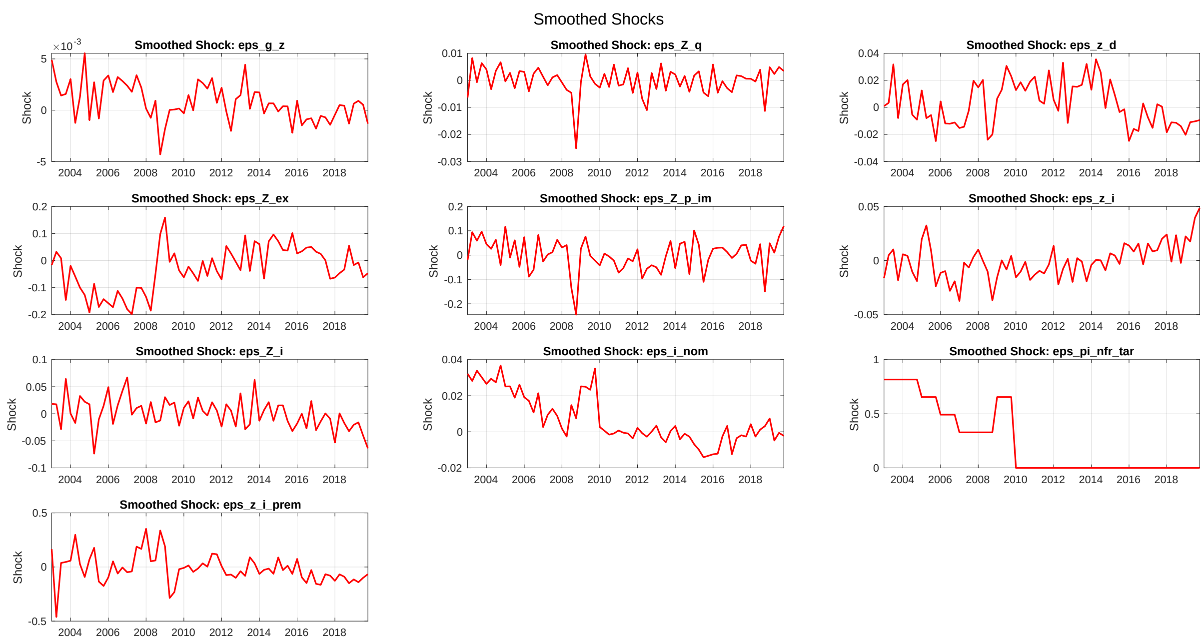


Figure 3.2: Time series for smoothed estimated shocks.

4 Results

We start by simulating 100,000 periods using the estimated persistencies and variances of the shocks. With the simulated variables, we calculate the means, standard deviations, correlations with GDP, and first-order autocorrelations, to replicate the moments in the data shown in Table 1.1. The results are reported in Table 4.1. First, we notice some discrepancies in the means as a result of model calibration since we assume steady-state growth to be lower than the sample average. Second, regarding volatilities, we find that consumption is about as volatile as GDP, although in the data it is somewhat less volatile. However, the volatility of investment and imports relative to that of GDP is very similar to what we find in the data, around four or five times higher. Our simulations also show that exports are more volatile than GDP, although slightly less so than the observed data. Third, consistent with what we observe in the data, the simulations show that all GDP components are procyclical, and that consumption is the type of expenditure most correlated with GDP, whereas exports are the least correlated, with a correlation that is practically equal to what we obtain in the data. Finally, with respect to autocorrelations, in the data (where variables are in quarterly log-differences), we do not observe any statistically significant pattern to replicate, except for the small negative autocorrelation of exports, which the model is able to reproduce.

Table 4.1: Business Cycle Moments

Moments of data					
Variables (QoQ)	$\overline{x}$	σ_x	$Corr(x,y)$	$Corr(x_t,x_{t-1})$	σ_x/σ_y
GDP	0.009	0.009	1.000	-0.02	1
Consumption	0.010	0.008	0.530 ***	-0.09	0.86
Investment	0.016	0.042	0.490 ***	-0.06	4.84
Imports	0.017	0.038	0.410 ***	-0.03	4.38
Exports	0.008	0.037	0.340 ***	-0.23	4.21

Moments of simulated variables *Model moments reported by Dynare*					
Variables (QoQ)	$\overline{x}$	σ_x	$Corr(x,y)$	$Corr(x_t,x_{t-1})$	σ_x/σ_y
GDP	0.007	0.015	1.000	0.18	1
Consumption	0.007	0.019	0.706	-0.03	1.27
Investment	0.015	0.073	0.661	0.34	4.91
Imports	0.015	0.085	0.633	0.10	5.72
Exports	0.006	0.041	0.349	-0.17	2.74

Note: For columns $Corr(x,y)$ and $Corr(x_t,x_{t-1})$, * p<0.1, ** p<0.05, *** p<0.01

Now, we report the impulse response functions for some of the main shocks in the model and the shock decompositions for the main macroeconomic aggregates.

4.1 Impulse Response Functions

Using the estimated parameters we report the impulse response functions (IRFs) for the most relevant shocks included in the model. We start by describing the foreign shocks: Demand, interest rate, and import prices. Then, we report the IRFs for domestic shocks: demand, productivity (transitory), and policy interest rate.

4.1.1 Foreign shocks

As shown in Figure 4.1, a positive foreign demand shock increases the growth rates of GDP, consumption, and investment. Higher foreign demand also increases headline inflation and leads to a rise in the policy interest rate.

The primary channel driving this positive effect on economic activity is exports. As foreign demand rises, the economy produces more domestic goods, which, in turn, raises the demand for factors of production and increases household income. Higher income further boosts consumption and investment. Since the shock is perceived as persistent, the resulting income effect significantly strengthens consumption and investment, while its impact on inflation and monetary policy remains relatively mild.

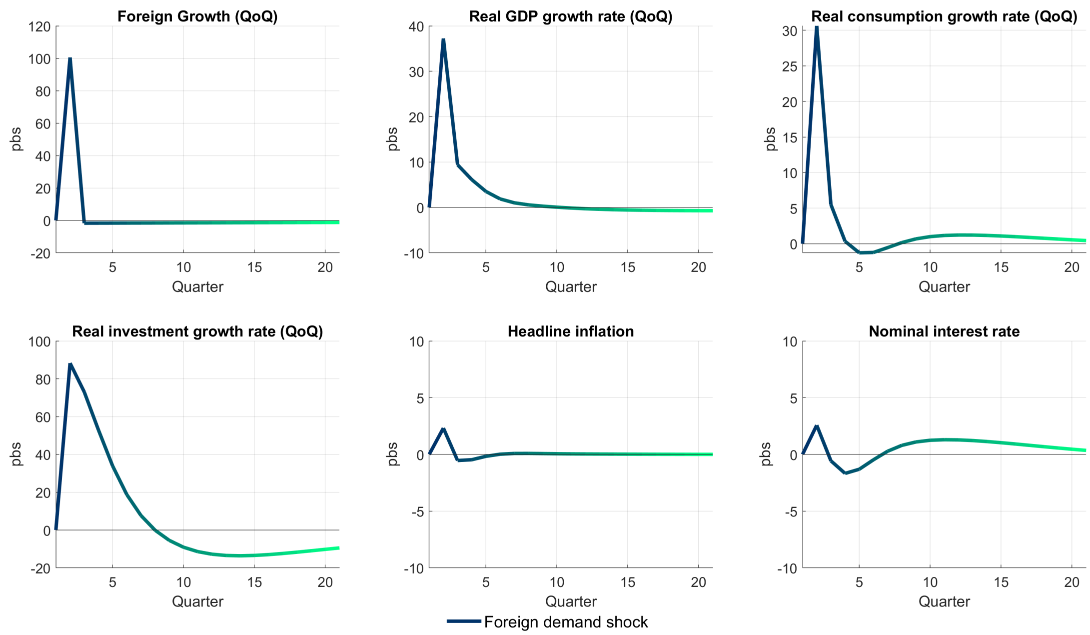


Figure 4.1: Foreign demand shock

We now analyze the effects of a shock to import prices in foreign currency. As shown in Figure 4.2, this shock leads to higher imported inflation. However, due to input adjustment costs in the production of consumption, investment, and raw materials, higher import inflation does not transfer one-to-one to domestic inflation. Regarding economic activity, higher import prices increase production costs, leading to a decline in GDP, consumption, and investment. The rise in inflation also prompts an increase in the policy interest rate.

The final shock analyzed in this section is a change in the foreign interest rate. As shown in Figure 4.3, an increase in foreign borrowing costs dampens economic activity, leading to declines in GDP, consumption, investment, and imports. Consequently, inflation initially declines, prompting the Central Bank to lower the interest rate. However, to uphold uncovered interest rate parity during the transition, the monetary authority eventually raises the policy interest rate.

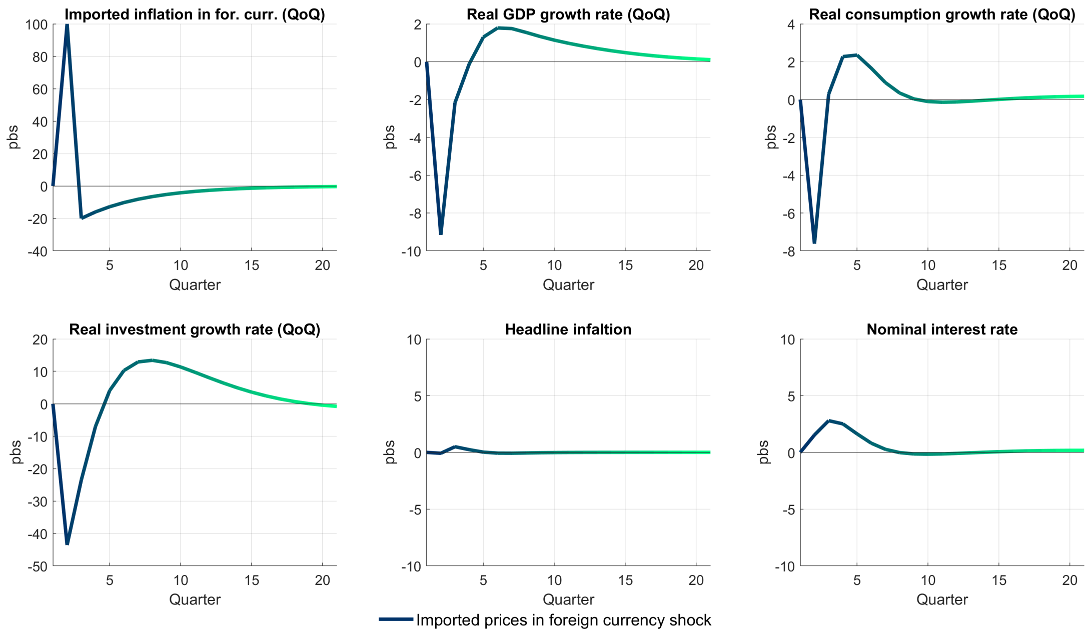


Figure 4.2: Imported price in foreign currency shock

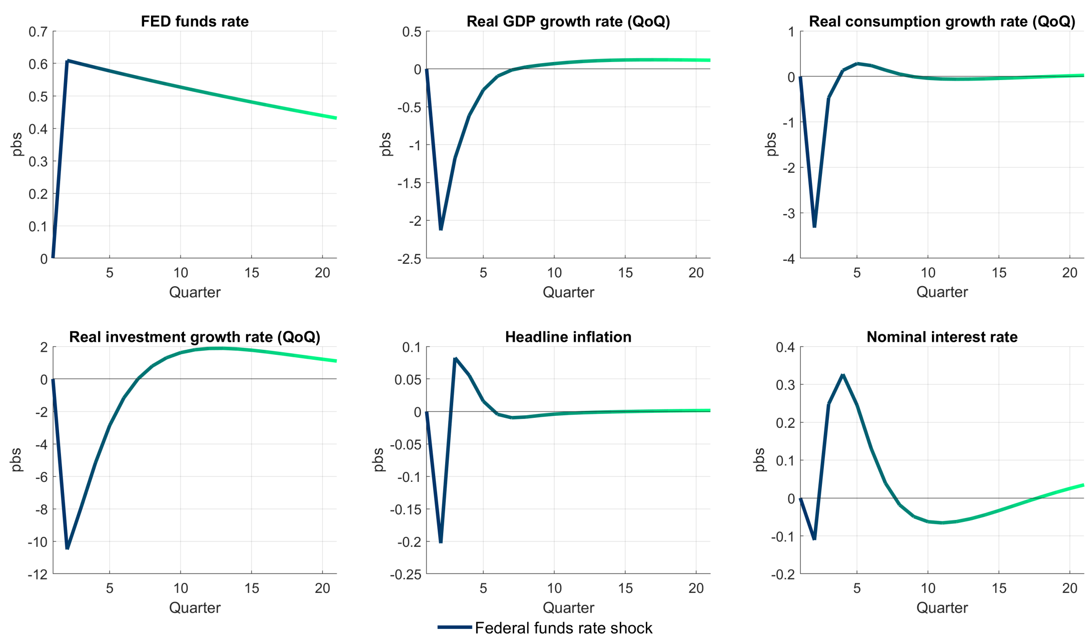


Figure 4.3: FED Funds rate shock

4.1.2 Domestic shocks

On the domestic side, we begin by describing the effects of a positive productivity shock (see Figure 4.4). A rise in productivity increases the marginal product of capital, labor, and raw materials. As a result, overall output increases, driving up the demand for production factors. The resulting income gains boost household consumption and investment. Additionally, greater productivity reduces production costs, which in turn lowers headline inflation. In response, the Central Bank reduces the policy interest rate. From a quantitative perspective, an efficiency shock significantly impacts inflation and monetary policy.

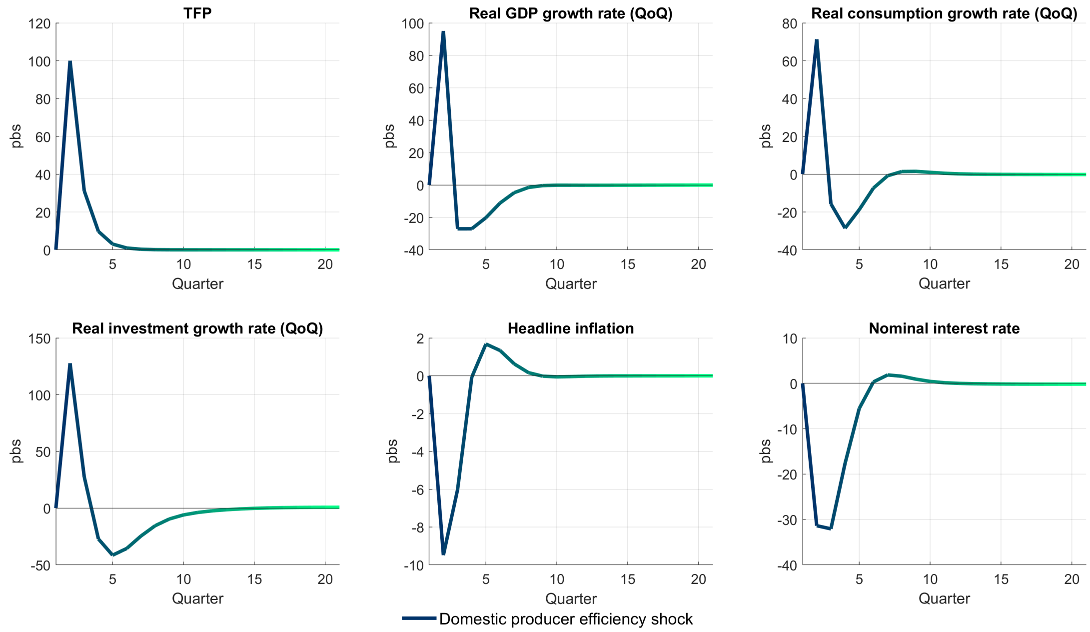


Figure 4.4: Productivity shock

Continuing with domestic shocks, we report a negative demand shock that reduces both consumption and investment. As shown in Figure 4.5, lower demand for consumption and investment reduces GDP growth and causes inflation to fall. In response, the Central Bank cuts the policy interest rate to boost economic activity and prices. In terms of quantities, this shock mainly affects investment demand and, to a lesser extent, consumption growth.

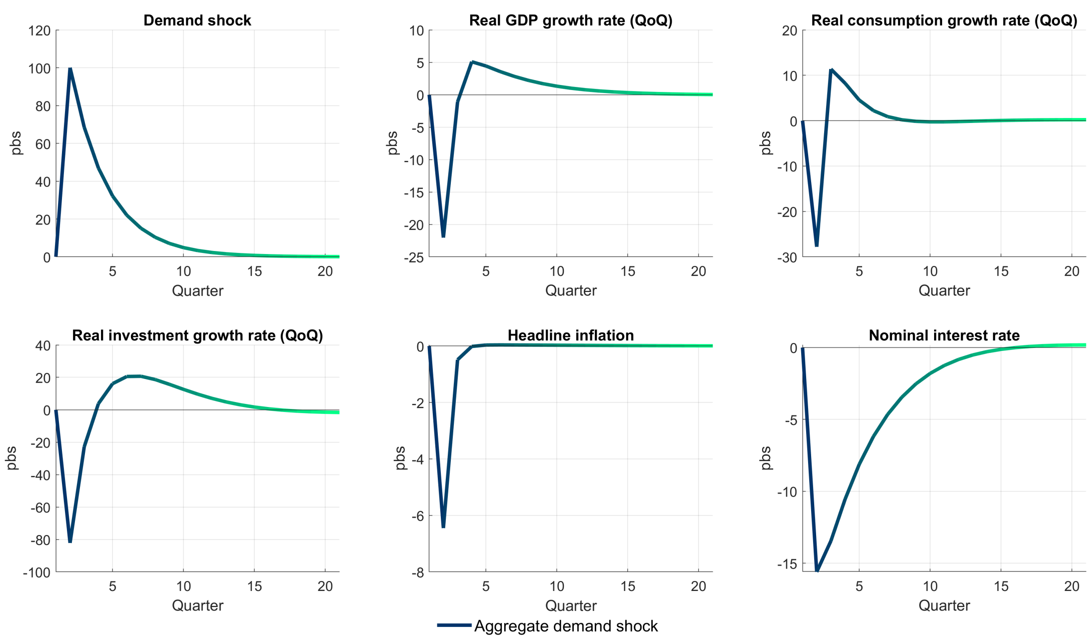


Figure 4.5: Demand contractionary shock

For identification purposes, we include an investment-specific efficiency shock, which increases productivity in the production of investment goods. As depicted in Figure 4.6, higher efficiency in investment production increases the demand for investment goods as their relative price drops. This, in turn, reduces the demand for consumption on impact. Higher investment expands production capacity through capital accumulation, thereby boosting economic activity. In terms of inflation, we observe a mild response in aggregate prices and the policy rate. Different from the aggregate demand shock, in this case we observe opposite responses on consumption and investment, and small effects on inflation.

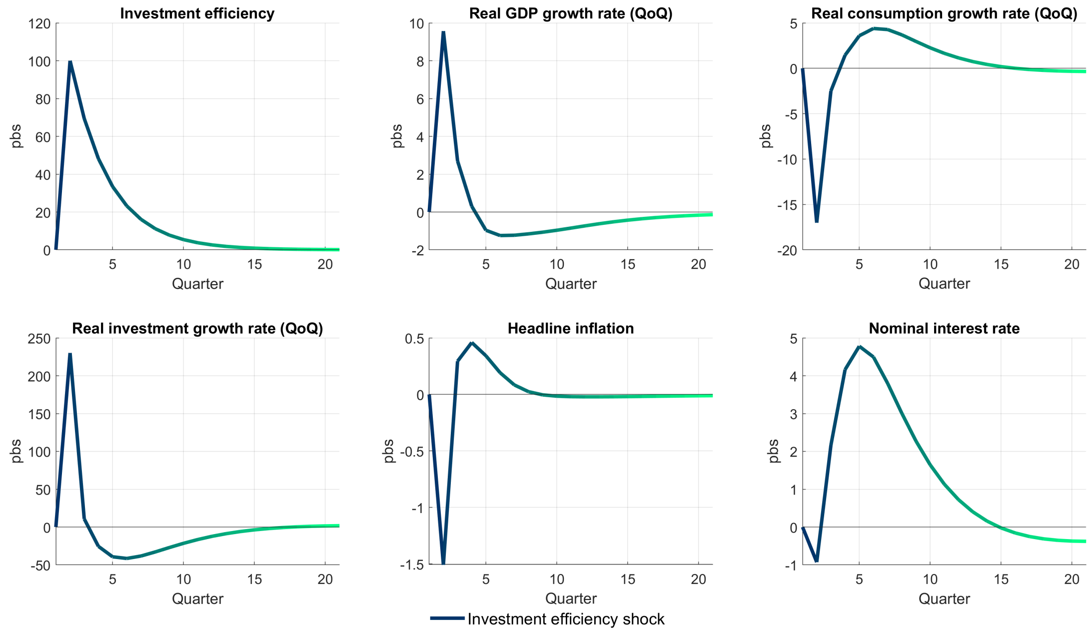


Figure 4.6: Investment efficiency shock

We finally report the aggregate effects of an increase in the monetary policy interest rate. As shown in Figure 4.7, the shock is relatively small and short-lived, primarily affecting prices by reducing inflation. Regarding economic activity, an increase in the interest rate leads to a decline in GDP, consumption, and investment.

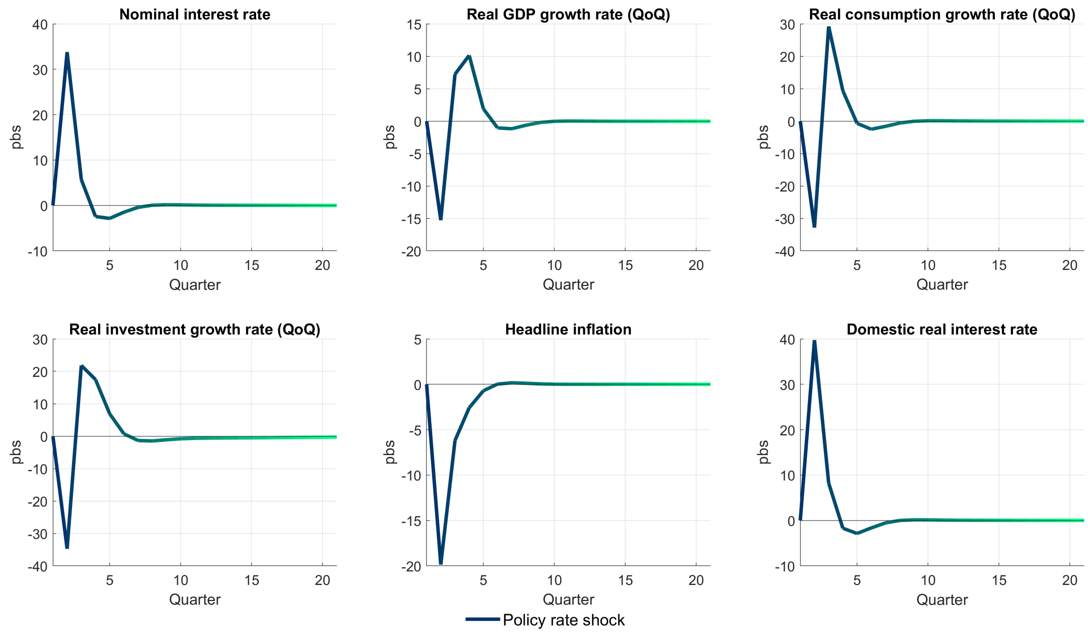


Figure 4.7: Monetary policy shock

4.2 Shock Decomposition

Analyzing business cycles through the model, we can segment the timeline from 2003 to 2019 into three distinct periods: the pre-financial crisis phase (2003-2008), the global crisis and the commodities boom (2009-2014), and the pre-pandemic phase (2015-2019). This classification allows us to analyze how different economic shocks have impacted the model's dynamics in each phase, identifying the main drivers of economic activity and their effects on key variables. Furthermore, it facilitates the comparison of shock effects across different contexts, providing a clearer understanding of economic evolution over time.

We report the shock decompositions for key macroeconomic aggregates, including the growth rates of GDP, consumption, investment, and exports, as well as CPI inflation and the policy interest rate. As shown in Figure 4.8a, real GDP growth in Colombia exceeded its potential level before the global financial crisis of 2008, driven mainly by trend shocks and favorable external conditions, such as import and export prices and strong foreign demand, particularly between 2003 and 2006. However, the foreign interest rate acted as a counteracting force, especially between 2004 and 2007.

During the global financial crisis, the decline in output growth was primarily explained by weaker foreign demand, falling foreign prices, and adverse domestic supply and demand shocks. In contrast, relaxed financial conditions in the U.S. provided some support for output growth during this period. Around 2011–2012, Colombia experienced another economic boom, this time fueled by a positive shock in export prices, especially oil. During these years, the economy once again grew above its potential level, driven by both permanent and transitory productivity shocks. The reversal in oil prices in 2014 explains the subsequent slowdown in GDP growth.

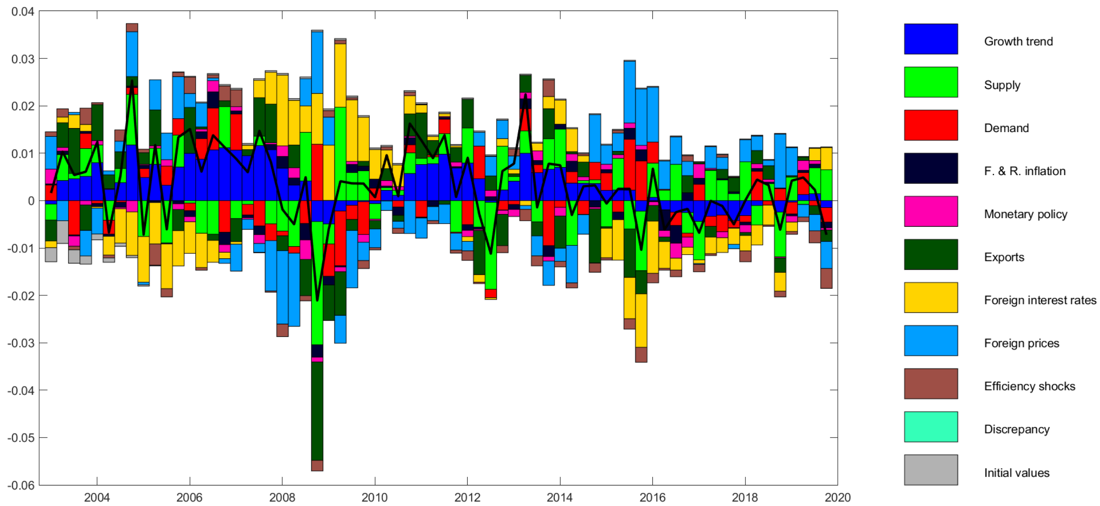


Figure 4.8: Shock decomposition: Real GDP Growth Rate (QoQ)

The aggregation of shocks includes: *Growth trend* (technological progress), *Supply* (Productivity), *Demand* (Demand, demand investment), *F. & R. inflation* (food and regulated inflation), *monetary policy* (policy rate and dynamic inflation target), *exports* (exports efficiency, export prices, foreign demand), *foreign interest rates* (FED fund rate, risk premium), *foreign prices* (foreign inflation, imported inflation, imports efficiency), *efficiency* (investment), *discrepancy* (consumption).

Regarding consumption dynamics (see Figure 4.9a), we observe a significant contribution from trend supply shocks, particularly between 2003 ad 2014. However, other foreign and domestic forces also play an important role. In particular, foreign financial conditions and import prices account for a substantial share of consumption volatility. Another key driver of consumption dynamics is domestic demand shocks. Unlike GDP growth, domestic monetary policy has a more pronounced influence on consumption dynamics. For accounting purposes consumption is also explained by a discrepancy terms that captures the additional volatility in consumption.

Focusing on specific periods, the consumption boom before 2007 was driven by trend shocks, whereas the slowdown and contraction during the global financial crisis were primarily explained by weaker foreign demand and negative supply shocks. Between 2005 and 2007, a decline in import prices also contributed significantly to consumption growth. However, this trend reversed in 2014, becoming a negative force for consumption. A similar pattern is observed in investment dynamics (see Figure 4.10a).

Investment, as reported in Table 4.1, is the most volatile component of GDP. Its behavior is influenced by the domestic growth trend but is primarily driven by foreign financial conditions and import prices. The additional volatility can be attributed to investment-specific technology shocks. Domestic demand shocks also play a significant role during certain periods, such as the pre-crisis boom and the global financial crisis. In 2006, domestic demand helped offset the impact of less favorable global financial conditions.

As expected, export growth is primarily driven by foreign factors, such as foreign demand and foreign prices. The foreign interest rate also influences exports through its impact on the real exchange rate. Other shocks that are not fully captured by foreign prices and U.S. GDP growth are included in the efficiency of export growth, which, in certain periods, serves as the main driver of export dynamics. During the oil price boom around 2012, exports grew mainly due to rising foreign prices. Similarly, the decline in exports during 2014–2015 was driven by falling oil prices.

Headline inflation, presented in Figure 4.12a, exhibits different patterns throughout the analyzed period. During the first phase, between 2003 and 2008, inflation was primarily driven by rising food and regulated goods

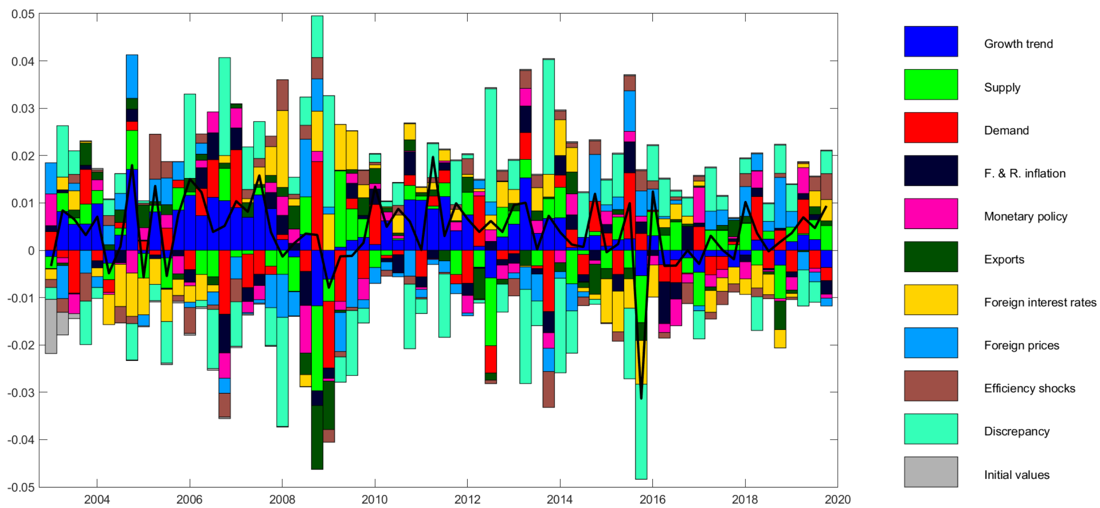


Figure 4.9: Shock decomposition - Real Consumption Growth Rate (QoQ)

The aggregation of shocks includes: *Growth trend* (technological progress), *Supply* (Productivity), *Demand* (Demand, demand investment), *F. & R. inflation* (food and regulated inflation), *monetary policy* (policy rate and dynamic inflation target), *exports* (exports efficiency, export prices, foreign demand), *foreign interest rates* (FED fund rate, risk premium), *foreign prices* (foreign inflation, imported inflation, imports efficiency), *efficiency* (investment), *discrepancy* (consumption).

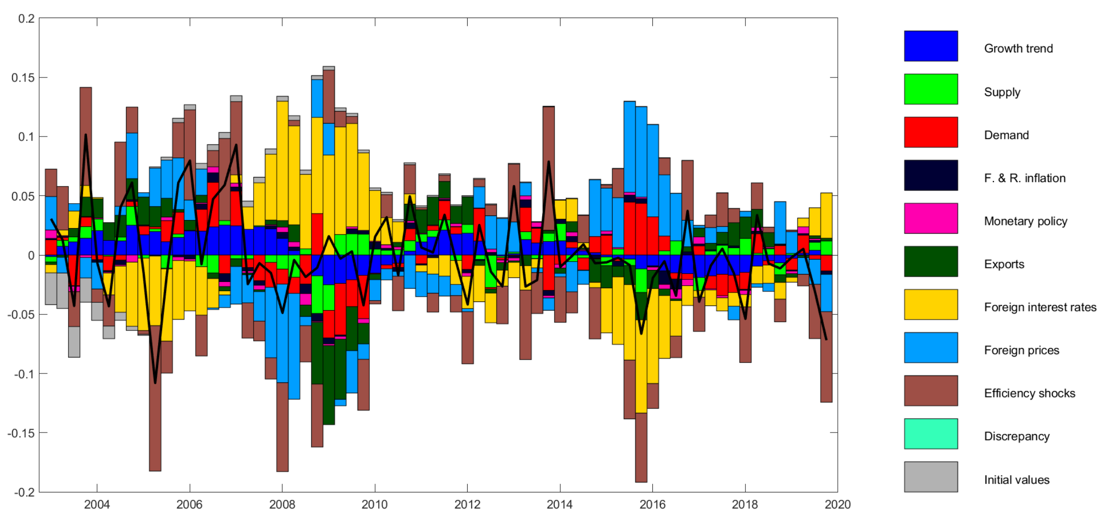


Figure 4.10: Shock decomposition - Real Investment Growth Rate (QoQ)

The aggregation of shocks includes: *Growth trend* (technological progress), *Supply* (Productivity), *Demand* (Demand, demand investment), *F. & R. inflation* (food and regulated inflation), *monetary policy* (policy rate and dynamic inflation target), *exports* (exports efficiency, export prices, foreign demand), *foreign interest rates* (FED fund rate, risk premium), *foreign prices* (foreign inflation, imported inflation, imports efficiency), *efficiency* (investment), *discrepancy* (consumption).

prices—shocks to which monetary policy does not react—and, to a lesser extent, by domestic and external demand pressures. Subsequently, between 2009 and 2015, inflation became less volatile, reflecting weaker demand pressures and stabilized food prices. In 2016, climate-related shocks (El Niño) affected the food basket, driving up CPI inflation. In response, monetary policy increased the interest rate while remaining expansionary. Toward the end of the period, as the effects of this event dissipated, inflation dynamics were shaped by demand pressures, counterbalanced by supply shocks. On balance, headline inflation has responded more strongly to transitory

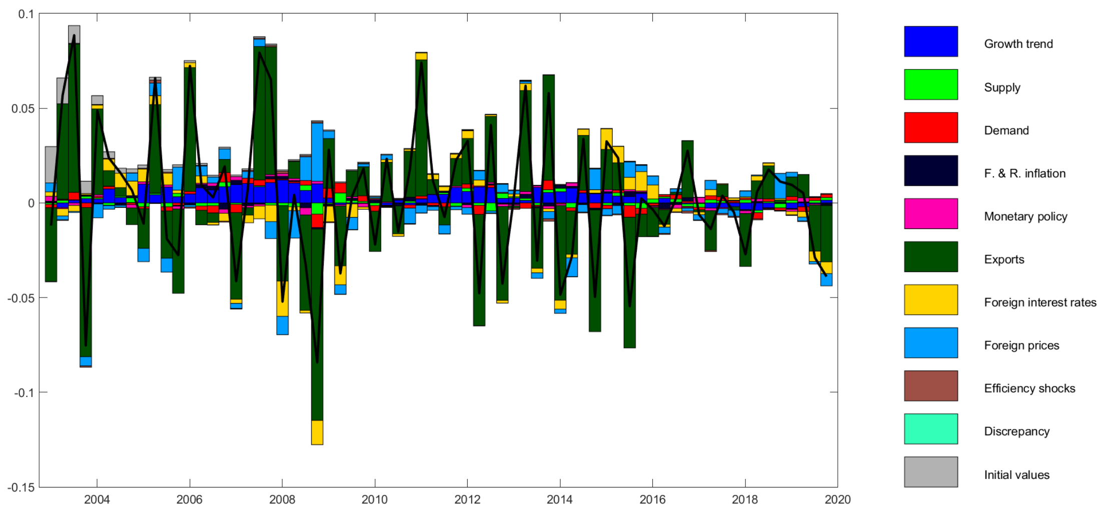


Figure 4.11: Shock decomposition - Real Exports Growth Rate (QoQ)

The aggregation of shocks includes: *Growth trend* (technological progress), *Supply* (Productivity), *Demand* (Demand, demand investment), *F. & R. inflation* (food and regulated inflation), *monetary policy* (policy rate and dynamic inflation target), *exports* (exports efficiency, export prices, foreign demand), *foreign interest rates* (FED fund rate, risk premium), *foreign prices* (foreign inflation, imported inflation, imports efficiency), *efficiency* (investment), *discrepancy* (consumption).

supply shocks, such as those affecting food and regulated goods, which typically do not trigger a reaction in monetary policy. Finally, the growth of the economy and transitory demand shocks have played a more limited role in explaining inflation.

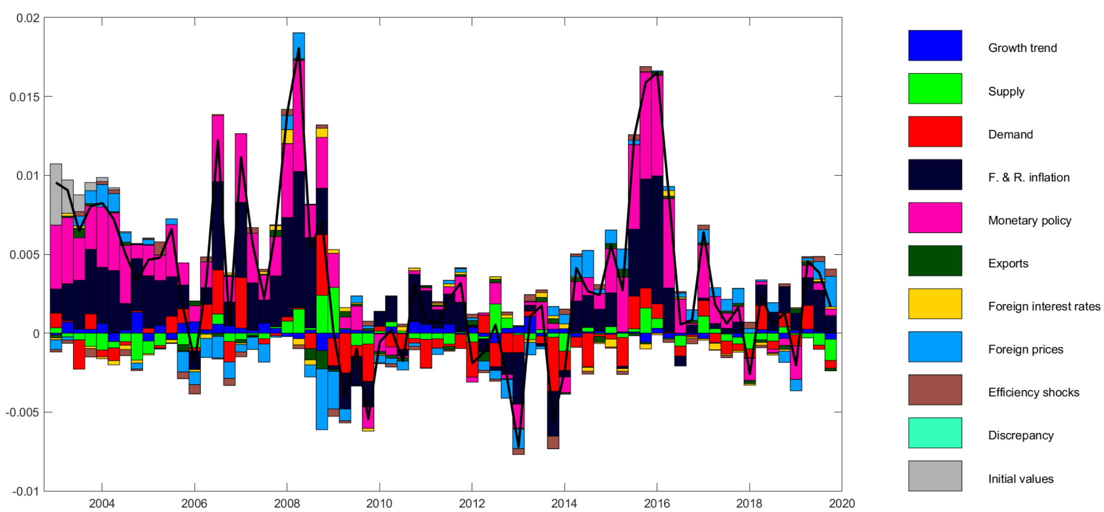


Figure 4.12: Shock decomposition - Headline inflation (QoQ)

The aggregation of shocks includes: *Growth trend* (technological progress), *Supply* (Productivity), *Demand* (Demand, demand investment), *F. & R. inflation* (food and regulated inflation), *monetary policy* (policy rate and dynamic inflation target), *exports* (exports efficiency, export prices, foreign demand), *foreign interest rates* (FED fund rate, risk premium), *foreign prices* (foreign inflation, imported inflation, imports efficiency), *efficiency* (investment), *discrepancy* (consumption).

Finally, the monetary policy rate, presented in Figure 4.13a, varies over the analyzed period. Initially, it remains close to its steady-state level, reflecting a balance between technological progress, demand pressures, supply factors, and foreign price dynamics. Subsequently, it falls below its long-term level, driven by a contraction in domestic demand and adverse external conditions, despite rising supply-side pressures from higher foreign prices. In the period preceding the pandemic, a reversal of these dynamics—stronger domestic demand and easing external constraints—kept the policy rate below its long-term equilibrium.

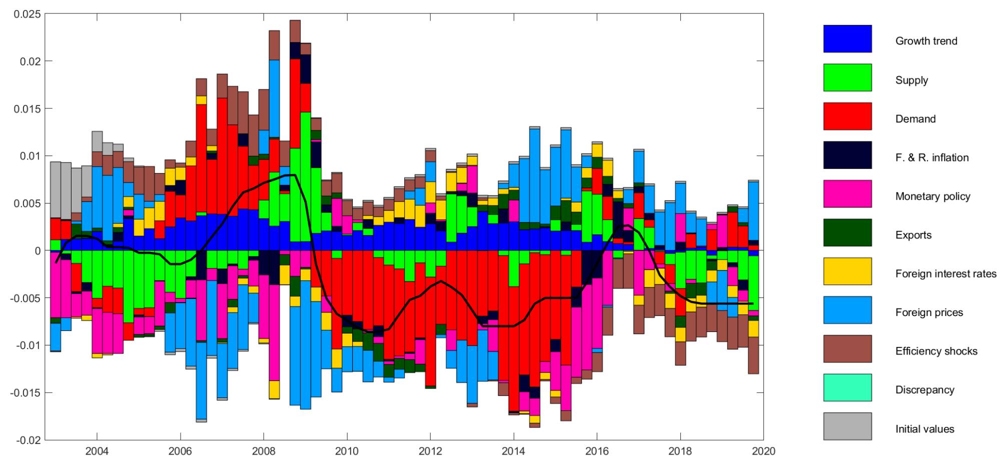


Figure 4.13: Shock decomposition - Policy Rate

The aggregation of shocks includes: *Growth trend* (technological progress), *Supply* (Productivity), *Demand* (Demand, demand investment), *F. & R. inflation* (food and regulated inflation), *monetary policy* (policy rate and dynamic inflation target), *exports* (exports efficiency, export prices, foreign demand), *foreign interest rates* (FED fund rate, risk premium), *foreign prices* (foreign inflation, imported inflation, imports efficiency), *efficiency* (investment), *discrepancy* (consumption).

4.3 Filtering Covid

Even though the model is estimated over the period 2000Q1–2019Q4, we filter the observed data through the Covid period and beyond, up to 2024Q3. The pandemic shock induced substantial deviations from the steady state that cannot be fully captured by structural shocks due to the mean-reverting nature of the model solution. To account for these disruptions, we introduce tailored *Covid shocks*, which absorb the disturbances in key exogenous variables, following the approach of Faria-e-Castro (2024) inspired by Ferroni et al. (2022).

The Covid deviations are introduced as i.i.d. shocks to the processes of certain exogenous variables, ensuring that they do not exhibit persistence beyond their general equilibrium effects. We define a transformation to seven exogenous variables, the aggregate demand ζ_t^d, investment demand ζ_t^i, investment combination efficiency Z_t^i, domestic total factor productivity Z_t^q, monetary policy shock $\zeta_t^{i^{nom}}$, foreign demand $y^\star$, and the foreign risk-free interest rate $i^{nom,Fed}$.

$$\zeta_t = \tilde{\zeta}_t(1+\epsilon_t^{\zeta,covid}) \quad \text{with} \quad \zeta_t \in \{\zeta_t^d, \zeta_t^i, Z_t^i, Z_t^q, \zeta_t^{i^{nom}}, y^\star, i^{nom,Fed}\} \tag{4.1}$$

where $\tilde{\zeta}_t$ is the model-consistent definition for each exogenous variable (e.g. a multiplicative AR-1 process) and the model incorporates ζ_t in its equations. We treat $\epsilon_t^{\zeta,covid}$ as an observable variable whose value is missing during the quarters between 2020Q1 and 2021Q2, the window in which the lockdown and subsequent covid-related policies were the strongest, and equal to zero in every other period. This allows the Kalman smoother to estimate the contribution of the pandemic to each exogenous variable as a further external source of deviation to the structural shocks.

Figure 4.14 displays the smoothed Covid shocks during the pandemic (2020Q1–2021Q2). The model attributes the

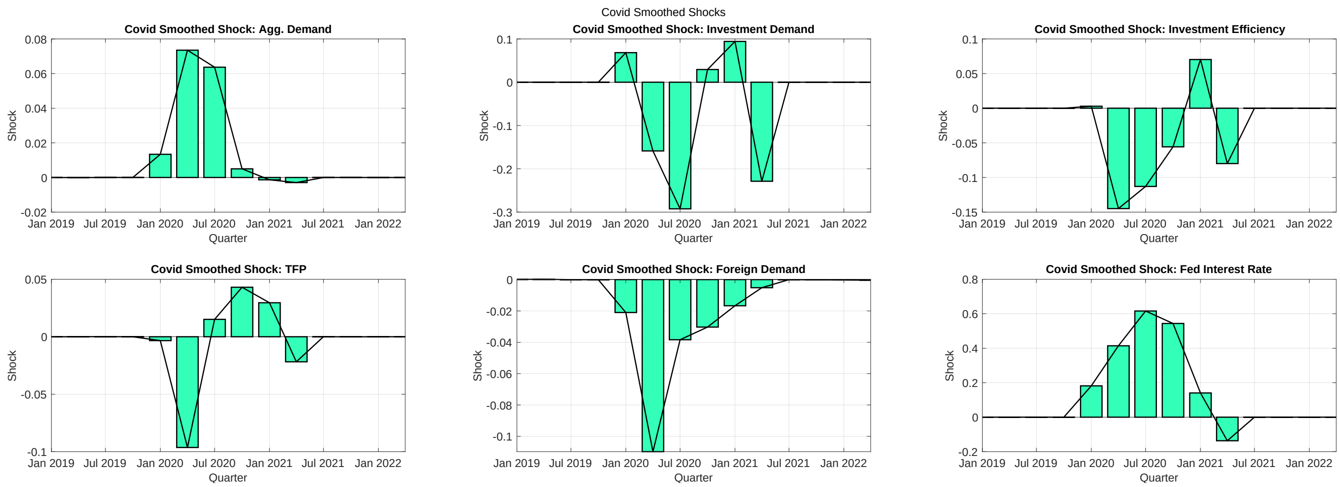


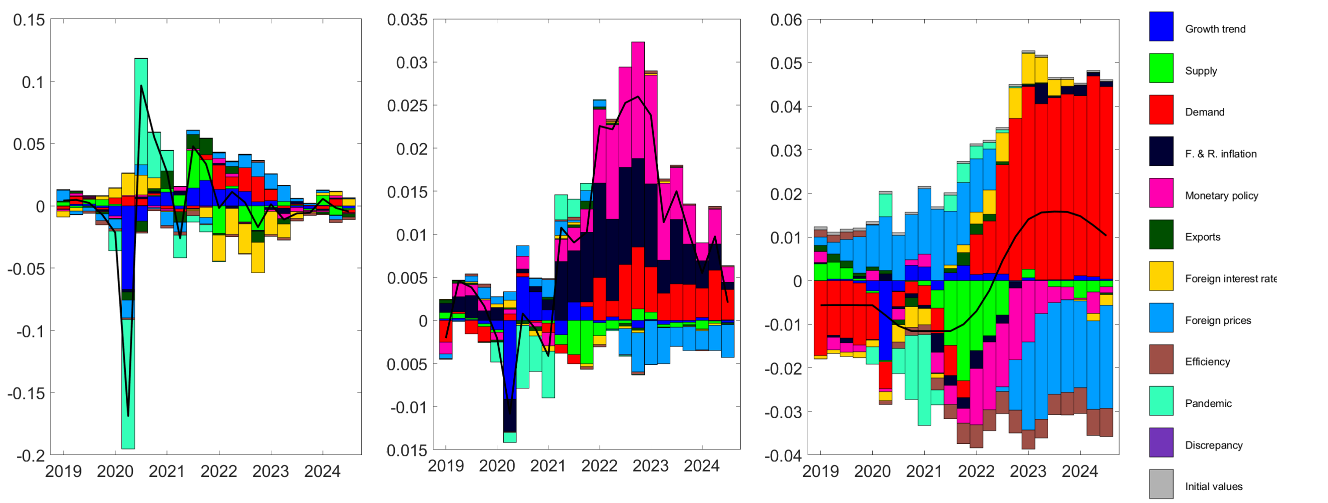
Figure 4.14: Time series for smoothed Covid shocks.

deviation in aggregate demand to rising consumption prices, a decline in investment demand and its efficiency, and a negative productivity shock. This aligns with the immediate contraction of both demand and supply curves due to lockdowns. Regarding the exogenous series for foreign demand and the Fed interest rate, the model captures a drop in demand alongside a positive interest rate shock, reflecting the reduction in interest rates that lowered international funding costs.

On the other hand, Figure 4.15 plots the shock decomposition for the real output growth rate, headline inflation and the policy rate. Note that during the 2020Q2 fall, most of the output growth rate series is explained by the combination of the pandemic shocks, which drive the greater volatility during the covid quarters. It also explains part of the drop in prices and policy rate.

Figure 4.15: Pre & post pandemic shock decomposition

(a) Real output growth rate (b) Headline inflation (c) Policy rate



The aggregation of shocks includes: *Growth trend* (technological progress), *Supply* (Productivity), *Demand* (Demand, demand investment), *F. & R. inflation* (food and regulated inflation), *monetary policy* (policy rate and dynamic inflation target), *exports* (exports efficiency, export prices, foreign demand), *foreign interest rates* (FED fund rate, risk premium), *foreign prices* (foreign inflation, imported inflation, imports efficiency), *efficiency* (investment), *discrepancy* (consumption).

5 Concluding Remarks

We develop a New Keynesian DSGE model for policy analysis and forecasting in Colombia. The model aligns with recent advancements in the economic literature and incorporates novel features, such as dynamic trade elasticity, to better capture Colombia's business cycle. We calibrate and estimate the model using Colombian and U.S. data.

Filtering the data with estimated parameters, we find that before the 2008 global financial crisis, Colombia's economy expanded beyond its potential level, mainly due to trend shocks. During this period, foreign demand also contributed positively, particularly between 2003 and 2006, though foreign interest rates partially offset these effects. The 2008 crisis led to a slowdown, primarily driven by declining foreign demand, falling foreign prices, and temporary total factor productivity (TFP) shocks. However, eased financial conditions in the U.S. helped cushion the impact on output growth. In 2011–2012, Colombia experienced another economic boom, this time fueled by a favorable shock in export prices, especially oil.

The Covid-19 pandemic introduced an unprecedented shock to the Colombian economy, significantly increasing macroeconomic volatility. To capture these disruptions, we extend the model's filtering process beyond its estimation period (2000Q1–2019Q4) up to 2024Q3, incorporating *Covid shocks* that absorb deviations in key exogenous variables. The shock decomposition reveals that pandemic-related shocks were the primary drivers of the sharp contraction in output during 2020Q2, as well as the drop in inflation and the policy rate, reflecting the combined effects of weakened demand and policy responses. By explicitly modeling these deviations, the framework ensures a more accurate assessment of the economy's post-pandemic recovery.

Impulse response analysis confirms key economic mechanisms. A positive foreign demand shock stimulates economic activity through increased exports, raising income and boosting consumption and investment. In contrast, rising import prices and higher foreign interest rates reduce output. On the domestic side, higher productivity lowers marginal costs, spurring economic growth while reducing inflation and the policy interest rate. Conversely, a positive demand shock increases consumption, investment, and inflation, prompting the Central Bank to raise interest rates.

Further research will focus on enhancing the model's forecasting capabilities, particularly in assessing Colombia's economic recovery after the Covid-19 shock. Future improvements include refining the government sector representation, incorporating a hand-to-mouth agent to better reflect the financial constraints of many Colombian households, and exploring alternative pandemic shock specifications to better understand the long-term implications of extreme macroeconomic disruptions.

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Appendix

A Complete model: Consistent with Dynare equations

A.1 Model equations (consistent with Dynare code)

Trend growth:

$$1 + g_t^{nz} = (1+\bar{n})(1+g_t^z) \tag{A.1}$$

Household Problem:

$$\delta(u_t^k) = \bar{\delta}_0 + \delta_1(u_t^k - 1) + \frac{\delta_2}{2}(u_t^k - 1)^2 \tag{A.2}$$

$$u_t^k = 1 \tag{A.3}$$

$$k_t = \left[1 - \delta_t - \frac{\kappa}{2}\left(\frac{i_t(1+g_t^{nz})}{k_{t-1}} - (\delta + g^{nz})\right)^2\right]\frac{k_{t-1}}{1+g_t^{nz}} + i_t \tag{A.4}$$

$$\Delta z_t = \Delta s_t + \pi_t^{\star} - \pi_t \tag{A.5}$$

$$\tilde{\beta}_t \equiv \beta(1+\overline{n})(1+g_t^z)^{1-\sigma} \tag{A.6}$$

$$\tau_t = z_t rem_t \tag{A.7}$$

$$\zeta_t^D \lambda_t = \left(c_t - \frac{\hbar c_{t-1}}{1+g_t^z}\right)^{-\sigma} - \mathbb{E}_t \frac{\hbar \tilde{\beta}_{t+1}}{1+g_{t+1}^z}\left(\frac{\zeta_{t+1}^u}{\zeta_t^u}\right)\left(c_{t+1} - \frac{\hbar c_t}{1+g_{t+1}^z}\right)^{-\sigma} \tag{A.8}$$

$$SDF_{t,t+1} \equiv \tilde{\beta}_{t+1}\frac{\lambda_{t+1}}{\lambda_t} \tag{A.9}$$

$$1 = \mathbb{E}_t \frac{SDF_{t,t+1}}{(1+g_{t+1}^{nz})}\left(\frac{\zeta_{t+1}^u}{\zeta_t^u}\right)\left(\frac{1+i_t^{nom}}{1+\pi_{t+1}}\right) \tag{A.10}$$

$$1 = \mathbb{E}_t \frac{SDF_{t,t+1}}{(1+g_{t+1}^{nz})}\left(\frac{\zeta_{t+1}^u}{\zeta_t^u}\right)\left(\frac{1+i_t^{\star,nom}}{1+\pi_{t+1}^{\star}}\right)\frac{z_{t+1}}{z_t} \tag{A.11}$$

$$\lambda_t^i = \mathbb{E}_t(\beta(1+\bar{n}))\frac{\zeta_{t+1}^u}{\zeta_t^u}(1+g_{t+1}^z)^{1-\sigma}\Bigg\{\lambda_{t+1}\left(\frac{r_{t+1}^k u_{t+1}^k}{1+g_{t+1}^{nz}}\right)$$
$$+\lambda_{t+1}^i\left[\kappa\left(\frac{i_{t+1}(1+g_{t+1}^{nz})}{k_t} - (\delta+g^{nz})\right)\frac{i_{t+1}}{k_t} + \left(1 - \delta_{t+1} - \frac{\kappa}{2}\left(\frac{i_{t+1}(1+g_{t+1}^{nz})}{k_t} - (\delta+g^{nz})\right)^2\right)\frac{1}{1+g_{t+1}^{nz}}\right]\Bigg\} \tag{A.12}$$

$$\lambda_t \zeta_t^i \zeta_t^D p_t^i = \lambda_t^i\left[1 - \kappa\left(\frac{i_t(1+g_t^{nz})}{k_{t-1}} - (\delta+g^{nz})\right)\right] \tag{A.13}$$

$$\Upsilon_t^w = \frac{\phi_w}{2}\left(\frac{\varpi_t(l)(1+\pi_t)(1+g_t^z)}{\varpi_{t-1}(l)(1+\pi_{t-1})(1+g_{t-1}^z)} - 1\right)^2 \tag{A.14}$$

$$(1+\pi_t)(1+g_t^z) = 1 + \pi_t^z \tag{A.15}$$

(A.16)

Foreign interest rate and risk premium:

$$(1+i_t^{nom,\star}) = (1+i_t^{nom,FED})(1+i_t^{prem}) \tag{A.17}$$

$$i_t^{prem} = \zeta_t^{i^{prem}} + \psi^{i^{prem}}\left[\exp\left(z_t\frac{b_t^\star}{y_t} - z\frac{b^\star}{y}\right) - 1\right] \tag{A.18}$$

New Keynesian Phillips Curve for wages:

$$\begin{aligned}\left(\frac{H_t}{Z_t^w}(\zeta_t^w)^{\frac{\varepsilon_{w,t}}{\varepsilon_{w,t-1)}}}\right)^{1+\eta}\frac{\psi_h\varepsilon_{w,t}}{\omega_t\lambda_t} &= (\varepsilon_{w,t}-1)\frac{H_t}{Z_t^w}(\zeta_t^w)^{\frac{\varepsilon_{w,t}}{\varepsilon_{w,t-1}}} \\ &+ \phi_w\left(\frac{\omega_t}{\omega_{t-1}}\frac{(1+\pi_t^z)}{(1+\pi_{t-1}^z)} - 1\right)\frac{(1+\pi_t^z)}{\omega_{t-1}(1+\pi_{t-1}^z)}p_t^q q_t \\ &- \mathbb{E}_t SDF_{t,t+1}\frac{\zeta_{t+1}^u}{\zeta_t^u}\left[\phi_w\left(\frac{\omega_{t+1}(1+\pi_{t+1}^z)}{\omega_t(1+\pi_t^z)} - 1\right)\frac{\omega_{t+1}(1+\pi_{t+1}^z)}{\omega_t^2(1+\pi_t^z)}p_{t+1}^q q_{t+1}\right]\end{aligned} \tag{A.19}$$

Production of Consumption Final Goods:

$$\begin{aligned}Z_t^c\left((\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^D\right)^{\frac{\omega_c-1}{\omega_c}} + (1-\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^{IM}\right)^{\frac{\omega_c-1}{\omega_c}}\right)^{\frac{1}{\omega_c-1}}(\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^D\right)^{-\frac{1}{\omega_c}} &= p_t^q + \phi_D^c\left(\frac{c_t^D}{c_{t-1}^D} - 1\right)\frac{1}{c_{t-1}^D} \\ &- \mathbb{E}_t SDF_{t,t+1}\frac{\zeta_{t+1}^u}{\zeta_t^u}(1+\pi_{t+1})\left(\phi_D^c\left(\frac{c_{t+1}^D}{c_t^D} - 1\right)\frac{c_{t+1}^D}{(c_t^D)^2}\right)\end{aligned} \tag{A.20}$$

$$\begin{aligned}Z_t^c\left((\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^D\right)^{\frac{\omega_c-1}{\omega_c}} + (1-\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^{IM}\right)^{\frac{\omega_c-1}{\omega_c}}\right)^{\frac{1}{\omega_c-1}}(1-\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^{IM}\right)^{-\frac{1}{\omega_c}} &= p_t^{IM} + \phi_{IM}^c\left(\frac{c_t^{IM}}{c_{t-1}^{IM}} - 1\right)\frac{1}{c_{t-1}^{IM}} \\ &- \mathbb{E}_t SDF_{t,t+1}\frac{\zeta_{t+1}^u}{\zeta_t^u}(1+\pi_{t+1})\left(\phi_{IM}^c\left(\frac{c_{t+1}^{IM}}{c_t^{IM}} - 1\right)\frac{c_{t+1}^{IM}}{(c_t^{IM})^2}\right)\end{aligned} \tag{A.21}$$

$$c_t + \frac{\phi_d^c}{2}\left(\frac{c_t^D}{c_{t-1}^D} - 1\right)^2 + \frac{\phi_{im}^c}{2}\left(\frac{c_t^{IM}}{c_{t-1}^{IM}} - 1\right)^2 = Z_t^c\left[(\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^D\right)^{\frac{\omega_c-1}{\omega_c}} + (1-\gamma^c)^{\frac{1}{\omega_c}}\left(c_t^{IM}\right)^{\frac{\omega_c-1}{\omega_c}}\right]^{\frac{\omega_c}{\omega_c-1}} \tag{A.22}$$

Production of Investment Final Goods:

$$\begin{aligned}Z_t^i\left(\left(\gamma^i\right)^{\frac{1}{\omega_i}}\left(i_t^D\right)^{\frac{\omega_i-1}{\omega_i}} + \left(1-\gamma^i\right)^{\frac{1}{\omega_i}}\left(i_t^{IM}\right)^{\frac{\omega_i-1}{\omega_i}}\right)^{\frac{1}{\omega_i-1}}\left(\gamma^i\right)^{\frac{1}{\omega_i}}\left(i_t^D\right)^{-\frac{1}{\omega_i}} &= \frac{p_t^q}{p_t^i} + \phi_D^i\left(\frac{i_t^D}{i_{t-1}^D} - 1\right)\frac{1}{i_{t-1}^D} \\ &- \mathbb{E}_t SDF_{t,t+1}\frac{\zeta_{t+1}^u}{\zeta_t^u}\frac{p_{t+1}^i}{p_t^i}(1+\pi_{t+1})\left(\phi_D^i\left(\frac{i_{t+1}^D}{i_t^D} - 1\right)\frac{i_{t+1}^D}{(i_t^D)^2}\right)\end{aligned} \tag{A.23}$$

$$\begin{aligned}Z_t^i\left(\left(\gamma^i\right)^{\frac{1}{\omega_i}}\left(i_t^D\right)^{\frac{\omega_i-1}{\omega_i}} + \left(1-\gamma^i\right)^{\frac{1}{\omega_i}}\left(i_t^{IM}\right)^{\frac{\omega_i-1}{\omega_i}}\right)^{\frac{1}{\omega_i-1}}\left(1-\gamma^i\right)^{\frac{1}{\omega_i}}\left(i_t^{IM}\right)^{-\frac{1}{\omega_i}} &= \frac{p_t^{IM}}{p_t^i} + \phi_{IM}^i\left(\frac{i_t^{IM}}{i_{t-1}^{IM}} - 1\right)\frac{1}{i_{t-1}^{IM}} \\ &- \mathbb{E}_t SDF_{t,t+1}\frac{\zeta_{t+1}^u}{\zeta_t^u}\frac{p_{t+1}^i}{p_t^i}(1+\pi_{t+1})\left(\phi_{IM}^i\left(\frac{i_{t+1}^{IM}}{i_t^{IM}} - 1\right)\frac{i_{t+1}^{IM}}{(i_t^{IM})^2}\right)\end{aligned} \tag{A.24}$$

$$i_t + \frac{\phi_d^i}{2}\left(\frac{i_t^D}{i_{t-1}^D} - 1\right)^2 + \frac{\phi_{im}^i}{2}\left(\frac{i_t^{IM}}{i_{t-1}^{IM}} - 1\right)^2 = Z_t^i\left[\left(\gamma^i\right)^{\frac{1}{\omega_i}}\left(i_t^D\right)^{\frac{\omega_i-1}{\omega_i}} + \left(1-\gamma^i\right)^{\frac{1}{\omega_i}}\left(i_t^{IM}\right)^{\frac{\omega_i-1}{\omega_i}}\right]^{\frac{\omega_i}{\omega_i-1}} \tag{A.25}$$

Production of Raw Materials:

$$Z_t^m \left((\gamma^m)^{\frac{1}{\omega_m}} \left(m_t^D\right)^{\frac{\omega_m - 1}{\omega_m}} + (1-\gamma^m)^{\frac{1}{\omega_m}} \left(m_t^{IM}\right)^{\frac{\omega_m-1}{\omega_m}} \right)^{\frac{1}{\omega_m - 1}} (\gamma^m)^{\frac{1}{\omega_m}} \left(m_t^D\right)^{-\frac{1}{\omega_m}} = \frac{p_t^q}{p_t^m} + \phi_D^m \left(\frac{m_t^D}{m_{t-1}^D} - 1 \right) \frac{1}{m_{t-1}^D} \tag{A.26}$$

$$-\mathbb{E}_t SDF_{t,t+1} \frac{\zeta_{t+1}^u}{\zeta_t^u} \frac{p_{t+1}^m}{p_t^m} (1+\pi_{t+1}) \left(\phi_D^m \left(\frac{m_{t+1}^D}{m_t^D} - 1 \right) \frac{m_{t+1}^D}{(m_t^D)^2} \right)$$

$$Z_t^m \left((\gamma^m)^{\frac{1}{\omega_m}} \left(m_t^D\right)^{\frac{\omega_m - 1}{\omega_m}} + (1-\gamma^m)^{\frac{1}{\omega_m}} \left(m_t^{IM}\right)^{\frac{\omega_m-1}{\omega_m}} \right)^{\frac{1}{\omega_m - 1}} (1-\gamma^m)^{\frac{1}{\omega_m}} \left(m_t^{IM}\right)^{-\frac{1}{\omega_m}} = \frac{p_t^{IM}}{p_t^m} + \phi_{IM}^m \left(\frac{m_t^{IM}}{m_{t-1}^{IM}} - 1 \right) \frac{1}{m_{t-1}^{IM}} \tag{A.27}$$

$$-\mathbb{E}_t SDF_{t,t+1} \frac{\zeta_{t+1}^u}{\zeta_t^u} \frac{p_{t+1}^m}{p_t^m} (1+\pi_{t+1}) \left(\phi_{IM}^m \left(\frac{m_{t+1}^{IM}}{m_t^{IM}} - 1 \right) \frac{m_{t+1}^{IM}}{(m_t^{IM})^2} \right)$$

$$m_t + \frac{\phi_d^m}{2} \left(\frac{m_t^D}{m_{t-1}^D} - 1 \right)^2 + \frac{\phi_{im}^m}{2} \left(\frac{m_t^{IM}}{m_{t-1}^{IM}} - 1 \right)^2 = Z_t^m \left[(\gamma^m)^{\frac{1}{\omega_m}} \left(m_t^D\right)^{\frac{\omega_m - 1}{\omega_m}} + (1-\gamma^m)^{\frac{1}{\omega_m}} \left(m_t^{IM}\right)^{\frac{\omega_m-1}{\omega_m}} \right]^{\frac{\omega_m}{\omega_m - 1}} \tag{A.28}$$

Production of Domestic Intermediates:

$$\Upsilon_t^q = \frac{\phi_q}{2} \left(\frac{p_t^q(j)(1+\pi_t)}{p_{t-1}^q(j)(1+\pi_{t-1})} - 1 \right)^2 \tag{A.29}$$

$$k_t^s = u_{t+1}^k k_t \tag{A.30}$$

$$r_t^k = \alpha(1+g_t^{nz}) \left(mc_t^q \frac{q_t}{k_{t-1}^s} \right) \tag{A.31}$$

$$p_t^m = \mu \left(mc_t^q \frac{q_t}{m_t} \right) \tag{A.32}$$

$$\varpi_t = (1-\alpha-\mu) \left(mc_t^q \frac{q_t}{h_t} \right) \tag{A.33}$$

$$mc_t^q = \frac{Z^{\pi^{nfr}}}{Z_t^q \psi_q} \left(\frac{r_t^k}{\alpha} \right)^{\alpha} \left(\frac{p_t^m}{\mu} \right)^{\mu} \left(\frac{\varpi_t}{1-\mu-\alpha} \right)^{1-\mu-\alpha} \tag{A.34}$$

New Keynesian Phillips Curve for Domestic Intermediates:

$$\frac{\varepsilon_{q,t} mc_t}{p_t^q} = (\varepsilon_{q,t} - 1) + \left[\phi_q \left(\frac{p_t^q (1+\pi_t)}{p_{t-1}^q (1+\pi_{t-1})} - 1 \right) \left(\frac{p_t^q}{p_{t-1}^q} \frac{(1+\pi_t)}{(1+\pi_{t-1})} \right) \right]$$

$$- \mathbb{E}_t SDF_{t,t+1} \frac{\zeta_{t+1}^u}{\zeta_t^u} \left[\phi_q \left(\frac{p_{t+1}^q (1+\pi_{t+1})}{p_t^q (1+\pi_t)} - 1 \right) \left(\left(\frac{p_{t+1}^q}{p_t^q} \right)^2 \frac{(1+\pi_{t+1})}{(1+\pi_t)} \right) \frac{q_{t+1}}{q_t} \right] \tag{A.35}$$

$$\frac{p_t^q}{p_{t-1}^q} = \frac{1+\pi_t^q}{1+\pi_t} \tag{A.36}$$

$$\Pi_t^q = \left(p_t^q - mc_t^q \right) q_t - \Upsilon_t^q p_t^q q_t \tag{A.37}$$

Imports:

$$\Pi_t^{im} = (p_t^{im} - z_t p_t^{im,\star}) im_t - \Upsilon_t^{im} p_t^q q_t \tag{A.38}$$

$$im_t = c_t^{IM} + i_t^{IM} + m_t^{IM} \tag{A.39}$$

$$\Upsilon_t^{IM} = \frac{\phi_{IM}}{2} \left(\frac{p_t^{IM}(j)(1+\pi_t)}{p_{t-1}^{IM}(j)(1+\pi_{t-1})} - 1 \right)^2 \tag{A.40}$$

$$\tag{A.41}$$

New Keynesian Phillips Curve for Imports:

$$\begin{aligned}\left[(1-\varepsilon_{im,t}) im_t + \varepsilon_{im,t} z_t \frac{p_t^{IM,\star}}{p_t^{IM}} im_t\right] &= \phi_{IM} \left(\frac{p_t^{IM}(1+\pi_t)}{p_{t-1}^{IM}(1+\pi_{t-1})} - 1 \right) p_t^q q_t \frac{(1+\pi_t)}{p_{t-1}^{IM}(1+\pi_{t-1})} \\ &- \mathbb{E}_t SDF_{t,t+1} \frac{\zeta_{t+1}^u}{\zeta_t^u} \phi_{IM} \left(\frac{p_{t+1}^{IM}(1+\pi_{t+1})}{p_t^{IM}(1+\pi_t)} - 1 \right) p_{t+1}^q q_{t+1} \frac{p_{t+1}^{IM}(1+\pi_{t+1})}{(p_t^{IM})^2(1+\pi_t)}\end{aligned} \tag{A.42}$$

$$p_t^{IM,\star} = (p_{t-1}^{IM,\star})^{\rho^{p^{IM,\star}}} (p^{IM,\star})^{1-\rho^{p^{IM,\star}}} (1+\epsilon_t)^{\rho^{p^{IM,\star}}} \tag{A.43}$$

Exports:

$$ex_t^{real} = Z_t^{ex} \left(\frac{z_t p_t^{ex,\star}}{p_t^q} \right)^{\mu_x} (y_t^\star)^{\mu_y} \tag{A.44}$$

$$ex_t = p_t^q ex_t^{real} \tag{A.45}$$

Taylor rule:

$$i_t^{nom} = \phi_i i_{t-1}^{nom} + (1-\phi_i) \left[i_t^{nom,n} + \phi_\pi \widetilde{\pi}_t + \phi_y \widetilde{y}_t \right] + \epsilon_t^{i^{nom}} \tag{A.46}$$

$$i_t^{nom,n} = \frac{(1+\bar{\pi})(1+g_t^z)^\sigma}{\beta} - 1 \tag{A.47}$$

$$\widetilde{\pi}_t = \mathbb{E}_t \pi_{t+4}^{NFR,A} - \overline{\pi}_t^A \tag{A.48}$$

$$\widetilde{y}_t = y_t - y \tag{A.49}$$

$$\bar{\pi}_t = \bar{\pi}_{t-1}^{\rho_{\bar{\pi}}} \bar{\pi}^{(1-\rho_{\bar{\pi}})} (1+\epsilon_t^{\bar{\pi}}) \tag{A.50}$$

Aggregate Definitions and Equilibrium:

$$tb_t = ex_t - z_t p_t^{IM,*} im_t \tag{A.51}$$

$$\Pi_t = \Pi_t^q + \Pi_t^{im} \tag{A.52}$$

$$\Upsilon_t \equiv \Upsilon_t^{\varpi} + \Upsilon_t^q + \Upsilon_t^{IM} \tag{A.53}$$

$$q_t = c_t^D + i_t^D + m_t^D + ex_t + \Upsilon_t \tag{A.54}$$

$$GDP_t = p_t^q q_t - p_t^m m_t + \Pi_t^{im} \tag{A.55}$$

$$ca_t = -z_t b_t^{\star} + z_t \frac{b_{t-1}^{\star}}{(1+\pi_t^{\star})(1+g_t^{nz})} \tag{A.56}$$

$$-tb_t = z_t rem_t - ca_t - z_t \frac{i_{t-1}^{nom,\star} b_{t-1}^{\star}}{(1+\pi_t^{\star})(1+g_t^{nz})} \tag{A.57}$$

$$\frac{z_t}{z_{t-1}} = 1 + \Delta z_t \tag{A.58}$$

Additional Complementary Definitions:

$$y_t^{Ann} = y_t + \frac{y_{t-1}}{1+g_t^{nz}} + \frac{y_{t-2}}{(1+g_t^{nz})(1+g_{t-1}^{nz})} + \frac{y_{t-3}}{(1+g_t^{nz})(1+g_{t-1}^{nz})(1+g_{t-2}^{nz})} \tag{A.59}$$

$$b_t^{\star,Ann} = \frac{b_t^{\star}}{y_t^{Ann}} \tag{A.60}$$

$$\frac{p_t^{im}}{p_{t-1}^{im}} = \frac{1+\pi_t^{im}}{1+\pi_t} \tag{A.61}$$

$$i_t^{real} = \frac{1+i_t^{nom}}{1+\pi_{t+1}} - 1 \tag{A.62}$$

$$i_t^{real,\star} = \frac{1+i_t^{nom,FED}}{1+\pi_{t+1}^{\star}} - 1 \tag{A.63}$$

$$Z_t^{\pi^{nfr}} = \left(Z_t^{\pi^{foo}}\right)^{\phi^{\pi^{foo}}} \left(Z_t^{\pi^{reg}}\right)^{1-\phi^{\pi^{foo}}} \tag{A.64}$$

$$Z_t^{\pi^{foo}} = \epsilon_t^{\pi^{foo}} + \rho_{\pi^{foo}}(Z_{t-1}^{\pi^{foo}}) + (1-\rho_{\pi^{foo}})(Z^{\pi^{foo}}) \tag{A.65}$$

$$Z_t^{\pi^{reg}} = \epsilon_t^{\pi^{reg}} + \rho_{\pi^{e}g}(Z_{t-1}^{\pi^{reg}}) + (1-\rho_{\pi^{reg}})(Z^{\pi^{reg}}) \tag{A.66}$$

A.2 Model equations: Exogenous process

$$g_t^z = \rho_z g_{t-1}^z + (1-\rho_z) g^z + \epsilon_t^z \tag{A.67}$$
$$\zeta_t^u = \rho_u \zeta_{t-1}^u + (1-\rho_u)\zeta^u + \epsilon_t^u \tag{A.68}$$
$$\zeta_t^D = \rho_D \zeta_{t-1}^D + (1-\rho_D)\zeta^D + \epsilon_t^D \tag{A.69}$$
$$\zeta_t^i = \rho_i \zeta_{t-1}^i + (1-\rho_i)\zeta^i + \epsilon_t^i \tag{A.70}$$
$$\varepsilon_{w,t} = (\varepsilon_{w,t-1})^{\rho_{\varepsilon_w}} (\varepsilon_w)^{1-\rho_{\varepsilon_w}} (1+\epsilon_t^{\varepsilon_w}) \tag{A.71}$$
$$\varepsilon_{q,t} = (\varepsilon_{q,t-1})^{\rho_q} (\varepsilon_q)^{1-\rho_q} (1+\epsilon_t^q) \tag{A.72}$$
$$\varepsilon_{im,t} = (\varepsilon_{im,t-1})^{\rho_{im}} (\varepsilon_{im})^{1-\rho_{im}} (1+\epsilon_t^{im}) \tag{A.73}$$
$$Z_t^w = (Z_{t-1}^w)^{\rho_{Z^w}} (1+\epsilon_t^{Z^w}) \tag{A.74}$$
$$Z_t^c = (Z_{t-1}^c)^{\rho_{Z^c}} (1+\epsilon_t^{Z^c}) \tag{A.75}$$
$$Z_t^i = (Z_{t-1}^i)^{\rho_{Z^i}} (1+\epsilon_t^{Z^i}) \tag{A.76}$$
$$Z_t^m = (Z_{t-1}^m)^{\rho_{Z^m}} (1+\epsilon_t^{Z^m}) \tag{A.77}$$
$$Z_t^q = (Z_{t-1}^q)^{\rho_{z,q}} (Z^q)^{1-\rho_{z,q}} (1+\epsilon_t^{z,q}) \tag{A.78}$$
$$Z_t^{ex} = (Z_{t-1}^{ex})^{\rho_{Z^{ex}}} (1+\epsilon_t^{Z^{ex}}) \tag{A.79}$$
$$\pi_t = \phi^{\pi^{NFR}} \pi_t^{NFR} + \phi^{\pi^{foo}} \pi_t^{foo} + (1-\phi^{\pi^{NFR}} - \phi^{\pi^{foo}}) \pi_t^{reg} + \epsilon_t^\pi \tag{A.80}$$
$$\pi_t^{foo} = \rho_{\pi^{foo}} \pi_{t-1}^{foo} + (1-\rho_{\pi^{foo}}) \pi^{foo} + \epsilon_t^{\pi^{foo}} \tag{A.81}$$
$$\pi_t^{reg} = \rho_{\pi^{reg}} \pi_{t-1}^{reg} + (1-\rho_{\pi^{reg}}) \pi^{reg} + \epsilon_t^{\pi^{reg}} \tag{A.82}$$

Foreign Variables

$$p_t^{ex,\star} = (p_{t-1}^{ex,\star})^{\rho_{p^{ex,\star}}} (p^{ex,\star})^{1-\rho_{p^{ex,\star}}} (1+\epsilon_t^{p^{ex,\star}}) \tag{A.83}$$
$$\pi_t^\star = (\pi_{t-1}^\star)^{\rho_{\pi^\star}} (\pi^\star)^{1-\rho_{\pi^\star}} (1+\epsilon_t^{\pi^\star}) \tag{A.84}$$
$$\tag{A.85}$$
$$\tag{A.86}$$
$$rem_t = rem_t^{\rho_{rem}} rem_{ss}^{(1-\rho_{rem})} (1+\epsilon_t^{rem}) \tag{A.87}$$
$$i_t^{nom,FED} = \left(i_{t-1}^{nom,FED}\right)^{\rho_{i^\star,FED}} \left(i^{nom,FED}\right)^{1-\rho_{i^\star,FED}} \left(1+\epsilon_t^{i^{nom,FED}}\right) \tag{A.88}$$
$$\zeta_t^{i^{prem}} = \left(\zeta_{t-1}^{i^{prem}}\right)^{\rho^{i^{prem}}} \left(\zeta^{i^{prem}}\right)^{1-\rho^{i^{prem}}} (1+\epsilon_t^{i^{prem}}) \tag{A.89}$$
$$y_t^\star = (y_{t-1}^\star)^{\rho_{y^\star}} (y^\star)^{1-\rho_{y^\star}} (1+\epsilon_t^{y^\star}) \tag{A.90}$$

B Steady state & Calibration

B.1 Steady state

In this section we describe the steady state calibration and analytical solution. We start defining some ratios for the Colombian economy in the long run, reported on Table 3.1. Given these ratios and some normalizations, we can solve the steady state analytically.

B.1.1 Growth

$$g^{n,z} = (1.0265)^{\frac{1}{4}} - 1 \qquad \text{Quarterly TFP growth} \tag{B.1}$$

$$g^{z} = \frac{(1+g^{n,z})}{(1+\bar{n})} - 1 \qquad \text{Quarterly real GDP growth} \tag{B.2}$$

$$\tilde{\beta} = \beta * (1+\bar{n}) * (1+g^{z})^{(1-\sigma)} \qquad \text{Effective dynamic discount factor} \tag{B.3}$$

$$sdf = \tilde{\beta} \qquad \text{Stochastic discount factor} \tag{B.4}$$

$$g^{n,z,Annual} = \sum_{i=0}^{3} \frac{1}{(1+g^{n,z})^{i}} - 1 \qquad \text{Adjusted Cumulative Annual Growth} \tag{B.5}$$

B.1.2 AR(1) process

Efficiency and cost shocks are normalized to 1 in the steady state:

$$\zeta^{d} = 1 \qquad \text{Aggregate demand} \tag{B.6}$$

$$\zeta^{u} = 1 \qquad \text{Preferences} \tag{B.7}$$

$$\zeta^{i} = 1 \qquad \text{Cost pust investment} \tag{B.8}$$

$$\zeta^{w} = 1 \qquad \text{Cost pust wage} \tag{B.9}$$

$$Z^{c} = 1 \qquad \text{Efficiency of consumption} \tag{B.10}$$

$$Z^{w} = 1 \qquad \text{Efficiency of wage} \tag{B.11}$$

$$Z^{i} = 1 \qquad \text{Efficiency of investment} \tag{B.12}$$

$$Z^{m} = 1 \qquad \text{Efficiency of imported goods} \tag{B.13}$$

$$Z^{q} = 1 \qquad \text{Efficiency of gross producers} \tag{B.14}$$

B.1.3 Inflation and price levels

The steady-state domestic inflation rates are consistent with the inflation target defined by the Central Bank of 3%. Relative prices are normalized to 1 in the steady state, with exception of the relative price of imports in foreign currency, which is determined endogenously in the steady state.

$$\pi_{ss} = (1.03)^{\frac{1}{4}} - 1 \qquad \text{Inflation target} \qquad (B.15)$$

$$\pi = \pi_{ss} \qquad \text{Headline inflation} \qquad (B.16)$$

$$\pi^{nfr} = \pi_{ss} \qquad \text{Core inflation} \qquad (B.17)$$

$$\pi^{q} = \pi_{ss} \qquad \text{Gross producer inflation} \qquad (B.18)$$

$$\pi^{foo} = \pi_{ss} \qquad \text{Food inflation} \qquad (B.19)$$

$$\pi^{reg} = \pi_{ss} \qquad \text{Regulated inflation} \qquad (B.20)$$

$$\pi^{im} = \pi_{ss} \qquad \text{Imported inflation in domestic prices} \qquad (B.21)$$

$$\pi^{\star} = \pi_{ss} \qquad \text{Foreign inflation} \qquad (B.22)$$

$$p^{q} = 1 \qquad \text{Price level of domestic production Normalization} \qquad (B.23)$$

$$p^{i} = 1 \qquad \text{Price level of investment. Normalization} \qquad (B.24)$$

$$p^{m} = 1 \qquad \text{Price of materials. Normalization} \qquad (B.25)$$

$$p^{im} = 1 \qquad \text{Price of imports in domestic currency. Normalization} \qquad (B.26)$$

$$\pi^{nfr,z} = (1 + \pi^{nfr}) * (1 + g^{z}) - 1 \qquad \text{Inflation + growth} \qquad (B.27)$$

$$\pi^{z} = (1 + \pi) * (1 + g^{z}) - 1 \qquad \text{Headline inflation + growth} \qquad (B.28)$$

B.1.4 Investment and capital

$$i = \frac{i}{y} * y \qquad \text{Investment/GDP} \qquad (B.29)$$

$$k = \frac{k}{y^{Ann}} * y^{Ann} \qquad \text{Stock of capital (supply)} \qquad (B.30)$$

$$u^{k} = 1 \qquad \text{Capital utilization} \qquad (B.31)$$

$$k^{s} = u^{k} * k \qquad \text{Capital demand} \qquad (B.32)$$

$$\delta(u^{k}) = 1 - \left(1 - \frac{i}{k}\right) * (1 + \bar{n}) * (1 + g^{z}) \qquad \text{Capital accumulation law} \qquad (B.33)$$

$$r^{k} = p^{i} \left[\frac{1}{sdf} - \frac{1 - \delta(u^{k})}{1 + g^{n,z}}\right] * \frac{(1 + g^{n,z})}{u^{k}} \qquad \text{Capital (First orden condition)} \qquad (B.34)$$

$$\delta_{1} = \frac{r^{k}}{p^{i}} \qquad \text{Rent of capital} \qquad (B.35)$$

$$\overline{\delta} = \delta(u^{k}) \qquad \text{Endogenous depreciation} \qquad (B.36)$$

$$m = \frac{m}{y} * y \qquad \text{Materials/GDP} \qquad (B.37)$$

B.1.5 Nominal interest rates

$$i^{nom,\star} = \frac{1}{sdf} * (1 + g^{nz}) * (1 + \pi^{\star}) - 1 \quad \text{Foreign nominal interest rate} \quad \text{(B.38)}$$

$$i^{nom} = \frac{1}{sdf} * (1 + g^{nz}) * (1 + \pi^{nfr}) - 1 \quad \text{Monetary policy rate} \quad \text{(B.39)}$$

$$\zeta^{i,prem} = \zeta^{i,prem}_{ss} \quad \text{Risk premium shock} \quad \text{(B.40)}$$

$$i^{prem} = \zeta^{i,prem} \quad \text{Risk premium} \quad \text{(B.41)}$$

$$i^{nom,FED} = \frac{1 + i^{nom,\star}}{1 + i^{prem}} - 1 \quad \text{FED funds rate} \quad \text{(B.42)}$$

B.1.6 Trade balance, current account and foreign debt

$$y^{Ann} = y * g^{nz,Ann} \quad \text{Annual real GDP SS}$$

$$ca = \frac{ca}{y} y \quad \text{Current account SS} \quad \text{(B.43)}$$

$$z = 1 \quad \text{Real exchange rate} \quad \text{(B.44)}$$

$$b^{\star} = \frac{ca}{\left(z \left(\frac{1}{(1+\pi^{f})(1+g^{nz})} - 1 \right) \right)} \quad \text{Foreign debt Current account} \quad \text{(B.45)}$$

$$\frac{b^{\star}}{y^{Ann}} = z \frac{b^{\star}}{y^{Ann}} \quad \text{Annual Foreign debt}$$

$$tb = \frac{tb}{y} * gdp \quad \text{Trade balance SS} \quad \text{(B.46)}$$

$$rem = \left(ca - tb + z * i^{nom\star} * \frac{b^{\star}}{(1 + g^{n,z}) * (1 + \pi^{\star})} \right) \frac{1}{z} \quad \text{Remittances SS} \quad \text{(B.47)}$$

$$\Delta z = 0 \quad \text{Real depreciation} \quad \text{(B.48)}$$

$$\Delta s = \Delta z - \pi^{\star} + \pi \quad \text{Nominal depreciation} \quad \text{(B.49)}$$

B.1.7 Model equations

$$\gamma_c = 1 - \frac{c^{IM}}{c} * \left(p^{im} \right)^{\omega_c} \quad \text{1-home bias imported consumption} \quad \text{(B.50)}$$

$$\gamma_i = 1 - \frac{i^{IM}}{i} * \left(\frac{p^{im}}{p^{i}} \right)^{\omega_i} \quad \text{1-home bias imported investment} \quad \text{(B.51)}$$

$$\gamma_m = 1 - \frac{m^{IM}}{m} * \left(\frac{p^{im}}{p^{m}} \right)^{\omega_m} \quad \text{1-home bias imported materials} \quad \text{(B.52)}$$

$$c = gdp - p_i * i - tb \quad \text{Consumption} \quad \text{(B.53)}$$

$$c^{im} = \frac{c^{IM}}{c} * c \quad \text{Consumption: Imported} \quad \text{(B.54)}$$

$$c^{d} = \gamma^{c} * (p^{q})^{-\omega_c} * c \quad \text{Consumption: Domestic} \quad \text{(B.55)}$$

$$\lambda = \frac{1}{\zeta^d} * \left(c * \left(1 - \frac{\overline{h}}{(1+g^z)}\right)\right)^{-\sigma} * \left(1 - \tilde{\beta} * \left(\frac{\overline{h}}{(1+g^z)}\right)\right) \quad \text{Lagrange multiplier} \tag{B.56}$$

$$i^{im} = \frac{i^{IM}}{i} * i \quad \text{Investment: imported} \tag{B.57}$$

$$i^d = \gamma^i * \left(\frac{p^q}{p^i}\right)^{-\omega^i} * i \quad \text{Investment: Domestic} \tag{B.58}$$

$$m^{im} = \frac{m^{IM}}{m} * m \quad \text{Raw materials: imported} \tag{B.59}$$

$$m^d = \gamma^m * \left(\frac{p^q}{p^m}\right)^{-\omega^m} * m \quad \text{Raw materials: domestic} \tag{B.60}$$

$$im = c^{im} + i^{im} + m^{im} \quad \text{Total imports in units} \tag{B.61}$$

$$imp = \frac{imp}{y} * gdp \quad \text{Total imports as share of GDP} \tag{B.62}$$

$$p^{im,\star} = \frac{imp}{z * im} \quad \text{Price of imports in foreign currency} \tag{B.63}$$

$$p^{im,\star}_{ss} = p^{im,\star} \quad \text{Price of imports in for. curr. (SS)} \tag{B.64}$$

$$\varepsilon^{im} = \frac{p^{im}}{p^{im} - p^{im,\star} * z} \quad \text{Elas. subst. import varieties} \tag{B.65}$$

$$\varepsilon^{im}_{ss} = \varepsilon^{im} \quad \text{Elasticity of substitution import varieties (SS)} \tag{B.66}$$

$$\Pi^{im} = \frac{z * p^{im,\star_f} * im}{\varepsilon^{im} - 1} \quad \text{Profits of imported firms} \tag{B.67}$$

$$ex = tb + z * p^{im,\star} * im \quad \text{Exports} \tag{B.68}$$

$$y^\star = \left(ex \left(\frac{p^q}{zp^{ex,\star}}\right)^{\mu_x}\right)^{1/\mu_y} \quad \text{For. demand for non-oil exports} \tag{B.69}$$

$$y^\star_{ss} = y^\star \quad \text{For. demand for non-oil exports (SS)} \tag{B.70}$$

$$q = \frac{1}{p^q}\left(gdp - (prof^{im} - p^m * m)\right) \quad \text{Gross Output} \tag{B.71}$$

$$mc^q = \frac{\varepsilon^q - 1}{\varepsilon^q} p^q \quad \text{Marginal cost of gross output} \tag{B.72}$$

$$\Pi^q = (p^q - mc^q)q \quad \text{Profits of domestic producers} \tag{B.73}$$

$$\Pi = \Pi^q + \Pi^{im} \quad \text{Aggregate profits} \tag{B.74}$$

$$\mu = \frac{p^m * m}{mc^q * q} \quad \text{Share of raw materials in domestic production} \tag{B.75}$$

$$\alpha = \frac{r^k \zeta^q k^s}{(1 + g^{n,z}) * mc^q * q} \quad \text{Share of capital in domestic production} \tag{B.76}$$

$$\psi^q = \frac{q}{\left(\frac{k^s}{(1+g^{nz})}\right)^{\alpha} * m^{\mu} * H^{(1-\mu-\alpha)}} \quad \text{Final Goods Production Function Parameter SS} \tag{B.77}$$

$$w = \frac{(1 - \alpha - \mu) * q * mc^q}{H} \quad \text{Real wage} \tag{B.78}$$

$$\psi^h = \frac{\varepsilon^w}{\varepsilon^w - 1} \frac{w\lambda}{H^{\eta}} \quad \text{Disutility of labor} \tag{B.79}$$

$$\Upsilon^w = 0 \quad \text{Price adjustment cost wages} \tag{B.80}$$

$$\Upsilon^q = 0 \quad \text{Price adjustment cost domestic varieties} \tag{B.81}$$

$$\Upsilon^{im} = 0 \quad \text{Price adjustment cost imports} \tag{B.82}$$

$$\Upsilon = \Upsilon^w + \Upsilon^q + \Upsilon^{im} \quad \text{Total price adjustment cost} \tag{B.83}$$

$$\tau = z * rem \quad \text{Transfers} \tag{B.84}$$

$$\tilde{y} = 0 \quad \text{Monetary policy: Output gap reacts} \tag{B.85}$$

$$\tilde{\pi} = 0 \quad \text{Monetary policy: Inflation gap reacts} \tag{B.86}$$

$$y = gdp \quad \text{GDP} \tag{B.87}$$

$$y_{ss} = y \quad \text{GDP SS} \tag{B.88}$$

B.2 Model variables

Table B.1: Endogenous

Variable	LaTeX	Description
`g_z`	g^z	Growth rate of technological progress
`g_nz`	g^{nz}	Growth rate of technological progress and polulation
`ddelta`	$\delta(u^k)$	Endogenous depreciation
`d_z`	Δz	Real depreciation
`z`	z	Real exchange rate
`d_s`	Δs	Nominal depreciation
`pi_f`	$\pi^{\star}$	Foreign inflation
`tau`	τ	Transfers
`rem`	rem	Remittances
`i`	i	Investment
`bbeta_tilde`	$\tilde{\beta}$	Beta tilde (SDF)

Table B.1 – Continued

Variable	LaTeX	Description
sdf	sdf	Stochastic Descount Factor
lambda	λ	Marginal utility of consumption
lambda_i	λ^i	Shadow price of investment
c	c	Consumption
c_d	c^d	Domestic consumption
c_im	c^{im}	Imported consumption
r_k	r^K	Rent of capital
z_d	ζ^d	Demand shock
z_i	ζ^i	Investment demand shock
vareps_w	ε^w	Elasticity of substitution for wage shock
z_u	ζ^u	Preferences shock
Upsilon_w	Υ^w	Wage adjustment cost
w	ϖ	Wage
H	H	Hours
pi_z	π^z	Head. inflation + growth
p_q	p^q	Realtive price production to NFR
p_im	p^{im}	Realtive price production to NFR
q	q	Production
Z_c	Z^c	Efficiency consumption shock
z_i_prem	$\zeta^{i^{prem}}$	Premium risk shock
i_prem	i^{prem}	Premiun risk
Z_w	Z^w	Wage efficiency
pi	π	Headline inflation
u_k	u^k	Capital depreciation
k	k	Capital
b_f	$b^\star$	Foreign debt
b_f_y_Ann	$b^{\star,Ann}$	Annual foreign debt
p_i	p^i	Investment price
pi_nfr	π^{nfr}	Non food and regulated inflation
i_nom	i^{nom}	Nominal interest rate
i_nom_n	$i^{nom,n}$	Neutral Nominal interest rate
i_nom_f	$i^{nom,\star}$	Foreign nominal interest rate
i_real	i^{real}	Domestic real interest rate
i_real_f	$i^{real,\star}$	Foreign real interest rate
prof	Π	Total profits
prof_q	Π^q	Domestic producers profits
prof_im	Π^{im}	Importers profits
Z_i	Z^i	Investment efficiency
i_d	i^d	Domestic investment
i_im	i^{im}	Imported investment
pi_q	π^q	Gross producer inflation
m	m	Raw materials
Z_m	Z^m	Raw materials efficiency

Table B.1 – Continued

Variable	LaTeX	Description
m_d	m^d	Domestic raw materials
m_im	m^{im}	Imported raw materials
p_m	p^m	Raw materials price
ex_r	ex^r	Real exports
p_ex_f	$p^{ex,\star}$	Relative price of exports in foreign currency
Upsilon	Υ	Total adjustment cost
Upsilon_q	Υ^q	Gross prod. adjustment cost
Upsilon_im	Υ^{im}	Imported adjustment cost
mc_q	mc_q	Gross producer marg. cost
k_s	k^s	Capital demand
Z_q	Z^q	TFP
vareps_q	ε^q	Gross prod. elast. subs. shock
im	im	Imports
vareps_im	ε^{im}	Imported elast. subs. shock
y_f	$y^\star$	Foreign demand
ex	ex	Exports
Z_ex	Z^{ex}	Exports demand shock
y_tilde	$\tilde{y}$	Output gap for mon. pol.
y	y	GDP
y_Ann	y^{Ann}	Annual GDP
pi_tilde	$\tilde{\pi}$	Inflation gap for mon. pol.
tb	tb	Trade balance
ca	ca	Current account
pi_im	π^{im}	Imported inflation
pi_foo	π^{foo}	Food inflation
pi_reg	π^{reg}	Regulated inflation
i_nom_fed	$i^{nom,FED}$	FED funds rate
p_im_f	$p^{im,\star}$	Imported price index
GDP_Ann	GDP^{Ann}	Annual GDP
pi_nfr_tar	$\overline{\pi}^{Tar}$	Time-varying inflation target
Z_pi_nfr	$Z\pi^{nfr}$	Food and Reg. Inf. Indexated to Core
Z_pi_foo	$Z\pi^{foo}$	Food Inf. Indexated
Z_pi_reg	$Z\pi^{foo}$	Regulated Inf. Indexated
Z_p_im	$Z\pi^{foo}$	Imported Price Wedge
D_y_f_obs	$\Delta y^{obs,\star}$	Observed: Foreign Growth
y_f_obs	π^{obs}	Observed: Foreign Demand
pi_f_obs	π^{obs}	Observed: Headline Inflation
i_nom_fed_obs	$i^{nom,FED}_{obs}$	Observed: FED funds rate
D_i_nom_fed_obs	$\Delta i^{nom,FED}_{obs}$	Observed: FED funds growth rate
i_prem_obs	$i^{nom,FED}_{obs}$	Observed: Risk Premium
pi_im_f_obs	$\pi^{im,\star}_{obs}$	Observed: Imported inflation in for. curr.
p_im_f_obs	$p^{im,\star}_{obs}$	Observed: Imported prices in for. curr.
p_ex_f_obs	$p^{ex,\star}_{obs}$	Observed: Imported prices in for. curr.

Table B.1 – Continued

Variable	LaTeX	Description
pi_obs	π_{obs}	Observed: Headline infaltion
pi_nfr_obs	π^{nfr}_{obs}	Observed: Core inflation
pi_foo_obs	π^{foo}_{obs}	Observed: Food inflation
pi_reg_obs	π^{reg}_{obs}	Observed: Regulated inflation
i_nom_obs	i^{nom}_{obs}	Observed: Nominal Interest Rate
pi_nfr_tar_obs	$\overline{\pi}_{obs}$	Observed: Dynamic Inflation Target
D_y_bar_obs	$\Delta\bar{y}_{obs}$	Observed: Real GDP trend growth
D_y_obs	Δy_{obs}	Observed: Real GDP growth rate (QoQ)
D_c_obs	Δc_{obs}	Observed: Real consumption growth rate (QoQ)
D_i_obs	Δi_{obs}	Observed: Real investment growth rate (QoQ)
D_im_obs	Δim_{obs}	Observed: Real imports growth rate (QoQ)
D_ex_r_obs	Δex_{obs}	Observed: Real exports growth rate (QoQ)
D_ex_obs	Δex_{obs}	Observed: Total exports (QoQ)
BD_c_obs	BD_c_obs	Observed: Discrepancy
BD_i_obs	BD_i_obs	Observed: Discrepancy
BD_ex_r_obs	$BD\_ex\_r\_obs$	Observed: Discrepancy
BD_im_obs	BD_im_obs	Observed: Discrepancy

Table B.2: Exogenous

Variable	LaTeX	Description
eps_g_z	ϵ^{g^z}	Techno. progress shock
eps_w	ϵ^{w}	Wage mark-up shock
eps_im	ϵ^{im}	Imported firms mark-up shock
eps_q	ϵ^{q}	Domestic producer mark-up shock
eps_z_u	$\epsilon^{z,u}$	Preferences shock
eps_z_d	$\epsilon^{z,d}$	Aggregate demand shock
eps_Z_w	$\epsilon^{Z,w}$	Wage efficiency shock
eps_Z_c	ϵ^{Z_c}	Consumption efficiency shock
eps_Z_q	ϵ^{Z_q}	Domestic producer efficiency shock
eps_Z_i	ϵ^{Z_i}	Investment efficiency shock
eps_i_nom_fed	$\epsilon^{i^{nom,\star}}$	Federal funds rate shock
eps_z_i_prem	$\epsilon^{\zeta^{i,prem}}$	Premium risk shock
eps_y_f	$\epsilon^{y^{\star}}$	Foreign demand shock
eps_i_nom	$\epsilon^{i^{nom}}$	Policy rate shock
eps_p_im_f	$\epsilon^{p^{im,\star}}$	Imported prices in foreign currency shock
eps_rem	$\epsilon^{\pi^{rem}}$	Remittances shock
eps_pi_foo	$\epsilon^{\pi^{foo}}$	Food inflation shock
eps_pi_reg	$\epsilon^{\pi^{reg}}$	Regulated inflation shock
eps_pi_f	$\epsilon^{\pi^{\star}}$	Foreign inflation shock
eps_Z_m	ϵ^{Z^m}	Efficiency Raw Materials Shock

Table B.2 – Continued

Variable	LaTeX	Description
eps_Z_ex	$\epsilon^{Z^{ex}}$	Exports demand shock
eps_z_i	$\epsilon^{\zeta^{i}}$	Investment demand shock
eps_p_ex_f	$\epsilon^{p^{ex,\star}}$	Relative price of exports in for. curr. shock
eps_pi_nfr_tar	$\epsilon^{\overline{\pi}^{Tar}}$	Time-varying inflation target shock
eps_Z_p_im	$\epsilon^{Z^{p,im}}$	Imported Price Wedge Shock
eps_i_prem_obs	$\epsilon^{i^{prem,obs}}$	Observed: Risk Premium Shock
eps_BD_c_obs	$eps\_BD\_c\_obs$	eps_BD_c_obs
eps_BD_i_obs	$eps\_BD\_i\_obs$	eps_BD_i_obs
eps_BD_ex_r_obs	$eps\_BD\_ex\_r\_obs$	eps_BD_ex_r_obs
eps_BD_im_obs	$eps\_BD\_im\_obs$	eps_BD_im_obs

B.3 Model Parameters

Steady state values for calibrated parameters. Note that the model simulation uses the estimated values for the estimated parameters (Table 3.4) and not the ones stored here.

Table B.3: Parameter Values

Parameter	Value	Description
$\bar{n}$	0.003	Population quarterly growth rate
g^{z}	0.004	Technological progress (SS)
β	0.995	Discount factor
σ	1.010	Intertemporal elasticity of substitution (Inverse)
$\bar{h}$	0.100	Intensity of habits formation
κ	31.900	Capital adjustment cost
$\bar{\delta}_0$	0.018	Depreciation (SS)
δ_1	0.027	Depreciation for capital utilization
δ_2	0.002	Quadratic depreciation for capital utilization
ψ_h	163.993	Labor disutility
η	3.000	Inverse Frisch elasticity
ω_c	2.000	Elas. subs. of consumption H and F varieties
γ_c	0.961	1-home bias consumption
ω_i	2.000	Elas. subs. of investment H and F varieties
γ_i	0.753	1-home bias investment
ω_m	2.000	Elas. subs. of materials H and F varieties
γ_m	0.820	1-home bias imported
ϕ^{w}	0.100	Adjustment costs of wage goods
ϕ_q	0.100	Adjustment costs of domestic goods
ϕ_{im}	0.010	Adjustment costs of imported goods
ψ_q	2.177	Parameters that guarantees that GDP equals to 1 in steady state

Table B.3 – Continued

Parameter	Value	Description
ε^{q}_{ss}	5.000	Elasticity of substitution (SS)
ε^{w}_{ss}	5.000	Elasticity of substitution (SS)
ε^{im}_{ss}	6.256	Elasticity of substitution (SS)
$\phi^{i,nom}$	0.800	Policy rate persistence
ϕ_{π}	1.500	Effect of inflation on the policy rate
ϕ_{y}	0.000	Effect of real activity on policy rate
π_{ss}	0.007	Inflation target
$\overline{\pi}^{tar}_{ss}$	0.007	Time-varying inflation target
i^{nom}_{ss}	0.016	Nominal interest rate (SS)
$i^{nom,FED}_{ss}$	0.006	FED funds rate (SS)
$\zeta^{i,prem}_{ss}$	0.008	Risk premium (SS)
π^{f}_{ss}	0.005	Foreign inflation (SS)
π^{foo}_{ss}	0.007	Food inflation (SS)
π^{foo}_{ss}	0.007	Core inflation (SS)
π^{reg}_{ss}	0.007	Regulated inflation (SS)
p^{im}_{ss}	1.000	Imported inflation (SS)
$p^{im,\star}_{ss}$	0.840	Imported inflation/foreign curr (SS)
$p^{ex,\star}_{ss}$	1.000	Rel. Price of exports in foreign curr (SS)
$\phi_{\pi^{nfr}}$	0.680	Share of core in hedline inflation
$\phi_{\pi^{foo}}$	0.150	Share of food in hedline inflation
$g^{\star}$	0.005	Trend growth rate of foreign demand
μ^{x}	0.567	Exports elasticity of relative price
μ^{y}	10.800	Exports elasticity to foreign demand
$y^{\star}_{ss}$	0.839	Foreign gross domestic product (SS)
y_{ss}	1.000	Gross domestic product (SS)
ψ^{iprem}	0.016	Elasticity of debt to the risk premium
$b^{\star,Ann}_{ss}$	0.597	Foreign debt over Annual GDP (SS)
$\pi^{im,\star}_{ss}$	0.007	Imported inflation for. curr (SS)
α	0.166	Share of capital in production
μ	0.547	Share of raw materials in production
rem_{ss}	0.039	Remittances (SS)
$\rho_{g^{z}}$	0.780	Persistence of technological growth
$\rho_{g^{u}}$	0.750	Persistence of preferences shock
$\rho_{z^{u}}$	0.750	Persistence of preferences shock
$\rho_{z^{i}}$	0.750	Persistence of marginal eff. of investment
$\rho_{z^{d}}$	0.700	Persistence of demand shock
ρ_{rem}	0.750	Persistence of remittances shock
$\rho_{\pi^{foo}}$	0.400	Persistence of Food inflaiton shock
$\rho_{\pi^{foo}}$	0.600	Persistence of Regulated inflation shock
$\rho_{\pi^{nfr}}$	0.500	Persistence of core inflation shock
$\rho_{Z^{c}}$	0.750	Persistence of efficiency of consumption shock
$\rho_{Z^{w}}$	0.750	Persistence of wage efficiency shock

Table B.3 – Continued

Parameter	Value	Description
ρ_{ζ^w}	0.750	Persistence of wage demand shock
$\rho_{\pi^\star}$	0.100	Persistence of foreign inflation shock
$\rho_{z^{i,prem}}$	0.797	Persistence of risk premium shock
ρ_{Z^i}	0.750	Persistence of investment efficiency shock
ρ_{Z^q}	0.750	Persistence of TFP shock
ρ_{z^q}	0.750	Persistence
ρ_{Z^m}	0.750	Persistence of materials efficiency shock
ρ_{y^f}	0.983	Persistence of foreign demand shock
$\rho_{Z^{ex}}$	0.700	Persistence of Exports demand shock
ρ_{ε^q}	0.750	Persistence of domestic elasticity of substitution shock
$\rho_{p^{ex,\star}}$	0.953	Persistence of relative price of exports in for. curr.
$\rho_{\varepsilon^{im}}$	0.750	Persistence of imported elasticity of subsitution shock
$\rho_{i^{nom,FED}}$	0.982	Persistence of Fed funds rate shock
ρ_{ε^w}	0.750	Persistence
$\rho_{p^{im,\star}}$	0.800	Persistence of rel. price of imports shock
$\rho_{\overline{\pi}^{Tar}}$	0.000	Persistence of time-varying inflation target shock
$Z_{ss}^{\pi^{foo}}$	1.000	Food inflation residual SS
$Z_{ss}^{\pi^{reg}}$	1.000	Regulated inflation residual SS
$Z_{ss}^{\pi^{nfr}}$	1.000	NFR inflation residual SS
$\rho_{Z\pi^{foo}}$	0.100	Food inflation residual persistence
$\rho_{Z\pi^{reg}}$	0.100	Regulated inflation residual persistence
$\rho_{Z\pi^{foo}}$	0.100	NFR inflation residual persistence
$\rho_{Z^{im}}$	0.750	Imported inflation efficiency
ϕ_d^c	0.050	Food and regulated indexation to core inf.
ϕ_d^c	0.100	Adjustment cost domestic consumption
ϕ_{im}^c	0.100	Adjustment cost imported consumption
ϕ_d^i	0.100	Adjustment cost domestic investment
ϕ_{im}^i	0.100	Adjustment cost imported investment
ϕ_d^m	0.010	Adjustment cost domestic materials
ϕ_{im}^m	0.010	Adjustment cost imported materials
$\rho_{B\Delta c}$	0.000	Consumption Discrepancy Persistence
$B\Delta\bar{c}_{ss}^{obs}$	0.000	Consumption Growth Discrepancy Steady State
$\rho_{B\Delta i}$	0.000	Investment Discrepancy Persistence
$B\Delta\bar{i}_{ss}^{obs}$	0.008	Investment Growth Discrepancy Steady State
$\rho_{B\Delta ex}$	0.000	Exports Discrepancy Persistence
$B\Delta\bar{ex}_{ss}^{obs}$	-0.001	Exports Growth Discrepancy Steady State
$\rho_{B\Delta im}$	0.000	Imports Discrepancy Persistence
$B\Delta\bar{im}_{ss}^{obs}$	0.008	Imports Growth Discrepancy Steady State